Monogamy: exception or rule?[*]

Ingela Alger[†]

23rd April 2020

Abstract

Why are some societies monogamous and others polygynous? Most theories of polygyny invoke male heterogeneity as an explanation. Arguing that such heterogeneity depends on men's willingness to compete against each other in the first place, I propose an evolutionary game to model the evolution of this trait. Lack of competitiveness (and the associated monogamous unions) is shown to be compatible with evolution if male reproductive success decreases with the number of wives. In a model where the man and his spouse(s) make fertility and child care choices that aim at maximizing reproductive success, I show that, due to men's involvement in child care and female agency over her fertility, male reproductive success is decreasing in the number of wives under certain conditions and increasing in others. The model thus sheds light on the variation in polygyny rates across space and time in human societies.

Keywords: economics of the family, monogamy, polygyny, reproductive success, evolution

JEL codes: D13, C73

---

[*] I acknowledge funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 789111 - ERC EvolvingEconomics), as well as IAST funding from the French National Research Agency (ANR) under grant ANR-17-EURE-0010 (Investissements d'Avenir program). I am grateful to Donald Cox, Hillard Kaplan, and Michael Wilson for extensive comments and discussions. I also thank Alice Baniel, Alexis Chaine, Hanna Kokko, François Salanié, Jonathan Stieglitz, and seminar participants at IAST and Freie Universität Berlin for comments.

[†] Toulouse School of Economics, CNRS, University of Toulouse Capitole, and Institute for Advanced Study in Toulouse, France. ingela.alger@tse-fr.eu

1 Introduction

The oldest known laws, including the Codes of Ur-Nammu and Hammurabi, show that humans have devised laws governing marriage for at least 4000 years. Marital patterns may affect economic outcomes through a host of channels, including demographic effects, intra-family transfers, intergenerational transmission of material wealth and human capital, female labor market participation, gender differences in well-being, and more.[1] It is therefore not surprising that economists have produced an important body of theoretical research to understand the forces that may have driven the historical evolution of marital patterns.[2] This paper contributes to this literature.

Laws governing marriage have varied across space and time. In broad terms, the imposition of monogamous marriage has gained in prevalence over time, and polyandry as a marriage form has been virtually non-existent. Rules imposing monogamous marriage on all men appeared already in ancient Greece and Rome (Scheidel, 2008): "Marriage in Rome was monogamous; *mating* was polygynous" (Betzig, 1992). Moreover, although polygynous marriage is still legal in about one fourth of all countries (Zeitzen, 2008) and only 190 of the 1231 traditional societies for which Murdock's ethnographic atlas provides data on marriage systems were classified as non-polygynous (Murdock, 1949, Gray, 1998, Murdock et al., 2000), anthropologists have reached the conclusion that even in societies classified as polygynous only a small number of men have more than one wife at a time: "the modal percentage of marriages that are polygynous among foragers is about 3 percent" (Kaplan and Lancaster, 2003); "ethnographic evidence indicates that most individuals within a society live in monogamous marriages that are generally, but not always, sexually exclusive [and] unions are commonly serially monogamous" (Schacht and Kramer, 2019).[3] This begs the question of whether laws imposing monogamous marriage in fact constrain behavior in a significant manner. I address this question, by analyzing a theoretical model of the evolution of decentralized household formation.[4]

The goal is to understand the effects on marital patterns of three universal (or near-universal) features of human life: (1) both men and women invest time and resources in offspring; (2) men's ability to acquire or destruct each other's productive resources has changed over the course of his-

---

[1] For empirical and theoretical research on how family structure affects economic outcomes, see, *inter alia*, Becker (1991), Lundberg and Pollak (2007), Tertilt (2005, 2006), Cox and Fafchamps (2008), Edlund and Lagerlöf (2006), Doepke and Tertilt (2009), Edlund and Kopczuk (2009), Edlund and Machado (2015), Grossbard (2015), Chiappori, Salanié, and Weiss (2016), and Bertocchi and Dimico (2019).

[2] See, in particular, Becker (1974, 1991), Grossbard (1976), Bergstrom (1994a,b), Lagerlöf (2005, 2010), Tertilt (2006), Gould, Moav, and Simhon (2008), De La Croix and Mariani (2015), and Francesconi, Ghiglino, and Perry (2016).

[3] Biological indicators further support the hypothesis that humans have tended to engage in long-term pair-bonding with a single partner for much of human history; while this precludes neither serial monogamy nor occasional uncommitted sexual relationships, most known extra-pair paternity rates in humans are low, indicating that long-term relationships tend to be sexually exclusive (see Møller, 2003).

[4] Whether formal laws are driven by the wishes of the rulers (Lagerlöf, 2010) and/or of the ruled (De La Croix and Mariani, 2015), grassroots individual preferences may influence laws because laws that adhere to underlying preferences are less likely to be contested. Using the terminology of Alexander (1987), here I focus on ecologically imposed rather than socially imposed behavioral norms.

tory; and (3) humans live in groups between which there is limited migration. The family being the unit of production of children, evolution by natural selection should have played a major role in shaping the human family. Accordingly, I apply evolutionary logic to build the model. The results lead to a simple point: even absent formal legislation, the combined effect of these three universal features should have favored monogamy rather than polygyny over the course of human history. The model also delivers novel insights for why marital norms vary across space and time.

It is widely recognized that male heterogeneity is a key driver of polygyny, and my model also focuses on this aspect.[5,6] The theoretical argument is that if women can choose whom they marry and if they rank men according to some attribute, women should accept to share high-quality men with other women, while low-quality men should attract at most one woman, or even share a woman with other men (Orians, 1969, Becker, 1974, 1991, Grossbard, 1976, Low, 1990, Bergstrom, 1994, Ross et al., 2018). While other attributes may matter, wealth—and more generally the control of productive resources—is often put forward as a key attribute.[7] However, previous theoretical models all take the heterogeneity in productive resources to be exogenously given, thereby missing the fact that any man's non-innate attributes at the moment he enters the marriage market depend on his behaviors in the years preceding that moment. I fill this gap by proposing a model in which male heterogeneity—and the ensuing polygyny rate—is endogenous, and determined by men's willingness to compete.[8]

The model builds on the idea that a man's desire to accumulate resources such as wealth or human capital during the years preceding marriage ultimately depends on how strongly he cares about achieving greater success in the marriage market. Specifically, a man's eagerness to compete with other men for the purpose of attracting more wives should depend on his preferred number of wives. If all men would prefer to have only one wife, one should expect no or little competition, no or low heterogeneity, and monogamy; by contrast, if all men would prefer to have many wives, one should expect men to compete against each other in some way that generates heterogeneity, and polygynous marriage for the winners of this race. A full understanding of the evolution of marriage systems thus requires an understanding of how evolution by natural selection shapes male *preferences* over

---

[5] In evolutionary biology *Bateman's principle* states that the variability in reproductive success should be larger among males than in females in most species simply because males can "go forth and multiply" descendants while females can only "go forth and add" them (Bateman, 1948).

[6] Female heterogeneity should also matter in this context. In particular, it has been shown that such heterogeneity should favor monogamy. Thus, in the model of Gould, Moav, and Simhon (2008), in societies where reproducing adults care about their children's human capital, the equilibrium "price" of a high-quality woman is high enough to induce monogamy even among rich men. In the anthropology and biology literatures it has been shown that the polygyny rate in a population may depend on the females' willingness to "trade" faithfulness for a lower polygyny rate (Kokko and Morrell, 2005, Fortunato and Archetti, 2010, Gavrilets, 2012). My model disregards this aspect by focusing solely on male heterogeneity.

[7] This is consistent with the fact that most human societies that allow polygyny also have the bride price custom, the bride price being what a man pays to marry a woman; see Gaulin and Boster (1990) and references therein, as well as Boserup (1970) and Betzig (1993).

[8] The question at hand is thus similar in spirit to Chiappori, Dias, and Meghir (2018), who examine a model in which education choices are partly driven by their effects on marriage market outcomes. The difference is that I adopt an evolutionary perspective, thus focusing on the long-run evolution of the desire of men to compete for women.

polygyny rates in the first place, and how these preferences affect the men' willingness to compete against each other.[9]

Following evolutionary logic, I posit that the ultimate goal of individuals is to maximize reproductive success, defined as the expected number of children that survive to sexual maturity.[10] For any given household composition, a man and his spouse(s) thus engage in parental care to this end. This is modeled as a non-cooperative game between the adults in the household. Characterization of equilibrium fertility and parental care choices allows me to characterize the number of wives a man would like to have if he could choose freely, and how this number depends on exogenously given factors of the environment, or the *ecology*, in which the population at hand evolves.[11] I then use this characterization to examine the evolution of the willingness of men to compete against other men. Assuming that male *degree of competitiveness* is a trait transmitted from father to son, I adopt a standard evolutionary game theoretic framework (Weibull, 1995) to determine which degrees of competitiveness are compatible with evolution by natural selection.[12] The strategies in the evolutionary game are the degrees of competitiveness. Men are matched randomly into pairs to play, and each man's strategy in any matched pair determines whether he competes or refrains from competing. This matching of males can be interpreted as village formation.[13] Men may differ in their degrees of competitiveness: in matched pairs with two non-competitive men, each man settles down and forms a monogamous household; by contrast, if at least one of the men in a matched pair is competitive the encounter ends with one winner, who takes over the women of the loser. I characterize the set of evolutionarily stable degrees of competitiveness. Two settings are considered: in one

---

[9] This approach rests on the idea, delineated by Bergstrom (1996) and Robson (2001, 2002), that economists may obtain valuable insights about human motivation by including evolutionary forces in their models. It is closely related to the growing literature on preference evolution (see, e.g., Frank, 1987, Güth and Yaari, 1992, Dekel, Ely, and Yilankaya, 2007, Heifetz, Shannon, and Spiegel, 2007, Rayo and Becker, 2007, Robson and Samuelson, 2011, Alger and Weibull, 2010, 2013, 2019).

[10] While mating success of offspring also matters, it is not modeled here. I also rule out unfaithfulness by assumption. Although this approach is reasonable for human societies where grand-parents closely monitor the behaviors of their sons or daughters in law, future research should allow for extra-pair matings.

[11] The quest for insights as to how ecological factors affect the evolution of mating systems is common in the biology literature (see, e.g., Bateman, 1948, Orians, 1969, Emlen and Oring, 1977, Clutton-Brock and Vincent, 1991, as well as Kokko and Jennions, 2008, and the references therein), as well as in the anthropology literature (see., e.g, Kaplan, Hooper, and Gurven, 2009, Nettle et al., 2013, and Moya, Snopkowski, and Sear, 2016). However, in this literature parental care is typically a one-dimensional variable (while in my model parents provide three goods to their offspring) and focus is often on the effects of spatial and temporal availability of mates (features that are disregarded here).

[12] While the mathematical model is silent as to whether the transmission is biological or cultural, recent evidence on the interacting effects of testosterone and cortisol levels on male willingness to compete (e.g., Knight et al., 2020) suggests that biological factors do matter (but since the mechanisms are not yet fully understood, father-son correlations are still unknown). Moreover, testosterone levels are on average lower in married than in unmarried men, and the lowest levels are found in married men with children (Alvergne, Faurie, and Raymond, 2009); Gettler et al., 2011). Theoretical analysis of transmission routes other than the purely vertical one considered here, which would be possible if transmission is (fully or partly) cultural, is left for future research.

