

Homo moralis goes to the voting booth:
a new theory of voter turnout

Ingela Alger and Jean-François Laslier



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Abstract

Why do voters incur costs to participate in large elections? This paper proposes an exploratory analysis of the implications of evolutionary Kantian morality for this classical problem in the economic theory of voting: the costly participation problem.

Keywords: voter turnout, voting, ethical voter, *homo moralis*, Kantian morality

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1 Introduction

While large democratic elections are a means for voters to influence the outcome, the rational theory literature (Downs 1957) has shown that purely consequentialist motives cannot explain the turnout rates observed in large political elections. This “paradox of voting” is due to the low probability for each voter of being pivotal (Ledyard 1981; Palfrey and Rosenthal 1985; Myerson 2000; Dhillon and Peralta 2002; Feddersen 2004). Several alternative theoretical approaches (Waas and Blais 2017) can be imagined, based on the empirical evidence that the decision to vote is correlated with an impressive set of variables (Smets and van Ham 2013). In particular, both social (Gerber et al. 2008; Rogers et al. 2017) and ethical (Blais 2000; Blais and Achen 2019) motives appear important. The ethical motive for voting was first modelled as satisfaction derived from complying with a norm (Riker and Ordeshook 1968; Fiorina 1976; Schuessler 2000).¹ Based on Harsanyi (1977, 1980), ethical voters were then modelled as civic-minded citizens who are *rule utilitarians*: they select a voting strategy that maximizes the total welfare of like-minded voters, which would obtain if all like-minded ethical voters selected it. Each ethical voter thus acts like a social planner for the group, generating positive equilibrium turnout rates that trade off the probability of winning against the group-aggregate cost of voting (Coate and Conlin 2002; Feddersen and Sandroni 2006; Feddersen et al 2009; Krishna and Morgan 2015; Bierbrauer et al. 2019).

We propose a novel formalization of ethical voters, which imposes a less demanding ethical standard than rule utilitarianism by taking voters to have a mix of purely selfish and moral concerns. Moreover, the voters in our model are equipped with preferences—dubbed *Homo moralis*—shown to be plausible from an evolutionary perspective (Alger and Weibull 2013).² In a two-player game, a *Homo moralis* using strategy x when the opponent uses strategy y achieves utility

$$U(x, y) = (1 - \kappa) \cdot \pi(x, y) + \kappa \cdot \pi(x, x),$$

where $\pi(x, y)$ is the individual’s material payoff, given the strategies effectively used, and $\pi(x, x)$ is the material payoff that the individual would obtain if, hypothetically, the opponent were instead to use the same strategy as him/her. The second term captures a Kantian moral concern, and $\kappa \in [0, 1]$ is the individual’s *degree of morality*. Immanuel Kant (1785) wrote “Act only according to that maxim whereby you can at the same time will that it should

¹A voter’s satisfaction may also stem from helping sustain democracy or expressing political views.

²Our approach is thus close in spirit to Conley et al. 2006.

become a universal law.” By maximizing the weighted sum of own material payoff and the Kantian moral concern, *Homo moralis* can be said to “act according to that maxim whereby you can at the same time will that others should do likewise with some probability.” Alger and Weibull (2013) show that evolution favors *Homo moralis* preferences with κ equal to the probability with which individuals carrying rare mutant preferences get to interact. We take this result as our starting point and examine voters’ decisions to participate in an election, assuming that they have *Homo moralis* preferences.

We model an election with two candidates. Prior to voting, each voter learns his own party preference but not the aggregate party preference distribution. Each voter chooses a costly level of effort, which determines the probability of managing to vote on time. In the *ex ante* setting (resp. *ex post* setting), voters select their voting strategies prior to (resp. after) learning their party preference. We model the electorate as a continuum, thus eliminating instrumental motives for voting. When contemplating a course of action, a *Homo moralis* evaluates what his material payoff would be if, hypothetically, a share κ of the population to which he belongs would follow the same course of action.³ Our findings are as follows.

In the symmetric setting—where the candidates enjoy equally large expected support—for any strictly positive κ there exists a unique equilibrium, with a strictly positive turnout rate (with one notable exception: the *ex ante* setting when the electorate is evenly divided in each state of Nature, not only in expected terms). With a convex effort cost function and a negligible marginal cost for small effort levels, voters can make a sizeable effort even if they attach a weight of only 0.1 or 0.2 to the Kantian moral concern.

Somewhat surprisingly, however, a positive degree of morality is not sufficient to generate a positive turnout from the supporters of both parties: in asymmetric settings where one party is highly likely to be supported by a strong majority, voters who support the other party do not vote.

Whether in the symmetric *ex ante* or *ex post* setting, the equilibrium turnout rate is increasing in the degree of morality: by bolstering the hypothetical effect of his effort envisaged by a *Homo moralis*, an increase in κ raises the utility benefit of exerting costly effort.

Finally, the equilibrium turnout is lower in the *ex ante* than in the *ex post* setting. Behind the veil of ignorance as to his eventual party preference, a *Homo moralis* internalizes the effect that his voting effort has on both of his incarnations. This consideration is absent in the *ex post* scenario, where *Homo moralis* preferences make voters adopt a partisan standpoint.

³We refer to our companion paper (Alger and Laslier, 2020) for an explanation for why the utility of *Homo moralis* in a continuum population takes this form.

2 The model

Consider an infinitely large population of voters, modelled as a continuum with mass 1. There are two political parties, A and B , and two states of Nature: in state ω_A a majority $q_A \geq 1/2$ prefers A over B ; in state ω_B a majority $q_B \geq 1/2$ prefers B over A . State ω_A occurs with probability $a \in (0, 1)$ and state ω_B with probability $b = 1 - a$. Prior to each election, Nature first draws the state and then a party preference $P(i) \in \{A, B\}$ for each voter i . Each voter observes his realized preference but not the state of Nature.

Each voter cares about the margin with which his preferred party wins or loses. This assumption is natural for parliamentary elections, where margins determine the number of seats obtained. But it may also be reasonable in winner-take-all elections, for example because such margins have effects on the ability of candidates to raise funds. Formally, let $v(\delta)$ denote the *material utility* that a voter obtains from margin $\delta \in [-1, 1]$, where v is strictly increasing. Material utility encompasses both the utility associated with the concrete effects of the election outcome on laws, taxes, and government spending, and psychological benefits or costs derived from the relative power of the two parties. We take the cost of seeing one's preferred party lose by a margin δ to equal the benefit of seeing one's preferred party win by the same margin δ :

$$v(\delta) = -v(-\delta) \quad \text{for all } \delta \in [-1, 1]. \quad (1)$$

Letting v be twice continuously differentiable and concave for $\delta > 0$, v is then convex for $\delta < 0$, and

$$v'(\delta) = v'(-\delta) \quad \text{and} \quad v''(\delta) = -v''(-\delta). \quad (2)$$

An example of such a function is

$$v(\delta) = \delta(2 + \delta)(2 - \delta), \quad (3)$$

for which $v'(\delta) = 4 - 3\delta^2$ and $v''(\delta) = -6\delta$.

A voter who participates in the election votes for his preferred party. The probability that a voter reaches the voting booth on time and manages to vote is determined by his *effort*, a one-dimensional aggregate measure of time devoted to registering, efforts to overcome "obstacles" that may appear on the way to the voting booth (family and/or professional obligations, illness, inclement weather, etc), and attention devoted to avoiding behaviors that would annul the vote. Letting the probability of voting success be linear in effort, we let effort be a number between 0 and 1. The cost of exerting effort $e \in [0, 1]$ is $c(e)$, where c

is a continuously differentiable and strictly convex function, with $c'(0) = 0$ and $c'(1) = +\infty$.

Denote by $e_{P(i)}(i) \in [0, 1]$ the effort of a voter i with party preference $P(i) \in \{A, B\}$. Let the mass of voters be described by the interval $[0, 1]$ with the Lebesgue measure. Because the allocation of preference to voters is random, the expected number of voters of type A who vote is:

$$\int_0^1 \mathbf{1}_{P(i)=A} e_A(i) di.$$

Since, by assumption, the probability of being of a given type depends only on the state of nature, we will slightly abuse notation and write the above integral as follows in state ω_A :

$$\int_0^{q_A} e_A(i) di.$$

With this notation in hand, party A 's expected margin in state ω_A is:

$$\delta_A = \int_0^{q_A} e_A(i) di - \int_{q_A}^1 e_B(i) di, \quad (4)$$

and party B 's expected margin in state ω_B is:

$$\delta_B = \int_0^{q_B} e_B(i) di - \int_{q_B}^1 e_A(i) di. \quad (5)$$

We will characterize type-homogenous equilibria, in which all the voters with the same party preference exert the same effort. If e_A and e_B denote these efforts, the expected margins in state A and B take the following simple forms:

$$\delta_A = q_A e_A - (1 - q_A) e_B \quad (6)$$

and

$$\delta_B = q_B e_B - (1 - q_B) e_A. \quad (7)$$

We distinguish between the *ex ante* and the *ex post* scenario, depending on whether each voter chooses a voting strategy before or after learning his party preference.

3 The *ex ante* scenario

In this scenario each voter i chooses a pair of efforts $(e_A(i), e_B(i)) \in [0, 1]^2$ before learning his party preference $P(i) \in \{A, B\}$. Our goal is to characterize symmetric equilibria, where

all voters choose the same effort pair. If all the other voters use some effort pair (e_A, e_B) , the *expected material utility* of a voter i who selects the effort pair $(e_A(i), e_B(i))$ is:

$$\begin{aligned} \pi(e_A(i), e_B(i), e_A, e_B) &= aq_A v(\delta_A) + b(1 - q_B)v(-\delta_B) - [aq_A + b(1 - q_B)]c(e_A(i)) \\ &\quad + a(1 - q_A)v(-\delta_A) + bq_B v(\delta_B) - [a(1 - q_A) + bq_B]c(e_B(i)). \end{aligned} \quad (8)$$

The first line is the expected material utility that the voter achieves if he turns out preferring party A , in which case he exerts effort $e_A(i)$ and enjoys material benefit $v(\delta_A)$ in state ω_A and $v(-\delta_B)$ in state B . Since each voter has measure zero, the expected margins are indeed independent of i 's efforts $(e_A(i), e_B(i))$. The second line is his expected material utility if he turns out preferring party B .

Prior to defining *Homo moralis* preferences, we state the following proposition (whose trivial proof is omitted).