[13] While the baseline model takes to the matching to be uniform random, I also analyze the more realistic case where it is random but assortative.

the only stake of the competition is women (who are all identical), and in the other the stake also includes productive resources.

If a man's reproductive success were always increasing in the number of women, it is unclear how natural selection could be compatible with men refraining from competing. My model provides a novel answer to this puzzle by revealing that in certain ecologies a man's reproductive success is strictly decreasing in the number of women. This result is explained by men's involvement in food production and women's agency over their fertility.[14] Whenever the man provisions food the number of mouths he has to feed is increasing in the number of wives, implying a quantity-quality trade-off with respect to the number of wives.[15] I show that the trade-off can be so severe that the man prefers to have one rather than several wives.[16] This result contrasts starkly with the existing literature, where men are systematically assumed to desire more wives (when their price is nil).

The said quantity-quality trade-off is more severe the more resources the man dedicates to producing food for his offspring. This decision in turn depends on the *ecology* in which the population evolves, which in the model is a vector of parameters that includes the marginal returns to time devoted to food production for men and women, and the relative importance of the food items brought back home by men and women for children's survival. In any given ecology, the male's decision to engage in food production also depends on how many wives he has. I show that if a man engages in food production, this must happen for low polygyny rates: indeed, because food is a fully rival good, a man is better off shifting his efforts away from food production to other (somewhat) non-rival forms of child care as the number of children he has to raise grows. For high enough polygyny rates, the man ceases to engage in food production, and a further increase in the polygyny rate then enhances reproductive success. If the man's productive resources are independent of the polygyny rate, the polygyny quantity-quality trade-off is thus more severe for low than for high polygyny rates. Depending on the ecology, men then either prefer both monogamy and high polygyny rates to intermediate polygyny rates, or always want more wives. While these preferences are typically altered when both women and productive resources are at stake in the competition between men, the ecology still plays a role: an identical gain or loss in productive resources for the winner has different effects in different ecologies, and male reproductive success may still be decreasing or non-monotonic in the polygyny rate.

Whether or not the male-male competition concerns only women or also productive resources,

---

[14] While paternal provisioning is rare among mammals (Clutton-Brock, 1991), it did evolve at some point among our hominin ancestors, and I situate my model in the period of our evolutionary past that follows this ground-breaking development (see Gavrilets, 2012, and Alger et al., 2020, for theories of male provisioning in the hominin lineage). Regarding women's agency over fertility decisions, infanticide by men appears to have been applied mostly to non-related children (van Schaik and Janson, 2000).

[15] The term "wife" is used for convenience only, since marriage has no function *per se* in the model. The key assumption is that both males and females engage in parental care, which is a reasonable assumption for humans and some other species (see Alger and Cox, 2013, for a review of the biology literature on parental care).

[16] Clearly, this result hinges on the assumption that a man cannot impose different fertility rates on his wives, i.e., he cannot use some for reproduction and others for production.

the qualitative nature of male preferences over polygyny rates has deep implications for the set of evolutionarily stable male degrees of competitiveness. In a population where the prevailing strategy consists in being peaceful and where a man's reproductive success is higher with one than with two wives, it would not pay off to mutate towards a higher degree of competitiveness in order to acquire more wives (and possibly also more or less productive resources). Hence, when conditions are such that male reproductive success is higher with one than with two wives, natural selection is compatible with lack of male-male competition. Nonetheless, the highest degree of competitiveness is also evolutionarily stable, in all ecologies: if the prevailing strategy consists in competing as much as possible, it would not pay off to mutate towards a lower degree of competitiveness, as this would entail a loss of reproductive resources with certainty.

The result that the highest degree of competitiveness is evolutionarily stable under all conditions does, however, not survive once the model is extended to allow for the third of the three universal features of human societies on which my analysis focuses. Our ancestors lived in populations structured into groups of small size, which extended beyond the nuclear family and between which there was limited migration.[17] It is well known that a key implication of such population structure is that mutants, even when rare, face a higher probability of interacting with mutants than residents do, even absent any ability of individuals to choose the individuals with whom they interact. In the biology literature, this effect is usually quantified by the *coefficient of relatedness* (Wright, 1931), which measures the probability that interacting individuals share a common ancestor.[18] I show that when the coefficient of relatedness is high enough, the highest degree of competitiveness fails to be evolutionarily stable. The reason is that polygyny is always inefficient in the sense that overall average reproductive success is lower under polygyny than under monogamy. If the prevailing strategy consists in competing as much as possible, relatedness implies that rare mutants who compete less can reap the benefits of the reduced competition with a non-negligible probability, and thus outperform the more competitive men.

When brought together, the results of my model indicate that (absent inter-generational transmission of wealth) polygynous marriage is consistent with evolution by natural selection only if (a) the ecology is generous enough for the male adult household member to provision little or no food to the offspring, (b) the competition does not reduce the winning male's productive resources too much, and (c) relatedness is low enough. Importantly, it is the combination of the three factors, rather than each factor alone, that is necessary for polygyny to be compatible with evolution by natural selection. In a discussion section I argue that this combination may have been rare in human history.

My findings complement those in the existing literature on the historical evolution of polygyny rates, which examines factors that I do not consider. Thus, according to Bergstrom (1994a), monog-

---

[17] See, e.g., Grueter, Chapais, and Zinner (2012), Malone, Fuentes, and White (2012), van Schaik (2016), and Layton et al. (2012). Van Schaik (2016) even argues that such population structure is part of the environment of evolutionary adaptation of the human lineage.

[18] See, e.g., Hamilton (1964), Grafen (1985), Frank (1998), and Rousset (2004). For economics models having analyzed the effects of such relatedness, see Bergstrom (1995, 2003), Alger and Weibull (2010, 2013, 2016), and Alger, Weibull, and Lehmann (2020).

amous unions among the wealthy arose as a response to increasing returns to wealth: monogamous marriage was a means to commit to primogeniture. In Lagerlöf's (2005) model, the greater dilution of land ownership over time in polygynous than in monogamous households implies that polygyny compresses male income heterogeneity over time, thus reducing the prevalence of polygynous marriages over time.[19] Bergstrom (1994b) studies how male heterogeneity in income potential affects the equilibrium bride price and the ensuing polygyny rate. By analyzing a model with an age gap between husbands and wives, Tertilt (2006) finds that monogamy is favored by low population growth rates, which limit the supply of women. According to Gould, Moav, and Simhon (2008), monogamy could have arisen as a result of heterogeneity in women's human capital together with a male preference for the human capital of their offspring; under these conditions men may prefer to have one high-quality wife rather than several low-quality ones.[20] Lagerlöf (2010) proposes that institutionally imposed monogamy could have been a response to social unrest among the poor, some of whom would be spouseless in a polygynous society. In a similar vein, De La Croix and Mariani (2015) show that the political support for polygynous marriage should decline as the share of the poor in a society decreases. Francesconi, Ghiglino, and Perry (2016) show that long-life monogamy is superior to both polygyny and serial monogamy because of the free-riding in paternal care that arises between the men if a woman mates with more than one man.

The closest model to mine is probably that by Ross et al. (2018), because it features men who provide both a rival and a non-rival good to their offspring, and the relative importance of these goods is linked to the ecology in which the population evolves. Importantly, however, their model, as well as the models cited above, all have exogenously given male preferences over polygyny rates and all disregard men's willingness to compete for wives as a fundamental driver of male heterogeneity. Moreover, they all assume that men would always want more wives, should their price be nil. By contrast, the model proposed here derives male preferences over polygyny rates from first principles and analyzes men's willingness to compete for wives as an inherited trait subject to natural selection. The central finding is that male reproductive success can be decreasing in the polygyny rate. This model thus provides a novel explanation for why a complete lack of male-male competition and the associated monogamy can be consistent with evolution by natural selection.

In the next section I analyze the *household game* played by the adults in any given household, and characterize male preferences over polygyny rates under constant productive resources. In Section 3 I turn to analysis of the evolutionary *competition game* between males, both under constant and non-constant productive resources, and with and without relatedness. Section 4 discusses the theoretical results in light of spatial and temporal variation of the model parameters over the course of human history. Section 5 concludes. All the mathematical proofs are in the Appendix.

---

[19] The model being dynamic, male heterogeneity changes over time. However, the law of motion is exogenous and, unlike in my model, men do not differ in their willingness to compete.

[20] See also Kanazawa and Still (1999).

2 The household game

I model a population in which each individual lives for at most two periods; as a non-productive and non-reproductive child in the first period, and—conditional on surviving childhood—as a productive and reproductive adult in the second period. In each generation the sex ratio is assumed to be balanced at birth, as evolutionary theory would predict (Fisher, 1930), and for simplicity also at the beginning of the adult period. The analysis focuses on behaviors in the adult period, which has two stages: a male-male competition stage followed by a child production stage.

In the male-male competition stage, men compete for women, and possibly also for wealth. The competition sorts men into two categories: those with and those without mates, where those with mates all have the same number of mates. At the beginning of the child production stage, there is thus a number of households, each composed of one man and his (identical) spouse(s). Within each such household the man and his spouse(s) then engage in tasks, the goal of each individual being to maximize his or her reproductive success, defined as the expected number of his or her offspring that survive to sexual maturity.[21]

While several alternative male-male competition scenarios will be analyzed (see Sections 3 and 4), the child production stage is the same for all of them. Since the reproductive success a man obtains for any given household composition impacts the benefit of engaging in competitive behavior in the male-male competition stage, I proceed by "backward induction" and first analyze the child production stage. This will allow me to characterize how male reproductive success varies with the number of wives.

2.1 Description of the household game

Consider a household with one man and his $k \geq 1$ spouse(s) entering the child production stage.[22] While a woman has no say when it comes to the number of wives in the household, I assume that she has full agency over her fertility and childcare decisions.[23] Each woman and each man is endowed with some resources that they devote to parental investment. While in reality these resources may be multi-dimensional, for simplicity I assume that they can be aggregated into one dimension, and I will refer to this resource as time. Let the time budgets available for parental investment be $X \in (0,1]$ for each woman and $Y \in (0,1]$ for the man. I assume that there is sexual division of labor: the man

---

[21] In reality mating success of offspring who have survived to sexual maturity also matters for an adult's reproductive success. I disregard this here, by letting mate matching be random rather than based on choice.

[22] For simplicity, interactions between households, divorce, and unfaithfulness are ruled out by assumption. It would clearly be desirable to endogenize the degree of unfaithfulness, but this has to be left for future research. For the time being, one interpretation of the benchmark model considered here is that there is strong social control (for instance, although they are not explicitly modeled here, there may be grand-parents who monitor how their children behave in the adult stage, or the females monitor each other). Moreover, most estimates of current extra-pair paternity rates are low, ranging between 0 and 11% across societies (see Simmons et al., 2004, and Anderson, 2006).