Proposition 1 *Voters with Homo oeconomicus preferences, represented by the expected material utility in (8), exert no voting effort.*

Each voter being atomistic, his effort has no effect on the election outcome. Since voting is costly, purely instrumentally motivated voters do not participate.⁴

A voter with *Homo Moralis* preferences and *degree of morality* $\kappa \in [0, 1]$ achieves the following utility from the effort pair $(e_A(i), e_B(i))$ when the other voters use the effort pair (e_A, e_B) :

$$\begin{aligned} U^{(\kappa)}(e_A(i), e_B(i), e_A, e_B) &= aq_A v(\delta_A^{(\kappa)}) + b(1 - q_B)v(-\delta_B^{(\kappa)}) - [aq_A + b(1 - q_B)]c(e_A(i)) \\ &\quad + a(1 - q_A)v(-\delta_A^{(\kappa)}) + bq_B v(\delta_B^{(\kappa)}) - [a(1 - q_A) + bq_B]c(e_B(i)), \end{aligned} \quad (9)$$

where

$$\delta_A^{(\kappa)} = \delta_A + \kappa [q_A(e_A(i) - e_A) - (1 - q_A)(e_B(i) - e_B)] \quad (10)$$

and

$$\delta_B^{(\kappa)} = \delta_B + \kappa [q_B(e_B(i) - e_B) - (1 - q_B)(e_A(i) - e_A)]. \quad (11)$$

The first line in (9) is the voter's expected material payoff if he turns out preferring party A , evaluated at the expected margins $\delta_A^{(\kappa)}$ and $-\delta_B^{(\kappa)}$ that would obtain if, hypothetically, a share κ of the other voters selected $(e_A(i), e_B(i))$ instead of (e_A, e_B) . For example, if $e_A(i) > e_A$,

⁴For the same reason, any consequentialistic utility function—even if it represents altruistic inclination towards the other voters rather than pure material self-interest—would generate zero effort.

the expected margin of party A would thus (hypothetically) increase by $\kappa q_A [e_A(i) - e_A]$ in state of nature ω_A , while that of party B would be reduced by $\kappa (1 - q_B) [e_A(i) - e_A]$ in state of nature ω_B . Because i 's effort cost does not depend on the voting strategies of other voters, however, it is the true expected cost that appears in the utility in (9). The special case $\kappa = 0$ yields the familiar *Homo oeconomicus* preferences, examined above. The remainder of this section restricts attention to strictly positive degrees of morality $\kappa \in (0, 1]$.

Using the symmetry of v (see (1)) we rewrite (9) as follows:

$$U^{(\kappa)}(e_A(i), e_B(i), e_A, e_B) = a(2q_A - 1)v(\delta_A^{(\kappa)}) + b(2q_B - 1)v(\delta_B^{(\kappa)}) - [aq_A + b(1 - q_B)]c(e_A(i)) - [a(1 - q_A) + bq_B]c(e_B(i)). \quad (12)$$

Hence, the marginal utilities of efforts $e_A(i)$ and $e_B(i)$ are, respectively:

$$\frac{\partial U^{(\kappa)}(e_A(i), e_B(i), e_A, e_B)}{\partial e_A(i)} = \kappa \left[aq_A(2q_A - 1)v'(\delta_A^{(\kappa)}) - b(1 - q_B)(2q_B - 1)v'(\delta_B^{(\kappa)}) \right] - [aq_A + b(1 - q_B)]c'(e_A(i)) \quad (13)$$

$$\frac{\partial U^{(\kappa)}(e_A(i), e_B(i), e_A, e_B)}{\partial e_B(i)} = \kappa \left[bq_B(2q_B - 1)v'(\delta_B^{(\kappa)}) - a(1 - q_A)(2q_A - 1)v'(\delta_A^{(\kappa)}) \right] - [bq_B + a(1 - q_A)]c'(e_B(i)). \quad (14)$$

These equations immediately reveal that the expected marginal benefit (the first line in each equation) is nil if the voters are evenly split between the two parties in both states ($q_A = q_B = 1/2$); equilibrium efforts are then nil. The reason is clear: given the symmetry on v , if $q_A = q_B = 1/2$ the benefit a voter garners from making effort when incarnated as an A -supporter would be exactly offset by the reduction in benefit this effort generates for the voter incarnated as a B -supporter, and *vice versa*. Costly effort is therefore not warranted. Does this also mean that effort is nil in any setting where the voter is equally likely to end up as an A -supporter or a B -supporter (i.e., when $a(2q_A - 1) = (1 - a)(2q_B - 1)$)? No: the following proposition shows that, except in the special case where the voters are always evenly split ($q_A = q_B = 1/2$), a voter with *Homo moralis* preferences with positive degree of morality $\kappa > 0$ exerts a strictly positive effort, for at least one of the party preference realizations.

Proposition 2 *For any degree of morality $\kappa \in (0, 1]$, the zero-effort strategy profile $(e_A^*, e_B^*) = (0, 0)$ is a Nash equilibrium if and only if $q_A = q_B = 1/2$.*

This result obtains because the marginal cost is nil at zero effort and even the smallest strictly positive value of κ makes the voter evaluate the consequences of voting effort as if it had an

impact on the outcome of the vote. Intuitive though as this result may seem, it is surprising that some strictly positive effort is undertaken even in the setting where the voter is equally likely to end up as an A -supporter or a B -supporter, i.e., when $a(2q_A - 1) = (1 - a)(2q_B - 1)$. Indeed, as mentioned above, from an *ex ante* perspective any voting effort for one particular party then stands an equal chance of benefiting and harming the voter. Given that voting is costly, one could have expected that voters would then have preferred to refrain from voting. We will next deepen the analysis of the symmetric setting to better understand this result. Prior to this, however, we use a numerical example to show that a positive κ does not necessarily induce a positive effort for both party preference realizations.

Example 1 *Let $v(\delta) = \delta(2 + \delta)(2 - \delta)$ and $c(e) = we/(1 - e)$. With these functional forms, the algebraic computations can be handled by Mathematica. Take $w = 2/10$, $q_A = 8/10$, and $q_B = 7/10$. In the *ex ante* scenario:*

1. *For $\kappa = 1/10$ and $a = 1/2$, the only equilibrium is interior: $(e_A^*, e_B^*) \simeq (.219, .142)$.*
2. *If κ increases, turnout increases and for $\kappa = 1$ and $a = 1/2$ it reaches $(e_A^*, e_B^*) \simeq (.613, .535)$.*
3. *If the asymmetry increases sufficiently, turnout for one of the parties vanishes. For $\kappa = 1/10$ and $a = 3/4$, $(e_A^*, e_B^*) \simeq (.282, 0)$.*

We now analyze further the symmetric setting, in which both states of the world are equally likely and the share of voters who support party A in state ω_A equals the share of voters who support party B in state ω_B , i.e., $a = 1/2$ and $q_A = q_B \equiv q$. Given the previous proposition, we restrict attention to the non-trivial case where a strict majority of voters prefer party P in state ω_P , i.e., $q \in (1/2, 1]$. In this setting each voter votes with a strictly positive probability, which does not depend on the voter's party preference.

Proposition 3 *Consider a symmetric setting ($a = 1/2$, $q_A = q_B \equiv q$) in which $q > 1/2$. For any degree of morality $\kappa \in (0, 1]$, there exists a unique symmetric Nash equilibrium (e_A^*, e_B^*) . This equilibrium entails a positive effort that is independent of party preference, i.e., $e_A^* = e_B^* \equiv e^* > 0$.*

To see why this result obtains, consider the equation that implicitly defines e^* (see (37)):

$$\kappa(2q - 1)^2 v'((2q - 1)e^*) = c'(e^*). \quad (15)$$

A positive voting effort e^* makes the voters achieve a positive margin

$$\delta^* = (2q - 1)e^* \tag{16}$$

for the majority at hand, a margin that generates an expected material benefit $[q - (1 - q)]v(\delta^*)$ to the voter (with probability q he is in the majority, and thus enjoys a positive margin of victory, but with the complementary probability he is in the minority and suffers from a loss). A positive degree of morality $\kappa > 0$ provides the voter with the satisfaction associated with the marginal effect $\kappa(2q - 1)$ that his level of effort would have on this margin, should a share κ of the other voters make the same effort. Hence, a costly effort is warranted.

The following proposition reports comparative statics results for the symmetric setting.

Proposition 4 *Consider a symmetric setting ($a = 1/2$, $q_A = q_B \equiv q$). If $v''' \leq 0$:*

- *for any $q \in (1/2, 1]$, e^* is strictly increasing in κ ;*
- *for any $\kappa \in (0, 1]$ there exists a threshold value $\tilde{q} \in (1/2, 1]$ such that e^* is strictly increasing in q for $q \leq \tilde{q}$ and strictly decreasing in q for $q > \tilde{q}$.*

Ceteris paribus an increase in the degree of Kantian morality induces a higher participation rate, because an increase in κ bolsters the margin that would obtain if, hypothetically, a share κ of the other voters exerted the same effort as the voter at hand, and thus bolsters the satisfaction that this voter derives from exerting effort. By contrast, since the marginal cost of voting effort is increasing while its marginal material benefit is decreasing, an increase in the degree of partisanship q would not necessarily raise participation, for a given degree of morality κ .

4 The *ex post* scenario

In this scenario each voter i chooses an effort $e_{P(i)}(i) \in [0, 1]$ upon learning her party preference $P(i) \in \{A, B\}$. Our goal is to derive results for *type-homogenous* equilibria (\hat{e}_A, \hat{e}_B) , where all the voters with a preference for party A (resp. B) exert the same effort \hat{e}_A (resp. \hat{e}_B). By contrast to the *ex ante* analysis, here a voter updates her beliefs about the state of nature upon observing her party preference, and the material utility is evaluated given the realized party preference. Thus, given that all the other A -supporters exert effort

e_A and all the B -supporters exert effort e_B , the expected material utility of an A -supporter i who exerts effort $e_A(i)$ is:

$$\hat{\pi}(e_A(i), e_A, e_B) = \hat{a} \cdot v(\delta_A) + (1 - \hat{a}) \cdot v(-\delta_B) - c(e_A(i)), \quad (17)$$

where \hat{a} is i 's posterior belief that the state is ω_A given that she prefers party A :

$$\hat{a} = \Pr[\omega_A | P(i) = A] = \frac{a \cdot q_A}{a \cdot q_A + b \cdot (1 - q_B)}. \quad (18)$$

Likewise, the expected material utility of a B -supporter who exerts effort $e_B(i)$, given the efforts e_A and e_B exerted by the other voters, is:

$$\hat{\pi}(e_B(i), e_A, e_B) = \hat{b} \cdot v(\delta_B) + (1 - \hat{b}) \cdot v(-\delta_A) - c(e_B(i)), \quad (19)$$

where \hat{b} is her posterior belief that the state is favorable for party B , i.e., ω_B :

$$\hat{b} = \Pr[\omega_B | P(i) = B] = \frac{b \cdot q_B}{b \cdot q_B + a \cdot (1 - q_A)}. \quad (20)$$

Before defining *Homo moralis* preferences, we observe that, as in the *ex ante* scenario, voters with *Homo economicus* preferences exert no voting effort (again the trivial proof is omitted).