[23] Alternative assumptions regarding fertility choice will be discussed below.

specializes in protection, women specialize in care, and both the man and the women provision the children with food.[24] Letting G_j and C_j denote the amounts of food and care that child j receives from its mother, and H_j and P_j the amounts of food and protection received from its father, I assume that the probability that the child (whether a boy or a girl) survives to sexual maturity is

$$\max\left\{0, 1 - bn_j\right\} \cdot S\left(C_j, G_j, H_j, P_j\right), \tag{1}$$

where n_j is the total number of children that j's mother gave birth to, and

$$S\left(C_j, G_j, H_j, P_j\right) = P_j^{\lambda} \cdot C_j^{\sigma} \cdot \left(\rho G_j + H_j\right)^{\tau}. \tag{2}$$

The first term in (1) reflects the physical toll of giving birth, where $1/b$ can be thought of as the maximum number of children a woman can have before she dies with certainty (in which case the children die since the mother's inputs are essential).[25] The function in (2) describes how the parental inputs affect the survival probability. It captures two realistic features. First, food, protection, and care are all essential goods: food is useless unless some protection and some care is provided, and *vice versa*. Second, the food provided by the mother and the father are perfect substitutes, although the food provided by the mother relative to that provided by the father may be more important ($\rho > 1$), equally important ($\rho = 1$), or less important ($\rho < 1$); this may depend on the nutritional attributes of different kinds of food in the location occupied by the population at hand. The parameters λ, σ, and τ measure how protection, care, and total food intake, respectively, impact the survival probability. These parameter values would typically also depend on the local environment: protection and care is relatively more important if there are many predators around or if there are many dangers that children need to learn to avoid.

The amounts of food, care, and protection that a child j receives, as captured by the vector $\left(C_j, G_j, H_j, P_j\right)$ (see (1) and (2)), depend on the production decisions of his or her parents, to which I now turn. Denote by $y \in [0, Y]$ the time that the man devotes to producing food, so that $Y - y$ is spent on protecting the household. Likewise, denote by x_i the time that wife $i = 1, ..., k$ devotes to producing food, and $X - \phi x_i$ the time that she spends on caring, where the parameter $\phi \in (0, 1)$ measures the extent to which a woman may produce food while providing care. Letting n_i denote wife i's number of offspring, the household's *child production allocation* is summarized by the vector $(\boldsymbol{n}, \boldsymbol{x}, y)$, where $\boldsymbol{n} = (n_1, ..., n_k)$ and $\boldsymbol{x} = (x_1, ..., x_k)$. If all the women adopt the same fertility and the same time allocation, i.e., if $n_i = n$ and $x_i = x$ for all $i \in \{1, 2, ..., k\}$, the child production allocation will be called *female-symmetric* and be denoted (n, x, y).

Assuming that each adult divides the goods he or she produces equally among his or her children,

---

[24] The terms protection and care should be interpreted broadly. Thus, protection may include shelter construction and maintenance, active protection against predators, as well as the transmission of human capital pertaining to such activities. Likewise, care may include the production and mending of clothes, storytelling, as well as the transmission of knowledge about social rules, plants, and animals.

[25] Here the physical toll is modeled as a scaling factor: *ceteris paribus*, the more children a woman has, the smaller is the survival probability of each of her children. Alternatively, one could let the physical toll have an impact on female productivity. This is left for future research.

I use an index i to refer to each of the n_i children of wife $i = 1,...,k$. Child i receives the amount

$$H_i = \frac{\theta y}{\sum_{j=1}^{k} n_j} \tag{3}$$

of food from its father, where $\theta > 0$ is the marginal return to male effort devoted to producing food, and the amount

$$G_i = \frac{\gamma x_i}{n_i}, \tag{4}$$

of food produced by its mother, where $\gamma > 0$ is the marginal return to female effort devoted to gathering.[26] In other words, the food brought home by the father is divided equally across all his children, while the food brought home by each mother is divided only across her own children. I assume that $\rho\gamma + \theta \leq 1$; as will be seen below, this ensures that S always takes a value between 0 and 1.

While food is a rival good, care and protection may be non-rival; for instance, a wall around the village protects all the children equally well, and a class about poisonous plants may benefit several children simultaneously. Formally, let

$$P_i = \frac{Y - y}{\left(\sum_{j=1}^{k} n_j\right)^{\alpha}} \tag{5}$$

be the amount of protection that each child of wife i receives, where $\alpha \in [0,1]$ is the degree of rivalry of protection. In the extreme case where $\alpha = 0$, protection is fully non-rival, and each child receives the full benefit of the total amount of protection produced by the father: $p(y, Y, \boldsymbol{n}) = Y - y$. At the other extreme, if $\alpha = 1$, protection is a fully rival good, and each child receives an equal share of the total amount produced: $p(y, Y, \boldsymbol{n}) = (Y - y)/\left(\sum_{j=1}^{k} n_j\right)$. Likewise, the amount of care that a child of wife i receives is

$$C_i = \frac{X - \phi x_i}{n_i^{\beta}}, \tag{6}$$

where $\beta \in [0,1]$ measures the degree of rivalry of care. If $\beta = 0$, care is fully non-rival, while if $\beta = 1$, it is a fully rival good.

I assume that all the adult members of the household make independent decisions, and that all seek to maximize own reproductive success. Thus, each woman $i = 1,...,k$ chooses her fertility n_i and her time allocation x_i, taking the man's and the other women's time allocations as given, and the man chooses his time allocation y, taking the women's fertility and time allocation decisions as given. Formally, given k, the situation at hand is thus a simultaneous-move game with $k+1$ players; each female player $i = 1,...,k$ chooses a strategy $(n_i, x_i) \in \mathbb{R}_+ \times [0, X]$, while the male player chooses a strategy $y \in [0, Y]$. The specification of each individual's reproductive success, which is his or her payoff in the game, as a function of the strategy profile $(\boldsymbol{n}, \boldsymbol{x}, y)$, completes the description of the

---

[26] Constant returns to effort are perfectly compatible with specialization, if, for instance, the marginal return to male effort devoted to gathering and to caring is strictly lower than that of a female, and the marginal return to female effort devoted to hunting and to protecting is strictly lower than that of the male.

game. The reproductive success of wife i writes

$$F\left(n_i, x_i, \boldsymbol{n}_{-i}, y\right) = n_i \cdot \max\left\{0, 1 - b n_i\right\} \cdot s\left(n_i, x_i, \boldsymbol{n}_{-i}, y\right), \tag{7}$$

where, from (2)-(6),

$$s\left(n_i, x_i, \boldsymbol{n}_{-i}, y\right) = \left[\frac{Y - y}{\left(\sum_{j=1}^{k} n_j\right)^{\alpha}}\right]^{\lambda} \cdot \left(\frac{X - \phi x_i}{n_i^{\beta}}\right)^{\sigma} \cdot \left[\frac{\rho \gamma x_i}{n_i} + \frac{\theta y}{\left(\sum_{j=1}^{k} n_j\right)}\right]^{\tau}, \tag{8}$$

while the reproductive success of the man is the sum of his wives' reproductive successes:

$$M\left(\boldsymbol{n}, \boldsymbol{x}, y\right) = \sum_{j=1}^{k} F\left(n_j, x_j, \boldsymbol{n}_{-j}, y\right) = \sum_{j=1}^{k} n_j \cdot \max\left\{0, 1 - b n_j\right\} \cdot s\left(n_j, x_j, \boldsymbol{n}_{-j}, y\right). \tag{9}$$

Let Γ denote the household game thus described.

This completes the description of the decisions taken within a household and their consequences. In the extremely long run, everything in life, including the features and the prevalence of animals and plants eaten by humans, is endogenous. However, the speeds at which different elements of a human society evolve, differ. In the model, I assume that the production technology, the degrees of rivalry of protection and care, the child survival probability function, the amount of female labor resources, and the physical toll due to childbirth are exogenous and fixed, and I refer to the associated set of parameters as *the ecology*. Formally, then, the ecology is the vector $\omega = (b, \phi, \sigma, \lambda, \tau, X, \alpha, \beta, \gamma, \theta, \rho)$. The ecology determines how parental time allocations and the fertility rate together determine male reproductive success, and below it will be seen how this in turn affects the set of sustainable polygyny rates. For further use below, let the set of ecologies be denoted Ω, i.e., $\Omega = \left\{\omega \in (0,1)^2 \times (0,1]^4 \times [0,1]^2 \times \mathbb{R}^3 \quad | \quad \rho\gamma + \theta \leq 1\right\}$.

2.2 Equilibria of the household game

Assuming the game is one of complete information and adopting Nash equilibrium as equilibrium concept, the following result obtains (the proof is in the appendix):

Proposition 1. *For any ecology ω, any number of wives $k \geq 1$, and any amounts of the male resources Y, there exists a unique Nash equilibrium of the household game Γ. Moreover, this equilibrium is female-symmetric.*

In words, in any household there is a unique Nash equilibrium strategy profile, and, moreover, at this equilibrium strategy profile all the women choose the same number of children and the same time allocation. While the model does not allow to obtain a closed-form solution for the equilibrium number of children, n^*, the expressions for the equilibrium time allocations, x^* and y^*, are as follows (x^* and y^* are stated as functions of the number of wives k and the amount of male resources Y, for these are the two variables that will be determined endogenously by the male-male competition). To simplify the notation let $\mu \equiv \theta/(\rho\gamma)$; this ratio measures the importance of food contributed by the man relative to that contributed by the women. Two cases arise: if $\frac{\tau}{\sigma+\tau} \leq \phi$,

$$(x^*(k,Y), y^*(k,Y)) = \begin{cases} \left(\frac{X\tau}{\phi(\sigma+\tau)}, 0\right) \text{ if } k \geq \frac{\mu\phi(\sigma+\tau)}{\lambda} \cdot \frac{Y}{X} \\ \left(\frac{X(\lambda+\tau)k - Y\sigma\phi\mu}{k\phi(\lambda+\sigma+\tau)}, \frac{Y\mu\phi(\sigma+\tau) - X\lambda k}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\mu\phi\sigma}{\lambda+\tau} \cdot \frac{Y}{X}, \frac{\mu\phi(\sigma+\tau)}{\lambda} \cdot \frac{Y}{X}\right] \\ \left(0, \frac{Y\tau}{\lambda+\tau}\right) \text{ if } k \leq \frac{\mu\phi\sigma}{\lambda+\tau} \cdot \frac{Y}{X}; \end{cases} \tag{10}$$

while if $\frac{\tau}{\sigma+\tau} > \phi$,

$$(x^*(k,Y), y^*(k,Y)) = \begin{cases} (X, 0) \text{ if } k \geq \frac{\mu\tau}{\lambda} \cdot \frac{Y}{X} \\ \left(X, \frac{Y\tau}{\lambda+\tau} - \frac{Xk\lambda}{\mu(\lambda+\tau)}\right) \text{ if } k \in \left[\frac{\mu\sigma\phi}{\lambda+\tau-\phi(\lambda+\sigma+\tau)} \cdot \frac{Y}{X}, \frac{\mu\tau}{\lambda} \cdot \frac{Y}{X}\right] \\ \left(\frac{X(\lambda+\tau)k - Y\sigma\phi\mu}{k\phi(\lambda+\sigma+\tau)}, \frac{Y\mu\phi(\sigma+\tau) - X\lambda k}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\mu\sigma\phi}{\lambda+\tau} \cdot \frac{Y}{X}, \frac{\mu\sigma\phi}{\lambda+\tau-\phi(\lambda+\sigma+\tau)} \cdot \frac{Y}{X}\right] \\ \left(0, \frac{Y\tau}{\lambda+\tau}\right) \text{ if } k \leq \frac{\mu\sigma\phi}{\lambda+\tau} \cdot \frac{Y}{X}. \end{cases} \tag{11}$$

Inspection of these expressions immediately reveals the following property, which will turn out to play an important role in the subsequent analysis:

Proposition 2. *For any ecology ω, any number of wives $k \geq 1$, and any amounts of the male resources Y, $\frac{\partial y^*(k,Y)}{\partial k} \leq 0$.*

In words, the amount of resources that the man spends on food production, $y^*(k,Y)$, is (weakly) decreasing in k. This property arises because an increase in the number of wives k implies that the man's food output is shared between a larger number of children. It is thus as if the man's ability to produce food relative to that of women was reduced. As a result, an increase in k reduces the man's incentive to engage in food production. For k large enough, all the food is produced by the women ($y^*(k,Y) = 0$). As will be seen below this feature is intimately linked with the qualitative characteristics of male preferences over polygyny rates.