Proposition 5 *Voters with Homo oeconomicus preferences, represented by the expected material utilities in (17) and (19), exert no voting effort.*

Turning now to *Homo Moralis* preferences, in the *ex post* scenario a voter with such preferences and a degree of morality $\kappa \in (0, 1]$ evaluates what her expected material utility would be, should a share κ of the voters *with the same party preference* choose the same effort (instead of a share of all the voters as in the *ex ante* scenario). Formally, for any type-homogenous effort profile $(e_A, e_B) \in [0, 1]^2$ used by the other voters, an A -supporter achieves the following utility from exerting effort $e_A(i)$:

$$U_A^{(\kappa)}(e_A(i), e_A, e_B) = \hat{a} \cdot v(\delta_A^{(A, \kappa)}) + (1 - \hat{a}) \cdot v(-\delta_B^{(A, \kappa)}) - c(e_A(i)) \quad (21)$$

where

$$\delta_A^{(A, \kappa)} = \delta_A + \kappa q_A [e_A(i) - e_A], \quad (22)$$

$$-\delta_B^{(A, \kappa)} = -\delta_B + \kappa(1 - q_B)[e_A(i) - e_A]. \quad (23)$$

If a share κ of the other A -supporters were to choose the same effort as the individual

herself instead of their actual effort e_A , the expected margin of party A would change by $\kappa q_A [e_A(i) - e_A]$ in state of nature ω_A (see (22)), while the expected margin of party B would change by $\kappa(1 - q_B) [e_A(i) - e_A]$ in state of nature ω_B (see (23)). As in the *ex ante* scenario, the utility cost of effort in (21) is unaffected by κ .

Likewise, a B -supporter achieves the following utility from exerting effort $e_B(i)$:

$$U_B^{(\kappa)}(e_B(i), e_A, e_B) = \hat{b} \cdot v(\delta_B^{(B,\kappa)}) + (1 - \hat{b}) \cdot v(-\delta_A^{(B,\kappa)}) - c(e_B(i)) \quad (24)$$

where

$$\delta_B^{(B,\kappa)} = \delta_B + \kappa q_B [e_B(i) - e_B] \quad (25)$$

and

$$-\delta_A^{(B,\kappa)} = -\delta_A + \kappa(1 - q_A) [e_B(i) - e_B]. \quad (26)$$

A type-homogenous equilibrium (\hat{e}_A, \hat{e}_B) solves the following fixed-point problem:

$$\begin{cases} \hat{e}_A \in \arg \max_{e_A(i) \in [0,1]} & U_A^{(\kappa)}(e_A(i), \hat{e}_A, \hat{e}_B) \\ \hat{e}_B \in \arg \max_{e_B(i) \in [0,1]} & U_B^{(\kappa)}(e_B(i), \hat{e}_A, \hat{e}_B). \end{cases} \quad (27)$$

We first prove that in any such equilibrium (if it exists), all voters exert a positive effort. To see this, consider first the marginal utility of effort of a voter i with party preference $P(i) = A$:

$$\kappa \left[q_A \hat{a} v'(\delta_A^{(A,\kappa)}) + (1 - q_B) (1 - \hat{a}) v'(-\delta_B^{(A,\kappa)}) \right] - c'(e_A(i)). \quad (28)$$

For any type-homogenous efforts e_A and e_B exerted by the other voters, the assumptions $v'(\cdot) > 0$ and $c'(0) = 0$ imply that when evaluated at $e_A(i) = 0$, this expression is strictly positive for any $\kappa \in (0, 1]$. Since the same remark applies to the marginal utility of a voter i with party preference $P(i) = B$, we have proved the following result.

Proposition 6 *For any degree of morality $\kappa \in (0, 1]$, any type-homogenous equilibrium (\hat{e}_A, \hat{e}_B) is such that all the voters exert a positive effort: $\hat{e}_A > 0$ and $\hat{e}_B > 0$.*

This result contrasts with the *ex ante* scenario, in which a positive degree of morality is not sufficient to generate a strictly positive effort for all parameter constellations.

Example 2 *For the same parameters as in Example 1, in the ex post scenario:*

1. For $\kappa = 1/10$ and $a = 1/2$, $(\hat{e}_A, \hat{e}_B) \simeq (.339, .320)$.
2. For $\kappa = 1$ and $a = 1/2$, $(\hat{e}_A, \hat{e}_B) \simeq (.718, .711)$.

3. For $\kappa = 1/10$ and $a = 3/4$, $(\hat{e}_A, \hat{e}_B) \simeq (.359, .280)$.

Equilibrium existence is more challenging to prove than in the *ex ante* scenario, and we had to add some conditions to do so; these additional conditions are, however, not necessary to prove that if an equilibrium exists it is unique, as shown in the following proposition.

Proposition 7 *Consider a symmetric setting ($a = 1/2$, $q_A = q_B \equiv q \geq 1/2$). For any $\kappa \in (0, 1]$ there exists a unique candidate for a symmetric type-homogenous equilibrium $(\hat{e}_A, \hat{e}_B) = (\hat{e}, \hat{e})$. For (\hat{e}, \hat{e}) to be an equilibrium it is sufficient that $v'''(\delta) \leq 0$ for $\delta \in [0, 1]$ and $c'''(e) \geq 0$ for all $e \in [0, 1]$.*

Prior to comparing the equilibrium effort in the *ex post* scenario to that in the *ex ante* scenario, we show that the comparative statics are qualitatively the same as in the *ex ante* scenario.

Proposition 8 *Consider a symmetric setting ($a = 1/2$, $q_A = q_B \equiv q$). If $v''' \leq 0$:*

- for any $q \in (1/2, 1]$, \hat{e} is strictly increasing in κ ;
- for any $\kappa \in (0, 1]$ there exists a threshold value $\hat{q} \in (1/2, 1]$ such that \hat{e} is strictly increasing in q for $q \leq \hat{q}$ and strictly decreasing in q for $q > \hat{q}$.

Propositions 6 and 7 imply that if an equilibrium exists in the symmetric case, it is implicitly defined by this equation:

$$\kappa [q^2 + (1 - q)^2] v'((2q - 1)\hat{e}) = c'(\hat{e}). \quad (29)$$

As in the *ex ante* scenario, a positive voting effort \hat{e} makes the independent voters achieve a positive margin

$$\hat{\delta} = (2q - 1)\hat{e} \quad (30)$$

for the majority at hand. However, the marginal benefit of this margin is not the same as in the *ex ante* setting. For the sake of illustration consider a voter who prefers party A . The term $[q^2 + (1 - q)^2]$ in (29) captures both the benefit of increasing the margin of victory when A is the majority party and the benefit of decreasing the victory margin when B is the majority party. Indeed, in the symmetric setting, the posterior belief of this voter that the state is ω_A is $\hat{a} = q$. Hence, $\kappa q^2 v'(\hat{\delta})$ is the marginal expected utility benefit from increasing the victory margin in state ω_A (in which an effort increase raises the hypothetical margin that *Homo moralis* ponders by κq). Similarly, $\kappa(1 - q)^2 v'(\hat{\delta})$ is the marginal expected utility

benefit for the A -supporter at hand from decreasing the victory margin of party B in state ω_B (in which an effort increase reduces the hypothetical margin envisaged by *Homo moralis* by $\kappa(1 - q)$).

Intuition suggests that voters exert more effort in the *ex post* than in the *ex ante* scenario, since in the latter the voters internalize the fact that they may end up preferring either party, whereas in the former preferences are purely partisan. A glance at the equations that implicitly define e^* (see (15)) and \hat{e} (see (29)) reveals that the comparison hinges on the terms $q^2 + (1 - q)^2$ and $(2q - 1)^2$. Since $q^2 + (1 - q)^2 > (2q - 1)^2$ for any $q \in [1/2, 1]$, we immediately obtain the following result, which confirms the intuition.

Proposition 9 *Consider a symmetric setting ($a = 1/2$, $q_A = q_B \geq 1/2$). For any degree of morality $\kappa \in (0, 1]$, the equilibrium effort \hat{e} in the *ex post* scenario strictly exceeds the equilibrium effort e^* in the *ex ante* scenario.*

5 Conclusion

Our exploratory analysis shows that the evolutionarily plausible class of *Homo moralis* preferences can lead to high turnout rates in large elections. Moreover, equilibrium turnout rates depend on the specifics of the election in hand. Since these preferences may be interpreted as capturing a partially Kantian motivation, the model is arguably plausible also in light of the ethical motive for voting often put forward by voters in surveys (Blais 2000).

6 Appendix

6.1 Proof of Proposition 2

Consider a voter who expects the other voters to exert no effort, $(e_A, e_B) = 0$. This implies that when evaluated at $e_A(i) = e_B(i) = 0$, $\delta_A^{(\kappa)} = 0 = \delta_B^{(\kappa)}$, so that $v'(\delta_A^{(\kappa)}) = v'(\delta_B^{(\kappa)})$. Simplification of (13) and (14) then lead to the following necessary and sufficient conditions for $(e_A^*, e_B^*) = (0, 0)$ to be a Nash equilibrium:

$$aq_A(2q_A - 1) - b(1 - q_B)(2q_B - 1) \leq 0 \tag{31}$$

and

$$bq_B(2q_B - 1) - a(1 - q_A)(2q_A - 1) \leq 0. \tag{32}$$

The statement in the proposition then follows from three observations. First, $q_A q_B > (1 - q_A)(1 - q_B)$ implies that at least one of the conditions is violated (to see this, note that $a q_A(2q_A - 1) - b(1 - q_B)(2q_B - 1) \leq 0$ then implies $b q_B(2q_B - 1) - a(1 - q_A)(2q_A - 1) > 0$, while $b q_B(2q_B - 1) - a(1 - q_A)(2q_A - 1) \leq 0$ then implies $a q_A(2q_A - 1) - b(1 - q_B)(2q_B - 1) > 0$). Second, since $q_A \geq 1/2$ and $q_B \geq 1/2$, $q_A q_B \leq (1 - q_A)(1 - q_B)$ if and only if $q_A = q_B = 1/2$. Third, the two necessary and sufficient conditions are satisfied for $q_A = q_B = 1/2$. **Q.E.D.**

6.2 Proof of Proposition 3

Since $q > 1/2$, Proposition 2 implies that either $e_A^* > 0$ or $e_B^* > 0$. Assume that there exists an equilibrium such that $e_A^* > e_B^* \geq 0$. At such an equilibrium, the necessary first-order condition for e_A^* is obtained by equating the expression in (13), evaluated at $e_A(i) = e_A^*$ and $e_B(i) = e_B^*$, to zero. Since the term multiplying κ in $\delta_A^{(\kappa)}$ and in $\delta_B^{(\kappa)}$ (see (10) and (11)) then equals zero, the first-order condition reduces to

$$\kappa(2q - 1) [qv'(\delta_A^*) - (1 - q)v'(\delta_B^*)] = c'(e_A^*), \quad (33)$$

where

$$\delta_A^* = [qe_A^* - (1 - q)e_B^*] \quad (34)$$

$$\delta_B^* = [qe_B^* - (1 - q)e_A^*]. \quad (35)$$

Likewise, $e_B^* \in [0, e_A^*)$ must satisfy the inequality

$$\kappa(2q - 1) [qv'(\delta_B^*) - (1 - q)v'(\delta_A^*)] \leq c'(e_B^*). \quad (36)$$