The other comparative statics are also intuitive. Thus, comparing two ecologies, with τ and $\tau' > \tau$, respectively, the marginal benefit from producing food for both men and women is larger in the latter and therefore both the man and his spouse(s) spend more time on food production in the latter ecology. Likewise, the man spends less time and the women more time on food production in ecologies where protection is more important (a higher λ), while the opposite occurs in ecologies where female care is more important (a higher σ). Similarly, in ecologies with greater economies of scope between the two female activities (a higher ϕ), the cost for women of allocating time away from caring is smaller, and hence, female food production is larger; this in turn entails a smaller marginal effect of male food production on child success, and hence the man devotes less time to food production.

In order to prepare the ground for the analysis of the male-male competition stage below, I ask the following question: if a man could freely choose the number of wives k, how many would he choose? The seemingly obvious answer is that the man must always benefit from an increase in the number of wives. However, this turns out not to be true in general.

The equilibrium child production allocation being unique (see Proposition 1) the man's reproductive success is uniquely determined for any number of wives k and any amount of male labor resources Y. Let $M^* : [0,+\infty)\times(0,1]$ denote the mapping that to each (k,Y) associates the equilibrium male reproductive success, i.e.,

$$M^*(k,Y) = M\left(\boldsymbol{n}^*(k,Y), \boldsymbol{x}^*(k,Y), y^*(k,Y)\right), \tag{12}$$

and note that $M^*(k,Y) > 0$ for any $k > 0$. The following proposition describes how M^* varies with k, for a given amount of male labor resources Y.

Proposition 3. *For any $Y \in (0,1]$ there exists a partition $\{\Omega_1, \Omega_2, \Omega_3\}$ of Ω (where Ω_1, Ω_2, and Ω_3 are all non-empty) such that:*
(1) for any $\omega \in \Omega_1$, $M^*(k,Y)$ is increasing in k;
(2) for any $\omega \in \Omega_2$, there exists $k^*(\omega) > 1$ such that $M^*(k,Y)$ is strictly decreasing in k for $k < k^*(\omega)$ and strictly increasing in k for $k > k^*(\omega)$;
(3) for any $\omega \in \Omega_3$, $M^*(k,Y)$ is strictly decreasing in k.

Men face a quantity-quality trade-off, because producing more children (by having more wives) entails a decrease in the survival probability of children. The proposition says that: (A) the trade-off can be so severe that male reproductive success declines if he adds more wives, and (B) the trade-off is most severe for low polygyny rates. Specifically, depending on the ecology, male reproductive success is either (1) monotonically increasing; (2) decreasing for low polygyny rates and increasing for high polygyny rates; or (3) monotonically decreasing. Interestingly, then, it cannot be increasing for low and decreasing for high polygyny rates.

The proof of the proposition reveals that the key variable behind this result is the man's involvement in food production. Any time spent by the man on food production means that he uses resources to produce a fully rival good. This in turn renders the quantity-quality trade-off more severe than if he spent all his resources on producing the somewhat non-rival good protection. Since, as discussed above (see Proposition 2), male food production declines with the number of wives, this severe trade-off can only appear for low polygyny rates. As the number of wives grows large enough, the man stops engaging in food production, and adding more wives must then be beneficial (unless protection is almost fully rival). This explains why, for given male labor resources Y, male reproductive success cannot be increasing for low and decreasing for high polygyny rates. To summarize the role played by the man's involvement in food production, this involvement is present in any ecology in $\Omega_2 \cup \Omega_3$, where the man's reproductive success is decreasing in k for some k.

The proof of the proposition further reveals that the degree of rivalry of protection, α, also plays a central role. The less rival is protection, the more the man stands to benefit from an increase in the number of wives. In the extreme case where protection is fully non-rival ($\alpha = 0$), he benefits from an increase in k at all polygyny levels. By continuity, the same result obtains as long as α is sufficiently small. Thus, α must be sufficiently large for male reproductive success to be decreasing for some polygyny rates.

Remark 1. *The proof of Proposition 3 shows that the result would still hold if the man could impose his preferred fertility on his wives, as long as this fertility is the same for all wives. Clearly, however, it would not hold if a man could impose zero fertility on some wives and use these as labor resources to raise the children he sires with the other wives. The female agency assumption is justified for several reasons, however. First, a woman who is told to have no children has extremely strong incentives to flee; hence, imposing zero fertility on a woman should entail significant costs for the man. Second, even if such a woman stays with her husband, she has no incentive to perform the production of food and care diligently. Third, absent efficient contraceptive methods, the man would have to refrain from consummating the marriage with these wives, and it is not clear how realistic such an assumption would be. Of course, one could counter-argue that the man can resort to consummation with all wives and discriminatory infanticide; however, this would likely simply worsen the two preceding issues.*

I conclude the analysis of the child production stage by noting that doubling the number of wives less than doubles reproductive success, except in the extreme case where protection is fully non-rival and the man devotes all his time to providing protection. Furthermore, male reproductive success is increasing in the productive resources he controls. Finally, a woman's reproductive success is strictly decreasing in the polygyny rate in her household, except in the extreme case where protection is fully non-rival and the man devotes all his time to providing protection. These results are stated formally in the following proposition (whose trivial proof is omitted):

Proposition 4. *For any given ecology* $\omega \in \Omega$*:*
(1) holding Y *fixed,* $M^*(2k, Y) \leq 2M^*(k, Y)$*, and the inequality holds as an equality if and only if male protection is fully non-rival (*$\alpha = 0$*) and the man devotes no time to food production (*$y^*(k, Y) = 0$ *for any* $k \geq 1$*);*
(2) holding $k \geq 1$ *fixed,* $M^*(k, Y)$ *is continuous and strictly increasing in male labor resources* Y*, and* $\lim_{Y \to 0} M^*(k, Y) = 0$*;*
(3) holding Y *fixed,* $\partial F^*(k, Y)/\partial k \leq 0$*, and the inequality is strict unless male protection is fully non-rival (*$\alpha = 0$*) and the man devotes no time to food production (*$y^*(k, Y) = 0$ *for any* $k \geq 1$*).*

I am now in a position to analyze the first stage of the adult period, namely, the male-male competition stage.

3 The male-male competition game

In the male-male competition stage, men compete for access to women. While competition can also affect productive resources, I first analyze the case where it does not. Throughout, all the women are taken to be identical. Men are also identical when entering the competition stage, except potentially in their eagerness to compete, which is the trait whose evolution I analyze.

3.1 Competition when only women are at stake

To model male-male competition I posit an evolutionary game—the *competition game*—which concerns the male part of the population. Adopting a standard evolutionary game theoretic approach (e.g., Weibull, 1995), I assume that there is a continuum of male individuals and that each man is "programmed" to play a certain strategy, which may be interpreted as his eagerness to compete, or *degree of competitiveness*, inherited from his father.[27] The competition game sorts men into two categories: "winners" and "losers." The winners get an equal number of wives each, while the losers remain mateless (or die, depending on the nature of the competition). Hence, this game endogenizes the allocation of women across men. The objective of the analysis is to determine the set of evolutionarily stable *degrees of competitiveness*.

To capture the main forces present in this model, it is sufficient to consider the simplest possible competition game, with only two strategies. Specifically, suppose that when entering the adult stage (from the teenage years) each man has one girlfriend, and that men are then (uniformly) randomly matched into pairs to play a simultaneous-move game with two pure strategies, *Compete* and *Surrender*. The strategy profile used in a matched pair determines the number of women accruing to each man. Thus, if both men play *Surrender* each gets to marry his teenage sweetheart. If at least one man plays *Compete*, then one of them gets to marry both girlfriends while the other one becomes mateless and remains so forever. A man who plays *Compete* wins with probability 1 if the other plays *Surrender*, and with probability 1/2 if the other plays *Compete*.[28]

In a matched pair where the strategy profile (*Surrender*, *Surrender*) is used, two monogamous households are thus formed. In the child production stage this gives rise to expected male reproductive success $M^*(1,Y)$ for each of these males. Likewise, in a matched pair where at least one player plays *Compete*, one bigynous household is formed, and the lucky man obtains expected male reproductive success $M^*(2,Y)$ while the unlucky man gets zero reproductive success. Accordingly, the payoffs in the evolutionary game are those in the matrix in Figure 1 (since this is a symmetric game, only the row player's payoffs are shown).

	Surrender	Compete
Surrender	$M^*(1,Y)$	0
Compete	$M^*(2,Y)$	$\frac{1}{2}M^*(2,Y)$

Figure 1. Payoffs in the competition game with constant productive resources

Let $r \in \{0,1\}$ denote a (pure) strategy in the evolutionary game, where $r=0$ means *Surrender* and $r=1$, *Compete*. Allow for mixed strategies and write $\zeta \in [0,1]$ for the probability of playing *Com-*

---

[27] In a more general model, each man could be equipped with preferences guiding his behavior in the competition game, and the transmitted trait would be the preferences. The simpler approach adopted here is in some settings equivalent to such preference evolution, and it also provides a useful benchmark (Alger and Weibull, 2013, 2019).

[28] The evolutionary stability properties of pure strategies in the competition game with more than two rounds of competition is analyzed in Appendix B.

pete. Now ponder the following thought experiment: suppose that a given strategy ζ, the "resident" strategy, is used by almost everyone in the population, except for a small share $\varepsilon > 0$ of individuals who use another strategy ζ', the "mutant" strategy. Is there any resident strategy ζ that outperforms every possible "mutant" strategy $\zeta' \in [0,1]$, $\zeta' \neq \zeta$, in the sense that those who carry the resident strategy get a strictly higher reproductive success on average than those who carry the mutant strategy? In other words, does the competition game have any evolutionarily stable strategies (ESS)? The following proposition answers this question.

Proposition 5. *In the competition game with constant productive resources:*
(i) $\zeta = 0$ *is evolutionarily stable if and only if the ecology* ω *is such that:*

$$M^*(1,Y) > M^*(2,Y). \tag{13}$$

(ii) No mixed strategy $\zeta \in (0,1)$ *is evolutionarily stable in any ecology* $\omega \in \Omega$.
(iii) $\zeta = 1$ *is evolutionarily stable in any ecology* $\omega \in \Omega$.

In other words, in some circumstances a population consisting of non-competitive men can resist the invasion of competitive men. Under constant productive resources, the ecology in which the population evolves is shown to be central: *Surrender* is evolutionarily stable in ecologies where men achieve a higher reproductive success with one than with two wives, given constant productive resources Y (condition (13)). To see why, suppose that *Surrender* is the resident strategy. Any mutant—who plays *Compete*—then almost surely achieves reproductive success $M^*(2,Y)$, because he is almost surely matched with a resident, who *Surrenders*. However, the vast majority of residents are matched with other residents, and they all achieve reproductive success $M^*(1,Y)$. Hence, residents who play *Surrender* outperform rare mutants who play *Compete* if, and only if, $M^*(1,Y) > M^*(2,Y)$.

Nevertheless, as shown in the last part of the proposition, *Compete* is always evolutionarily stable, i.e., even when condition (13) holds. To see why, suppose that *Compete* is the resident strategy. Any *Surrendering* mutant then almost surely achieves reproductive success equal to 0, because he is almost surely matched with a resident, who *Competes*. By contrast, almost all residents get reproductive success $M^*(2,Y)/2$. [29]

Finally, as indicated in the second part of the proposition, the competition game with constant productive resources admits no evolutionarily stable mixed strategy. To see, consider a population where some mixed strategy $\zeta \in (0,1)$ is the resident strategy and the mutant pure strategy *Compete*. An individual who is matched with a resident then does equally well whether he is a resident or a mutant (to see this, note that for a mixed strategy to be ESS it must be such that the residents achieve the same average reproductive success whether they play *Surrender* or *Compete*); however, an individual who is matched with a mutant does strictly better if he is a mutant than if he is a resident, since the latter then gets the same reproductive success as the mutant with probability $1-\zeta$ but 0 reproductive success with probability ζ.

---

[29] Note that this result would obtain even under the less stark assumption that playing *Surrender* against someone playing *Compete* would give a positive probability of winning, as long as this probability would be below 1/2.

In sum, when men compete for women alone there is either one or two evolutionarily stable strategies, depending on the ecology. The assumption that a man's productive resources (Y) do not depend on the number of rounds he competes is, however, highly unrealistic. Indeed, this would require both that the resources a man can ever hope to control are his own physical labor resources, and that these labor resources are unaffected by the amount of competition. In particular, competition cannot entail any risk of physical injury. While examples of male-male competitions satisfying these requirements are conceivable (think of song or poetry contests and chess tournaments), the quest for a model that could help reach a better understanding of the evolution of male-male competition calls for a more general version of the male-male competition game, one in which men compete for both reproductive and productive resources. I now turn to such a model.

3.2 Competition when both women and productive resources are at stake

In the competition game with endogenous productive resources, a male who has successfully competed one round has productive resources φ, which may differ from his initial resources Y. The payoff matrix of this evolutionary game is shown in Figure 2.

	Surrender	Compete
Surrender	$M^*(1,Y)$	0
Compete	$M^*(2,\varphi)$	$\frac{1}{2}M^*(2,\varphi)$

Figure 2. Payoffs in the competition game with endogenous productive resources

While many factors can affect the material resources accruing to the winner of a competition (such factors will be discussed in greater detail in Section 4), it seems reasonable to put both a lower and an upper bound on φ. Specifically, I assume that competition cannot fully deplete a man's resources, and that it can at most allow the winner to acquire all of the loser's resources, i.e., $\varphi \in (0,2Y]$. The following results obtain.

Proposition 6. *In the competition game with endogenous productive resources:*
(i) $\zeta = 0$ is evolutionarily stable if and only if:

(1) either the ecology ω is such that $M^*(1,Y) > M^*(2,Y)$ and $\varphi < Y + A$, where $A > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y+A)$;

(2) or the ecology ω is such that $M^*(1,Y) < M^*(2,Y)$ and $\varphi < Y - B$, where $B > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y-B)$.
(ii) No mixed strategy $\zeta \in (0,1)$ is evolutionarily stable in any ecology $\omega \in \Omega$.
(iii) $\zeta = 1$ is evolutionarily stable for all $\omega \in \Omega$ and all $\varphi \in (0,2Y]$.

As shown in the proof of the proposition, *Surrender* is evolutionarily stable if and only if $M^*(1,Y) > M^*(2,\varphi)$. Since the equilibrium male reproductive success function M^* is increasing and continuous in the man's productive resources, this implies that *Surrender* is evolutionarily stable if either (1) the ecology is such that *Surrender* is evolutionarily stable when productive resources are constant and winning one round of competition does not enhance productive resources by too much, or (2) the ecology is such that *Surrender* is not evolutionarily stable for constant resources but winning one round of competition entails a large enough drop in productive resources.

In sum, then, on top of ecological factors, any technological and/or institutional factors that affect the material resources accruing to the winner of a competition should also be expected to impact the evolutionary stability of non-competitive males. Remarkably, however, these factors have no impact on the evolutionary stability of competitive males: in all the settings analyzed above *Compete* is an evolutionarily stable strategy. This is true even though any population would be better off as a whole if males were non-competitive, in the sense that female reproductive success is strictly decreasing in the polygyny rate (this follows from Proposition 4 and the assumption that winning a competition cannot more than double a man's productive resources ($\varphi \leq 2Y$).[30] In other words, in any setting where both *Surrender* and *Compete* are evolutionarily stable, evolution could have led either to the (efficient) absence of male-male competition, or to the (inefficient) presence of male-male competition, where efficiency is measured in terms of female reproductive success. In the following subsection I extend the model to allow for a force that will be seen to render *Compete* evolutionarily unstable in certain settings.

3.3 The effects of relatedness on male-male competition

So far, the analysis has disregarded an important and ubiquitous feature of human societies, namely, the fact that our ancestors lived in groups of small size, which extended beyond the nuclear family and between which there was limited migration. A key implication of such population structure is that mutants, even when rare, face a higher probability of interacting with mutants than residents do, even absent any ability of individuals to choose the individuals with whom they interact. In the biology literature, this effect is usually quantified by the *coefficient of relatedness* (Wright, 1931), which measures the probability that interacting individuals share a common ancestor. In order to extend the model in this direction I adopt the formalization proposed by Bergstrom (2003) (see also Grafen, 1979, Alger and Weibull, 2013, and Jensen and Rigos, 2018).[31]

Thus, consider the evolutionary game analyzed in the preceding subsection, and denote the resident strategy by $\zeta \in [0,1]$ and the mutant strategy by $\zeta' \in [0,1]$, $\zeta' \neq \zeta$, the latter being present in a (small) share $\varepsilon > 0$ of the population. The *assortment function* $\sigma : (0,1) \to [-1,1]$ maps to each mutant population share $\varepsilon \in (0,1)$ the difference between the probability for a resident to be matched with a resident and a mutant to be matched with a resident (the assortment function is the same for all

---

[30] The only setting in which female reproductive success is not strictly decreasing in the polygyny rate is when paternal investment comes in the form of a purely public good ($\alpha = 0$) and all the productive resources are acquired by the winning male ($\varphi = 2Y$). It is then constant in the polygyny rate.

[31] While this formalization is less fine-grained than that obtained in the classic island model in evolutionary biology (see, e.g., Cavalli-Sforza and Bodmer, 1971, Frank 1998, Rousset 2004, Hartl and Clark, 2007), it is sufficient for my purposes. For a recent adoption of the island to preference evolution in srategic interactions, see Alger, Weibull, and Lehmann (2020).

$(\zeta, \zeta') \in [0,1]^2$):

$$\sigma(\varepsilon) = \Pr[\zeta|\zeta, \varepsilon] - \Pr[\zeta|\zeta', \varepsilon]. \tag{14}$$

In the special case of uniform random matching, there is no difference, i.e., $\sigma(\varepsilon) = 0$ for all $\varepsilon \in (0,1)$. I assume that both conditional probabilities $\Pr[\zeta|\zeta, \varepsilon]$ and $\Pr[\zeta|\zeta', \varepsilon]$ are continuous in ε, and denote by σ_0 the limit of the assortment function σ as the share of mutants tends to zero: $\sigma_0 = \lim_{\varepsilon \to 0} \sigma(\varepsilon)$. Noting that $\lim_{\varepsilon \to 0} \Pr[\zeta|\zeta, \varepsilon] = 1$ (because the population is infinitely large), it is clear that $\lim_{\varepsilon \to 0} \sigma(\varepsilon) = 1 - \lim_{\varepsilon \to 0} \Pr[\zeta|\zeta', \varepsilon] = \lim_{\varepsilon \to 0} \Pr[\zeta'|\zeta', \varepsilon]$. In other words, σ_0 is also the probability that a mutant is matched with another mutant in the limit as the share of mutants tends to 0. It follows that $\sigma_0 \in [0,1]$. For my purposes I will say that relatedness is present when $\sigma_0 > 0$ and absent when $\sigma_0 = 0$. The latter case having been analyzed above, I here assume $\sigma_0 > 0$. Letting

$$\tilde{\sigma}_0 \equiv \frac{M^*(2, \varphi)}{2 \cdot M^*(1, Y)}, \tag{15}$$

the following results obtain.

Proposition 7. *In the competition game with endogenous productive resources and relatedness:*
(i) for any value of $\sigma_0 \in (0,1]$ the set of parameter values for which $\zeta = 0$ is evolutionarily stable absent relatedness (see Proposition 6) is a proper subset of the set of parameter values for which $\zeta = 0$ is evolutionarily stable when the relatedness is σ_0.
(ii) No mixed strategy $\zeta \in (0,1)$ is evolutionarily stable in any ecology $\omega \in \Omega$.
(iii) $\zeta = 1$ is evolutionarily stable if $\sigma_0 < \tilde{\sigma}_0$ while it is not evolutionarily stable if $\sigma_0 > \tilde{\sigma}_0$.

Comparing these results to those reported for the setting without relatedness ($\sigma = 0$) in Proposition 6, it is clear that relatedness works in favor of the *Surrender* strategy and against the *Compete* strategy. The effect is twofold.

First, relatedness expands the set of parameter configurations for which *Surrender* is evolutionarily stable. Compared to the setting without relatedness, competition can entail a larger increase in productive resources $\varphi - Y$ without threatening the stability of *Surrender* in ecologies for which *Surrender* is evolutionarily stable under constant resources; likewise, a smaller drop in productive resources $Y - \varphi$ due to competition is sufficient to render *Surrender* evolutionarily stable in ecologies for which *Surrender* is evolutionarily stable under constant resources.

Second, and in stark contrast with Propositions 5 and 6, *Compete* fails to be evolutionarily stable if relatedness is pronounced enough. The threshold value for σ_0 (see (15)) is the ratio of male reproductive success obtained if all males *Compete* ($M^*(2, \varphi)/2$) to that obtained if all males *Surrender* ($M^*(1, Y)$). To understand why, suppose that *Compete* is the resident strategy and *Surrender* the mutant one. A necessary condition for *Compete* to be evolutionarily stable against *Surrender* is that the average reproductive success of residents be at least as large as that of mutants, when the share of mutants tends to zero, i.e.:

$$\frac{M^*(2, \varphi)}{2} \geq (1 - \sigma_0) \cdot 0 + \sigma_0 \cdot M^*(1, Y). \tag{16}$$

The right-hand side shows that, even in a population where essentially all individuals *Compete*, a vanishingly rare mutant, who *Surrenders*, faces a positive probability of interacting with another mutant, thereby being able to settle down with his teenage girlfriend. If *Compete* entails an inefficiency, i.e., if $M^*(2,\varphi)/2$ falls short of $M^*(1,Y)$, and if rare mutants have a sufficiently strong tendency to interact with each other, the necessary condition (16) is violated: the inefficient strategy *Compete* then cannot withstand the invasion of the more efficient strategy *Surrender*.

Thus, whenever the threshold value $\tilde{\sigma}_0$ for σ_0 falls short of 1, *Compete* fails to be evolutionarily stable for any $\sigma_0 \in (\tilde{\sigma}, 1]$. The ratio in (15) depends both on the ecology (ω) and on the effect of competition on productive resources ($\varphi - Y$). For any given effect of competition on productive resources, $\tilde{\sigma}$ is increasing in the effect of adding a second wife under constant productive resources ($M^*(2,Y) - M^*(1,Y)$). And for any given ecology, $\tilde{\sigma}$ is increasing in the productive resources that competition bestows on the winner (φ).