Now, $e_A^* > e_B^*$ implies $|\delta_A^*| > |\delta_B^*|$. The assumptions on v (see (2)) then imply $v'(\delta_B^*) > v'(\delta_A^*)$, which in turn implies $qv'(\delta_B^*) - (1 - q)v'(\delta_A^*) > qv'(\delta_A^*) - (1 - q)v'(\delta_B^*)$. Hence, the left-hand side of (36) strictly exceeds that of (33). But together with strict convexity of c , this contradicts $e_A^* > e_B^*$. Having thus proved that at any equilibrium $e_A^* = e_B^* \equiv e^* > 0$, any such e^* satisfies the necessary first-order condition

$$\kappa(2q - 1)^2 v'((2q - 1)e^*) = c'(e^*). \quad (37)$$

Existence and uniqueness follow from our assumptions on v and c . **Q.E.D.**

6.3 Proof of Proposition 4

By some abuse of notation, let e^* denote the function that to each pair $(\kappa, q) \in (0, 1] \times (1/2, 1]$ associates the value e^* implicitly defined by (15). By the implicit function theorem, in a neighborhood of e^* :

$$\frac{\partial e^*(\kappa, q)}{\partial \kappa} = \frac{(2q-1)^2 v'(\delta^*)}{c'(e^*) - \kappa^2(2q-1)^3 v''(\delta^*)}, \quad (38)$$

which is strictly positive,

$$\begin{aligned} \frac{\partial e^*(\kappa, q)}{\partial q} &= \frac{4\kappa(2q-1)v'(\delta^*) + 2\kappa^2(2q-1)^2 e^* v''(\delta^*)}{c'(e^*) - \kappa^2(2q-1)^3 v''(\delta^*)} \\ &= \frac{2\kappa(2q-1)}{c'(e^*) - \kappa^2(2q-1)^3 v''(\delta^*)} \cdot [2v'(\delta^*) + (2q-1)e^* v''(\delta^*)], \end{aligned} \quad (39)$$

and

$$\begin{aligned} \frac{\partial e^*(\kappa, q)}{\partial} &= \frac{\kappa(2q-1)^2 v'(\delta^*) + \kappa^2(2q-1)^3 e^* v''(\delta^*)}{c'(e^*) - \kappa^2(2q-1)^3 v''(\delta^*)} \\ &= \frac{\kappa(2q-1)^2}{c'(e^*) - \kappa^2(2q-1)^3 v''(\delta^*)} \cdot [v'(\delta^*) + (2q-1)e^* v''(\delta^*)]. \end{aligned} \quad (40)$$

The denominator being strictly positive,

$$\text{sign} \left(\frac{\partial e^*(\kappa, q)}{\partial q} \right) = \text{sign} [2v'(\delta^*) + (2q-1)e^* v''(\delta^*)]. \quad (41)$$

Since (41) is strictly positive for $q = 1/2$, by continuity it is also strictly positive for q close to $1/2$. For e^* to be increasing in q , the expression in (41) must be positive for $q \in [1/2, 1]$. Since δ^* is increasing in e^* and v is strictly concave, the first term would then decrease in q , while the absolute value of the second term would increase in q . Hence, there may exist some $\tilde{q} \in (1/2, 1]$ such that the sum of the two terms is nil for $q = \tilde{q}$. We show, by contradiction, that e^* must then be decreasing in q for $q > \tilde{q}$. Thus, consider some $\hat{q} > \tilde{q}$ and let \hat{e}^* denote the associated equilibrium effort. Suppose that there exists some $\varepsilon > 0$ such that $e^* \geq \hat{e}^*$ for all $q \in [\hat{q}, \hat{q} + \varepsilon]$. But then (41) would be strictly negative for $q \in (\hat{q}, \hat{q} + \varepsilon]$, contradicting the assumption that $e^* \geq \hat{e}^*$ for all $q \in [\hat{q}, \hat{q} + \varepsilon]$. **Q.E.D.**

6.4 Proof of Proposition 7

In the symmetric setting, $\hat{a} = \hat{b} = q$. Consider a voter i with party preference $P(i)$, and assume that all the other voters make effort $e^* \in (0, 1)$. Then $\delta_A = \delta_B = (2q - 1)e^* \equiv \delta$, and i 's expected utility as a function of $e_{P(i)}(i)$ is:

$$U_{P(i)}^{(\kappa)}(e_{P(i)}(i), e^*, e^*) = q \cdot v(\delta + \kappa q [e_{P(i)}(i) - e^*]) + (1 - q) \cdot v(-\delta + \kappa(1 - q) [e_{P(i)}(i) - e^*]) - c(e_{P(i)}(i)). \quad (42)$$

e^* is a symmetric equilibrium iff $e_{P(i)}(i) = e^*$ maximizes the utility in (42). Since

$$\frac{\partial U_{P(i)}^{(\kappa)}(e_{P(i)}(i), e^*, e^*)}{\partial e_{P(i)}(i)} = \kappa q^2 \cdot v'(\delta + \kappa q [e_{P(i)}(i) - e^*]) + \kappa(1 - q)^2 \cdot v'(-\delta + \kappa(1 - q) [e_{P(i)}(i) - e^*]) - c'(e_{P(i)}(i)), \quad (43)$$

replacing $e_{P(i)}(i)$ by e^* in (43) and equating the resulting expression to zero gives the necessary first-order condition for e^* to be a symmetric interior equilibrium. By symmetry of v (see (2)), this condition reduces to:

$$\kappa[q^2 + (1 - q)^2] \cdot v'((2q - 1)e^*) = c'(e^*). \quad (44)$$

The right-hand side is a strictly increasing function that equals 0 for $e^* = 0$ and tends to $+\infty$ as e^* tends to 1. For any $\kappa > 0$ the left-hand side is strictly positive for $e^* = 0$, and it is strictly decreasing since its derivative wrt e^* ,

$$\kappa^2(2q - 1)[q^2 + (1 - q)^2] \cdot v''((2q - 1)e^*),$$

is strictly negative. We conclude that (44) has a unique interior solution $e^* \in (0, 1)$. To prove the second statement in the proposition, we turn to the second partial derivative:

$$\frac{\partial^2 U_{P(i)}^{(\kappa)}(e_{P(i)}(i), e^*, e^*)}{\partial [e_{P(i)}(i)]^2} = (\kappa)^2 q^3 \cdot v''(\delta + \kappa q [e_{P(i)}(i) - e^*]) + (\kappa)^2 (1 - q)^3 \cdot v''(-\delta + \kappa(1 - q) [e_{P(i)}(i) - e^*]) - c''(e_{P(i)}(i)). \quad (45)$$