Taken together, all the results reported above suggest that, absent inter-generational transmission of wealth, in human history polygynous marriage would have been consistent with evolution by natural selection only in times and places where the following three conditions were met simultaneously. First, the ecology ω must be generous enough for the male adult household member not to be required to engage heavily in provisioning food (or some other rival good) to his offspring. Second, in the least generous ecologies the competition must increase the winning male's productive resources, while in more generous ecologies it cannot reduce them too much. Third, relatedness must be low enough. Importantly, it is the combination of the three factors, rather than each factor alone, that is necessary for polygyny to be compatible with evolution by natural selection. In the next section I argue that this combination may well have been uncommon over the course of human history.

4 Discussion

The theoretical model analyzed above provides a rich set of predictions. While it is beyond the scope of this paper to conduct a formal empirical analysis, in this section I discuss the model predictions in light of a few important sources of variation across human sub-populations over space and time. In the model a population is fully described by the following three components: the ecology ($\omega \in \Omega$), the effect of competition on male productive resources ($\varphi - Y$), and the coefficient of relatedness ($\sigma_0 \in [0,1]$). I first discuss in general how the ecology and the coefficient of relatedness may have varied across space and time, and then propose a structured comparison of different human sub-populations through history, a comparison which also discusses how technological change may have affected the effect of competition on male productive resources.

Prior to the industrial revolution, the vast majority of people were self-sustaining farmers, horticulturalists, pastoralists, or hunter-gatherers. As such, their livelihoods depended to a large extent on the climatic, geological, and ecological conditions in their local environment. Since *Homo sapiens* had colonized all continents except Antarctica already in pre-neolithic times, in terms of the model this means that different human sub-populations faced a rich set Ω of different ecologies both in pre- and in post-neolithic times. Sources of variation would have included the marginal returns to male and female food production efforts, the nutritional value of local food sources, environmental hazards—such as natural disasters, predators, and attacks by other groups—some of which it was possible to protect against, etc. Arguably, it is reasonable to assume that in pre-industrial times (and using again the term "ecology" as defined in the model): (1) at any point in time there would have been significant spatial variation between the ecologies $\omega \in \Omega$ faced by human sub-populations; and (2) in any given location, sub-populations that adopted agriculture would have experienced a substantial change in the ecology ω compared to the pre-neolithic one, because of changes in the food production technology and in the food sources, and possibly also in the prevalence of environmental hazards; there would nonetheless still be spatial variation in the ecologies faced by sub-populations having adopted agriculture, due to variation in geological and climatic conditions.

In the model the ecology matters to the extent that it affects the trade-offs in the production of reproductive success. For example, in pre-industrial times the effort demanded to produce the necessities for a family would have been quite different in a climate with long and harsh winters than in a tropical or sub-tropical climate. While lack of data renders a detailed comprehensive comparison of the ecologically imposed trade-offs faced by pre-industrial human sub-populations impossible, existing studies suggest a significant impact of the ecology (as defined in the model) on reproductive success, even for populations sharing several ecological conditions. For example, a comparison of agriculturalists and nomadic herders in Northern Finland born between 1641 and 1884 shows that the agriculturalists had, on average, a significantly higher number of offspring, about 0.3 children more per (monogamous) family, while there was no significant difference in mortality prior to adulthood (Helle et al., 2014). A study of contemporary Pumé, an indigenous group in Venezuela, revealed that women among the horticulturalist river Pumé have a significantly larger number of children who survive to adulthood than the foraging savannah Pumé (Kramer and Greaves, 2007).

Turning to relatedness, the key driver is migration. To see this, consider first a population in which each individual would migrate on his/her own from the group into which (s)he was born to a randomly chosen group of people anywhere else in the world. In such a population, the probability of interacting with individuals sharing a recent common ancestor would be close to zero. By contrast, if only some individuals disperse from their natal group, or if all disperse to a place close to the birthplace, the probability of interacting with individuals sharing a recent common ancestor is positive. Clearly, human behavior is best described by the second scenario. And while, again, it is impossible to get a precise and comprehensive comparison of migration rates across time and space in human history, Murdock's ethnographic atlas suggests that virilocality (the tendency for women to move to the husband's locality) has been more prevalent than uxorilocality (the tendency for men to move to the wife's locality): out of 1267 societies, 692 were classified as mainly virilocal and only 305 as mainly uxorilocal. This matters for the model predictions to the extent that virilocality implies a higher coefficient of relatedness among men than among women. Furthermore, some studies do provide precise information on the propensity to migrate. For example, in their study of dispersal of individuals born in Finland between 1749 and 1880, Nitsch, Lummaa, and Faurie (2016) find that around 20% of all individuals who survived to the age of 15 moved out of their birth parish, and that the average distance between the birth parish and the destination parish was only 65km. While these numbers were similar for men and women, a key difference between the sexes was that among the individuals who dispersed, women were almost twice as likely to move to the spouse's birth parish, implying a higher coefficient of relatedness among young men than among young women. Overall, then, relatedness has arguably been an ubiquitous feature of all human sub-populations, especially between men.

With these factors in mind, I structure the discussion of the model predictions around two qualitatively different types of society, according to the impact of competition on a man's productive resources. In this discussion I will assume that $\varphi = bcY$, where $c \in (0,1]$ and $b \in [1,2]$ are parameters representing, respectively, the resource depletion and resource accumulation associated with competition.

4.1 Competition without resource accumulation: pre-neolithic societies

Imagine a society where the only labor resources a man can ever hope to control are his own physical labor resources; that is, slavery is not an option, and no or little accumulation of material resources beyond those necessary for survival (such as basic tools and shelter) is possible. In such a society, the only prize that men compete for is access to women. If competition, moreover, entails some risk of physical injury, then labor resources may be expected to decline with competition, or at least not increase. In terms of the model, in this society, the accumulation factor b would be equal to or close to 1, while the depletion factor c would be small enough for competition to entail resource depletion, or at least not any resource accumulation ($bc \leq 1$).

Arguably, this scenario appears to correspond to most human societies prior to the neolithic revolution. Absent agricultural production, any group of *Homo sapiens* had to rely on the food provided by its surrounding natural ecology. In places where this natural pantry could not be replenished fast enough, humans would have had to live a nomadic lifestyle. Recall from the model analysis that male involvement in food production indicates that the benefit from having more than one wife is limited. While it is challenging to evaluate the extent to which men provisioned children with food in pre-neolithic times, it is a well-established fact that among modern hunter-gatherers and horticulturalists men heavily engage in such provisioning (Kaplan et al., 2001).[32] In most places climatic conditions further restricted the ability to store any food surpluses. Moreover, *Homo sapiens* and even earlier hominins having mastered the production and use of lethal tools such as axes and spears for hundreds of thousands of years, contests between men could entail significant physical injury.

Viewed through the lens of the model analyzed above, it thus appears plausible that pre-neolithic societies would have evolved to have non-competitive men and monogamous unions: challenging ecological conditions coupled with the absence of resource accumulation ($bc \leq 1$) could have been such that male reproductive success was higher with one than with two wives ($M^*(1,Y) > M^*(2,\varphi)$), thus rendering the *Surrender* strategy evolutionarily stable, i.e., non-competitive men compatible

---

[32] Moreover, a recent study suggests that male provisioning had already evolved when *Homo sapiens* appeared (Alger et al., 2020; see, in particular, the discussion of the paleontological and archeological evidence).

with evolution by natural selection.[33] If, moreover, relatedness among young men was high enough, the *Compete* strategy would not have been evolutionarily stable (see Proposition 7). Such a conclusion would be compatible with evidence from traditional societies that have been observed over the past few hundred years: an overwhelming majority of unions in these societies are monogamous (see the references in the second paragraph of the introduction).

4.2 Competition with resource accumulation: agricultural societies

In terms of the model, the advent of agriculture brought about two key changes. First, in any given location on Earth, the adoption of agriculture likely induced a change in the ecology ω, especially the marginal returns to male and female labor in food production (the parameters γ, θ, and ρ), but potentially also the extent to which the non-food contribution of the father was a rival good (the parameter α). Second, food production became based on transferable assets such as land and farmed animals. In the model the former change would typically have induced changes in the intra-household division of labor, while the latter change meant that, contrary to most pre-agricultural societies, the control of productive assets now also became a stake of male-male competition. I argue that in the model these two changes may have had either qualitatively similar or opposite effects on male-male competition.

First, the type of crops and animal husbandry that a sub-population adopted, together with the tools at hand as well as the local geological and climatic conditions, would all have mattered for the changes in the ecology ω that the adoption of agriculture would have induced. Hence, these factors could have led men to become more or less involved in the production of food (and other non-rival goods) than in pre-agricultural times. For example, the adoption of the plough enhanced men's involvement in food production (Goody, 1976). *Ceteris paribus*, the changes in ω due to the transition to agriculture could thus have either strengthened or weakened the evolutionary stability of the *Surrender* strategy.

Second, in an extreme variant of a society in which productive assets are seizable, the winner's productive resources double ($bc = 2$). If the protection that the man provides to the children in his household is not fully rival ($\alpha < 1$) then the reproductive success of a man with two wives is more than twice as large as that of a man in a monogamous union, while if it is fully rival ($\alpha = 1$) his reproductive success is exactly twice as large. In any event, only *Compete* would be evolutionarily stable, because the coefficient of relatedness required to prevent this would be excessively high ($\tilde{\sigma} \geq 1$, see equation (15)). A doubling of productive resources does not appear historically relevant, however: since productive resources would have included the man's own labor, a doubling of productive resources would have required the losing man to become the winner's slave. Hence, the most likely scenario is that in the early agricultural societies the winner would see his productive resources increase but not double ($1 < bc < 2$). The model predicts that this would have tilted the balance in

---

[33] An oft-invoked explanation for why pre-agricultural societies tend to be monogamous is that in such societies there is no surplus to divide (see, e.g., Lagerlöf, 2005). My analysis shows that this argument is not sufficient, because men could still compete directly for wives. The key issue, then, is whether men would benefit or not from having more than one wife.

favor of the *Compete* strategy.

Arguably, however, the advent of metallurgy further affected the trade-offs. Indeed, significant technological advances followed in the footsteps of the agricultural life-style. In particular, weapons became more sophisticated and lethal, especially with the advent of metallurgy. As a result, men would have had to sacrifice an increasing amount of resources to make or acquire weapons and adequate protective gear. In the model such technological advances correspond to a decrease in the depletion parameter c over time. Depending on the accumulation parameter b, this development could have induced a negative net effect of competition on productive resources (i.e., $bc < 1$). Hence, the advent of metallurgy should have tilted the balance back in favor of the *Surrender* strategy.

4.3 Three major paths?

In sum, my findings suggest that the evolution of the prevalence of polygynous unions in any given human sub-population is specific to the local constraints it faced together with the technology it adopted for food extraction (the "ecology" ω in the model), the seizability and/or vulnerability of productive resources due to male-male competition (the difference $\varphi - Y$ in the model), and the extent to which competing males share a recent common ancestor (relatedness σ_0 in the model). In particular, the discussion above indicates that the transition to an agricultural life-style would not necessarily have given rise to more competitive men and polygyny, especially in ecologies in which men chose to engage substantially in the production of food (and/or some other rival goods).[34] More generally, I hypothesize that the theoretical findings point to the possibility that most human sub-populations may have experienced one of the following three historical development paths, each path being defined by its most common marriage system in pre-neolithic times, in neolithic times, and during the bronze and iron ages.

- Path 1: Monogamy - Polygyny - Polygyny
- Path 2: Monogamy - Polygyny - Monogamy
- Path 3: Monogamy - Monogamy - Monogamy

In accordance with the discussion in subsection 4.1, all paths have monogamy as the main marriage system in pre-neolithic times. Due to the seizability of productive assets—especially land—that it implied, the adoption of agriculture would have led to significant levels of polygyny, but only in societies where ecological conditions where such that men did not need to engage heavily in the production of food (Paths 1 and 2). Hence, polygyny would not have arisen in human sub-populations which either did not adopt agriculture or which did transition to agriculture but faced harsh conditions that rendered strong male involvement in food production necessary (Path 3). Finally, among the populations that adopted agriculture, a distinction can be made between those that later adopted metallurgy and those that didn't. Those that didn't should not have experienced any change. Those

---

[34] Note that any effects of storability of wealth is absent from my model since there is no transfer of wealth between generations. Such storability could thus have had effects on top of those discussed here.

that did develop metallurgy may—due to the lethality of weapons—either have continued to be largely monogamous (Path 3), transitioned back to monogamy if the relatedness among competing males was high enough (Path 2), or remained polygynous if the relatedness among competing males was low enough (Path 1).[35]

5 Conclusion

This paper studies the evolution of marriage systems over the course of human history. The central idea is that through their effects on biological reproductive success, marriage systems were likely shaped to a significant extent by natural selection. The model focuses on the incentives for men to compete with each other to acquire more wives. It includes three ubiquitous features of human history that arguably had first-order effects on these incentives. First, local ecological factors, such as the effort needed to produce food and the returns from the father's and the mother's protection and care, was key to reproductive success in all pre-industrial times. Second, the advent of agriculture brought about a drastic change in the incentives to compete; however, by contrast to the storability of wealth that is often put forward as a key effect of agriculture, I focus on the seizability of productive assets that the advent of agriculture brought about. Finally, the model also incorporates the fact that in all human societies there was limited migration; importantly, if high enough, the associated relatedness is shown to prevent the highest degree of competitiveness from being compatible with natural selection. In other words, relatedness can induce a society to shift from inefficient polygyny to efficient monogamy.

My model predicts that the marriage system should be intimately linked with the extent to which men engage in food production. This prediction is compatible with Goody's (1976) observation that monogamy is correlated with the use of the plough (see also Boserup, 1970).[36] Interestingly, the model can further help explain why the correlation is not perfect. Indeed, the plough didn't develop until several millenia after the advent of agriculture, at a time when many societies could have become largely polygynous. My model shows that even though the plough led to greater involvment in food production by men, only some societies would have transitioned to monogamy whereas others would have remained polygynous (depending on the relatedness and the returns to competition on productive assets). More generally, the model generates testable predictions regarding how ecological constraints may have impacted the historical evolution of marriage systems. It remain to be

---

[35] Taken together, these three paths should have induced the share of men who were successful at reproducing to be the lowest in neolithic societies. This is consistent with the drop in Y-chromosome diversity for the period between approximately 10kya and 6kya, and the subsequent rise in this diversity, as inferred from contemporary data by Karmin et al. (2015). These paths would further be consistent with the relatively low levels of physical violence observed between men, and which has been the subject of many theories (see the overview by Seabright, 2004); the theory proposed here arguably sheds new light on this question.

[36] Notice the subtle difference with Boserup's (1970) argument that polygyny made it possible for a man to control more land and labor. In my model the ultimate driver of male behavior is reproductive success. Hence a man benefits from outcompeting other men only if the *combined effect* of more wives and productive resources on reproductive success is positive.

seen whether these predictions can be tested with data on the historical evolution of Y-chromosome diversity (Balaresque and Jobling, 2007) or on ecological constraints in today's traditional societies (Ross et al., 2018).

The analysis relies on a model which disregards some arguably important aspects, notably the transferability of wealth across generations (Bergstrom, 1994a, Lagerlöf, 2005). Moreover, as suggested by Henrich, Boyd, and Richerson (2012), a full understanding of the forces behind marriage systems may require modeling the evolution of such inter-group competition as well. This hypothesis finds support in the empirical analyses of historical Y-chromosome diversity by Balaresque et al. (2015) and Zeng, Aw, and Feldman (2018). Thus, while ecological constraints and relatedness may have led men to refrain from competing against each other within groups, as suggested by my model, competition between groups of related males for women and productive resources may have arisen instead. Finally, as pointed out by Betzig (1992), "marriage in Rome was monogamous; *mating* was polygynous". Hence, future analyses should also examine the consequences of sexual infidelity for the historical evolution of marriage systems.[37]

---

[37] For further inspiration on this and related topics, see Hrdy (1999) and Fisher (2016) and references therein.

6 Appendix A: Proofs

6.1 Proposition 1

To prove that any Nash equilibrium strategy profile is female-symmetric, assume by contradiction that there exists a Nash equilibrium strategy profile according to which k_1 women play strategy (n_1, x_1), $k_2 = k - k_1$ women play strategy $(n_2, x_2) \neq (n_1, x_1)$, and the man plays strategy y. For this to be an equilibrium, it must be that

$$(n_1, x_1) \in \arg \max_{(n,x)\in[1,+\infty)\times[0,X]} n \cdot a(n) \cdot s\Big(n, x, (n_1, n_2)^{(k_1-1,k_2)}, y\Big), \tag{17}$$

and

$$(n_2, x_2) \in \arg \max_{(n,x)\in[1,+\infty)\times[0,X]} n \cdot a(n) \cdot s\Big(n, x, (n_1, n_2)^{(k_1,k_2-1)}, y\Big), \tag{18}$$

where $(n_1, n_2)^{(k_1-1,k_2)}$ denotes the $(k-1)$-dimensional vector whose first $k_1 - 1$ components equal n_1 and the remaining k_2 components equal n_2, and $(n_1, n_2)^{(k_1,k_2-1)}$ the $(k-1)$-dimensional vector whose first k_1 components equal n_1 and the remaining $k_2 - 1$ components equal n_2. Furthermore, the strategies (n_1, x_1) and (n_2, x_2) must yield the same reproductive success, i.e.:

$$n_1 \cdot a(n_1) \cdot s\Big(n_1, x_1, (n_1, n_2)^{(k_1-1,k_2)}, y\Big) = n_2 \cdot a(n_2) \cdot s\Big(n_2, x_2, (n_1, n_2)^{(k_1,k_2-1)}, y\Big). \tag{19}$$

Without loss of generality, assume that $n_1 > n_2$. Now, suppose that one woman deviates from (n_1, x_1) to (n_2, x_2). Then she achieves reproductive success

$$n_2 \cdot a(n_2) \cdot s\Big(n_2, x_2, (n_1, n_2)^{(k_1-1,k_2)}, y\Big), \tag{20}$$

where

$$\begin{aligned} s\Big(n_2, x_2, (n_1, n_2)^{(k_1-1,k_2)}, y\Big) \quad = \quad & \left[\frac{Y-y}{[(k_1-1)\,n_1 + (k_2+1)\,n_2]^{\alpha}}\right]^{\lambda} \cdot \\ & \cdot \left(\frac{X - \phi x_2}{n_2^{\beta}}\right)^{\sigma} \cdot \left[\frac{\rho\gamma x_2}{n_2} + \frac{\theta y}{(k_1-1)\,n_1 + (k_2+1)\,n_2}\right]^{\tau}. \end{aligned} \tag{21}$$

Since $n_1 > n_2$, this is strictly greater than

$$s\Big(n_2, x_2, (n_1, n_2)^{(k_1,k_2-1)}, y\Big) = \left[\frac{Y-y}{(k_1 n_1 + k_2 n_2)^{\alpha}}\right]^{\lambda} \cdot \left(\frac{X - \phi x_2}{n_2^{\beta}}\right)^{\sigma} \cdot \left[\frac{\rho\gamma x_2}{n_2} + \frac{\theta y}{k_1 n_1 + k_2 n_2}\right]^{\tau}.$$

Together with (19), this in turn implies

$$n_2 \cdot a(n_2) \cdot s\Big(n_2, x_2, (n_1, n_2)^{(k_1-1,k_2)}, y\Big) > n_1 \cdot a(n_1) \cdot s\Big(n_1, x_1, (n_1, n_2)^{(k_1-1,k_2)}, y\Big),$$

which contradicts (17).

The above arguments prove that there exists no Nash equilibrium strategy profile in which women employ two different strategies. Similar arguments can be used to show that there exists no Nash equilibrium strategy profile in which women employ three or more different strategies.

Having thus proved that that any Nash equilibrium strategy profile is female-symmetric, I proceed to characterize the set of Nash equilibria. Any female-symmetric strategy profile (n^*, x^*, y^*) is a Nash equilibrium strategy profile if and only if

$$\left\{ \begin{array}{c} n^* \in \arg\max_{n\in[1,+\infty)} F\left(n, x^*, (n^*)^{(k-1)}, y^*\right) \\ x^* \in \arg\max_{x\in[0,X]} F\left(n^*, x, (n^*)^{(k-1)}, y^*\right) \\ y^* \in \arg\max_{y\in[0,Y]} M\left(\boldsymbol{n}^*, \boldsymbol{x}^*, y\right), \end{array} \right. \tag{22}$$

where $(n^*)^{(k-1)}$ denotes the $(k-1)$-dimensional vector whose components all equal n^*.

Define the mappings $g : [1, +\infty) \to \mathbb{R}_+$ and $\tilde{s} : [0, X] \times [0, Y] \to [0, 1]$ by

$$g(n) = n^{1-\alpha\lambda-\beta\sigma-\tau} \cdot a(n) \tag{23}$$

and

$$\tilde{s}(x, y) = (Y - y)^\lambda \cdot (X - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^\tau, \tag{24}$$

respectively. Then note that given that all women choose n^*, a woman's reproductive success is separable in her time allocation x and the number of children n^*, since F can then be written:

$$F\left(n^*, x, (n^*)^{(k-1)}, y^*\right) = k^{-\alpha\lambda} \cdot g(n^*) \cdot \tilde{s}(x, y^*). \tag{25}$$

Likewise, given that all women choose n^*, the man's reproductive success is also separable in his time allocation y and the number of children per woman n^*:

$$\begin{array}{rcl} M\left(\boldsymbol{n}^*, \boldsymbol{x}^*, y\right) & = & k \cdot F\left(n^*, x, (n^*)^{(k-1)}, y^*\right) \\ & = & k^{1-\alpha\lambda} \cdot g(n^*) \cdot \tilde{s}(x^*, y). \end{array}$$

Since $k^{1-\alpha\lambda} \cdot g(n^*) \geq k^{-\alpha\lambda} \cdot g(n^*) > 0$ for any $k \geq 1$ and any n^* (where the strict inequality follows from revealed preference) the last two equations in (22) are equivalent to

$$\left\{ \begin{array}{l} x^* \in \arg\max_{x\in[0,X]} \tilde{s}(x, y^*) \\ y^* \in \arg\max_{y\in[0,Y]} \tilde{s}(x^*, y), \end{array} \right. \tag{26}$$

implying that any equilibrium female-symmetric household time allocation $(x^*, y^*) \in [0, X] \times [0, Y]$ is independent of the number of children n^* per wife. The next part of the proof characterizes the set of female-symmetric equilibrium household time allocations $(x^*, y^*) \in [0, X] \times [0, Y]$.