When evaluated at $e_{P(i)}(i) = e^*$ this equals

$$(\kappa)^2 [q^3 - (1 - q)^3] \cdot v''((2q - 1)e^*) - c''(e^*).$$

This is strictly negative; to see this, note that $q^3 > (1 - q)^3$ and that v'' is strictly negative since the argument $(2q - 1)e^*$ is strictly positive, and recall that $c'' > 0$). Hence, we can first conclude that $e_{P(i)}(i) = e^*$ is a local maximum. However, for e^* to be an equilibrium it must also be a global maximum. This is what we show next.

e^* is a global maximum if the second derivative in (45) is negative for all $e_{P(i)}(i) \in [0, 1]$. Since $c'' > 0$ and $(\kappa)^2 > 0$, a sufficient condition for this to be true is:

$$q^3 \cdot v''(\delta^* + \kappa q [e_{P(i)}(i) - e^*]) + (1 - q)^3 \cdot v''(-\delta^* + \kappa(1 - q) [e_{P(i)}(i) - e^*]) < 0,$$

or

$$q^3 \cdot v''(x) + (1 - q)^3 \cdot v''(-x - \kappa [e^* - e_{P(i)}(i)]) < 0 \quad (46)$$

where

$$x = (2q - 1)e^* + \kappa q [e_{P(i)}(i) - e^*]. \quad (47)$$

A priori, condition (46) may fail to hold. To see this, let $\kappa = 1$ and $e_{P(i)}(i) = 0$; then $x < 0$, and it follows that $v''(x) > 0$ and $v''(-x - \kappa [e^* - e_{P(i)}(i)]) = v''(-qe^*) > 0$. Although the second partial derivative in (45) may thus be positive for some $e_{P(i)}(i) \in [0, 1]$, we will now show that it changes sign at most once. This implies that for any given e^* , U is quasi-concave in $e_{P(i)}(i)$ for $e_{P(i)}(i) \in [0, 1]$, a property which ensures that the local maximum $e_{P(i)}(i) = e^*$ is also a global maximum in $[0, 1]$.

For U to be quasi-concave, it suffices that the second partial derivative in (45) be decreasing in $e_{P(i)}(i)$. If $c'''(e) \geq 0$ for all $e \in [0, 1]$ (one of the conditions stated in the proposition), it suffices that the left-hand side of (46) is decreasing in $e_{P(i)}(i)$ for this to be true. If $v'''(e) \geq 0$ (the other condition stated in the proposition), then the left-hand side of (46) is decreasing in $e_{P(i)}(i)$ if the arguments of v'' (that is, x and $-x - \kappa[e^* - e_{P(i)}(i)]$) are increasing in $e_{P(i)}(i)$. This is clearly true, as revealed by inspection of the expression for x in (47) and the fact that $-x - \kappa[e^* - e_{P(i)}(i)] = -(2q - 1)e^* + \kappa(1 - q)[e_{P(i)}(i) - e^*]$. **Q.E.D.**

6.5 Proof of Proposition 8

By some abuse of notation, let \hat{e} denote the function that to each pair $(\kappa, q) \in (0, 1] \times [1/2, 1]$ associates the value \hat{e} implicitly defined by (29). By the implicit function theorem, in a neighborhood of \hat{e} :

$$\frac{\partial \hat{e}(\kappa, q)}{\partial \kappa} = \frac{[q^2 + (1 - q)^2] v'(\hat{\delta})}{c'(\hat{e}) - \kappa^2(2q - 1) [q^2 + (1 - q)^2] v''(\hat{\delta})}, \quad (48)$$

which is strictly positive, and

$$\begin{aligned} \frac{\partial \hat{e}(\kappa, q)}{\partial q} &= \frac{\kappa [2q - 2(1 - q)] v'(\hat{\delta}) + 2\kappa^2(2q - 1) [q^2 + (1 - q)^2] \hat{e}v''(\hat{\delta})}{c'(\hat{e}) - \kappa(2q - 1) [q^2 + (1 - q)^2] v''(\hat{\delta})} \\ &= \frac{2\kappa(2q - 1)}{c'(\hat{e}) - \kappa^2(2q - 1)^3 v''(\hat{\delta})} \cdot \left[v'(\hat{\delta}) + [q^2 + (1 - q)^2] \hat{e}v''(\hat{\delta}) \right]. \end{aligned} \quad (49)$$

The denominator being strictly positive, for any $q > 1/2$, we have

$$\text{sign} \left(\frac{\partial \hat{e}(\kappa, q)}{\partial q} \right) = \text{sign} \left[v'(\hat{\delta}) + [q^2 + (1 - q)^2] \hat{e}v''(\hat{\delta}) \right]. \quad (50)$$

Application of the same logic as the one used in the proof of Proposition 4 leads to the stated result. **Q.E.D.**

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