To begin, note that $y = Y$ cannot be part of an equilibrium strategy profile. Suppose, to the contrary, that $(x^*, y^*) = (x^*, Y)$ for some $x^* \in [0, X]$. Then $\tilde{s}(x^*, y^*) = 0$ for any $x^* \in [0, X]$, while for any $y \in (0, Y)$, $\tilde{s}(x^*, y) > 0$. Similarly, $(x^*, y^*) = (0, 0)$ cannot be an equilibrium strategy profile. Indeed, $\tilde{s}(0, 0) = 0$, while, for any $y \in (0, Y)$, $\tilde{s}(0, y) > 0$.

Next, it is straightforward to verify that, for each reproductive ecology ω and each $y \in [0, Y)$, $\tilde{s}(x, y)$ is strictly concave in x. Likewise, for each female time allocation $x \in [0, X]$, $\tilde{s}$ is strictly concave in y. Hence, it is sufficient to study the first-order partial derivatives of $\tilde{s}(x, y)$ to determine the best response functions.

Thus, for the man:

$$\begin{aligned}\frac{\partial \tilde{s}(x,y)}{\partial y} &= \frac{\tau\theta}{k}\cdot(Y-y)^{\lambda}\cdot(X-\phi x)^{\sigma}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau-1} \\ &\quad -\lambda(Y-y)^{\lambda-1}(X-\phi x)^{\sigma}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau}.\end{aligned} \tag{27}$$

Dividing this by the strictly positive term $(Y-y)^{\lambda-1}\cdot(X-\phi x)^{\sigma}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x,y)}{\partial y}$ is the same as the sign of

$$Y\theta\tau - k\lambda\rho\gamma x - \theta(\lambda+\tau)y. \tag{28}$$

This expression is strictly decreasing in y. It is non-negative for all $y \in [0,Y]$ iff it is non-negative for $y = Y$, i.e., if $Y\theta\tau - k\lambda\rho\gamma x - \theta(\lambda+\tau) \geq 0$, which is false. The expression in (28) is non-positive for all $y \in [0,Y]$ iff it is non-positive for $y = 0$, i.e., iff $Y\theta\tau - k\lambda\rho\gamma x \leq 0$, or

$$x \geq Y\theta\tau/(k\lambda\rho\gamma) \equiv x_1.$$

Thus, if $x \in (0,x_1)$, there exists a unique y such that the expression in (28) equals zero:

$$y = \frac{Y\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)}x. \tag{29}$$

Hence, the male's best response to the female strategy x (where x is chosen by each of the k women) is:

$$\begin{cases} y^{m} = 0 \text{ if } x \geq x_1 \\ y^{m} = \frac{Y\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)}x \text{ if } x \in [0,x_1]. \end{cases} \tag{30}$$

Turning now to the (representative) woman's best response:

$$\begin{aligned}\frac{\partial \tilde{s}(x,y)}{\partial x} &= \tau\rho\gamma\cdot(Y-y)^{\lambda}\cdot(X-\phi x)^{\sigma}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau-1} \\ &\quad -(Y-y)^{\lambda}\cdot\phi\sigma\cdot(X-\phi x)^{\sigma-1}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau}.\end{aligned} \tag{31}$$

Dividing this by the strictly positive term $(Y-y)^{\lambda}\cdot(X-\phi x)^{\sigma-1}\cdot\left(\rho\gamma x+\frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x,y)}{\partial x}$ is the same as the sign of

$$Xk\tau\rho\gamma - (\sigma+\tau)k\rho\gamma\phi x - \sigma\phi\theta y. \tag{32}$$

This expression is strictly decreasing in x. It is non-negative for all $x \in [0,X]$ iff it is non-negative for $x = X$, i.e., if $Xk\tau\rho\gamma - (\sigma+\tau)k\rho\gamma\phi X - \sigma\phi\theta y \geq 0$, i.e., iff

$$y \leq \left[\frac{k\tau\rho\gamma}{\sigma\phi\theta} - \frac{(\sigma+\tau)k\rho\gamma}{\sigma\theta}\right]X \equiv y_0.$$

Note that $y_0 \geq 0$ iff $\frac{\tau}{\tau+\sigma} \geq \phi$. The expression in (32) is non-positive for all $x \in [0, X]$ iff it is non-positive for $x = 0$, i.e., iff $Xk\tau\rho\gamma - \sigma\phi\theta y \leq 0$, or

$$y \geq Xk\tau\rho\gamma/(\sigma\phi\theta) \equiv y_1.$$

Thus, if $y \in (\max\{0, y_0\}, y_1)$, there exists a unique x such that the expression in (32) equals zero:

$$x = \frac{X\tau}{(\sigma+\tau)\phi} - \frac{\sigma\theta}{(\sigma+\tau)k\rho\gamma}y. \tag{33}$$

Hence, a female's best response to the male strategy y is:

$$\begin{cases} x^f = 0 \text{ if } y \geq y_1 \\ x^f = \frac{X\tau}{(\sigma+\tau)\phi} - \frac{\sigma\theta}{(\sigma+\tau)k\rho\gamma}y \text{ if } y \in [\max\{0, y_0\}, y_1] \\ x^f = 1 \text{ if } y \leq \max\{0, y_0\}. \end{cases} \tag{34}$$

Noting that y^m is linear and strictly decreasing in x (whenever $x \in [0, x_1]$), and that x^f is linear and strictly decreasing in y (whenever $y \in [y_0, y_1]$), a necessary and sufficient condition for equilibrium to be unique is that the (absolute value of the) slope of the man's best response curve be strictly smaller than the (absolute value of the) slope of the (representative) woman's best response curve, which is true: $\left|\frac{dy^m}{dx}\right| = \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} < \frac{(\sigma+\tau)k\rho\gamma}{\sigma\theta} = \left|1/\left(\frac{dx^f}{dy}\right)\right|$.

Combining (30) and (34), two cases may be distinguished, depending on whether $y_0 \leq 0$ or $y_0 > 0$. First, if $\frac{\tau}{\sigma+\tau} \leq \phi$ (i.e., if $y_0 \leq 0$),

$$(x^*, y^*) = \begin{cases} \left(\frac{X\tau}{\phi(\sigma+\tau)}, 0\right) \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\phi(\sigma+\tau)} \cdot \frac{X}{Y} \\ \left(\frac{X(\lambda+\tau)\rho\gamma k - Y\sigma\phi\theta}{\rho\gamma k\phi(\lambda+\sigma+\tau)}, \frac{Y\theta\phi(\sigma+\tau) - X\rho\gamma\lambda k}{\theta\phi(\lambda+\sigma+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\phi(\sigma+\tau)} \cdot \frac{X}{Y}, \frac{k(\lambda+\tau)}{\sigma\phi} \cdot \frac{X}{Y}\right] \\ \left(0, \frac{Y\tau}{\lambda+\tau}\right) \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\sigma\phi} \cdot \frac{X}{Y}. \end{cases} \tag{35}$$

Second, if $\frac{\tau}{\sigma+\tau} > \phi$ (i.e., if $y_0 > 0$)

$$(x^*, y^*) = \begin{cases} (X, 0) \text{ if } \frac{\theta}{\rho\gamma} \leq \frac{k\lambda}{\tau} \cdot \frac{X}{Y} \\ \left(X, \frac{Y\tau\theta - Xk\lambda\rho\gamma}{\theta(\lambda+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k\lambda}{\tau} \cdot \frac{X}{Y}, \frac{k[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}{\sigma\phi} \cdot \frac{X}{Y}\right] \\ \left(\frac{X(\lambda+\tau)\rho\gamma k - Y\sigma\phi\theta}{\rho\gamma k\phi(\lambda+\sigma+\tau)}, \frac{Y\theta\phi(\sigma+\tau) - X\rho\gamma\lambda k}{\theta\phi(\lambda+\sigma+\tau)}\right) \text{ if } \frac{\theta}{\rho\gamma} \in \left[\frac{k[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}{\sigma\phi} \cdot \frac{X}{Y}, \frac{k(\lambda+\tau)}{\sigma\phi} \cdot \frac{X}{Y}\right] \\ \left(0, \frac{Y\tau}{\lambda+\tau}\right) \text{ if } \frac{\theta}{\rho\gamma} \geq \frac{k(\lambda+\tau)}{\sigma\phi} \cdot \frac{X}{Y}. \end{cases} \tag{36}$$

The last part of the proof concerns the equilibrium number of children per woman. To begin, note that if there is only one wife ($k = 1$) she chooses the number of children n that maximizes

$$n^{1-\alpha\lambda-\beta\sigma-\tau} \cdot a(n) \cdot (Y - y^*)^{\lambda} \cdot (X - \phi x^*)^{\sigma} \cdot (\rho\gamma x^* + \theta y^*)^{\tau}, \tag{37}$$

or $g(n) \cdot \tilde{s}(x^*, y^*)$, where $g(n)$ was defined in (23). Since (x^*, y^*) does not depend on n, this amounts to choosing n to maximize $g(n)$. If interior ($n > 1$), the solution must satisfy the first-order condition

$$g'(n) = (1 - \lambda\alpha - \sigma\beta - \tau)(1 - bn) \cdot n^{-\lambda\alpha-\sigma\beta-\tau} - bn^{1-\lambda\alpha-\sigma\beta-\tau} = 0. \tag{38}$$

Hence, two cases arise. First, if $1-\lambda\alpha-\sigma\beta-\tau \leq 0$, then $g'(n) < 0$ for all $n \in [1,+\infty)$, in which case $n^* = 1$. Second, if $1-\lambda\alpha-\sigma\beta-\tau > 0$, then the second derivative

$$g''(n) = (1-\lambda\alpha-\sigma\beta-\tau)\left[-(\lambda\alpha+\sigma\beta+\tau)(1-bn)n^{-\lambda\alpha-\sigma\beta-\tau-1} - 2bn^{-\lambda\alpha-\sigma\beta-\tau}\right] \tag{39}$$

is strictly negative, so that either $n^* = 1$ (if $g'(1) \leq 0$), or there exists $n > 1$ that satisfies the necessary first-order condition for an interior solution, $g'(n) = 0$. Since $g'(n) = 0$ iff $n = \frac{1-\lambda\alpha-\sigma\beta-\tau}{(2-\lambda\alpha-\sigma\beta-\tau)b}$, the solution in the case $k = 1$ writes:

$$n^*(1) = \max\left\{1, \frac{1-\lambda\alpha-\sigma\beta-\tau}{(2-\lambda\alpha-\sigma\beta-\tau)b}\right\}. \tag{40}$$

Finally, if $k > 1$, an interior equilibrium $n^* > 1$ must satisfy the first-order condition:

$$\left[g'(n)\cdot s\left(n, x^*, (n^*)^{(k-1)}, y^*\right) + g(n)\cdot s_1\left(n, x^*, (n^*)^{(k-1)}, y^*\right)\right]_{|n=n^*} = 0, \tag{41}$$

where s_1 denotes the partial derivative of s with respect to its first argument. Since $s(\cdot) > 0$, $g(n) > 0$, and $s_1(\cdot) < 0$, this equation implies that at an interior equilibrium n^*, $g'(n^*) > 0$, which in turn implies that for any k for which $n^*(k) > 0$, $n^*(k) < n^*(1)$. Note that this further means that if if $1-\lambda\alpha-\sigma\beta-\tau \leq 0$, then $n^*(k) = 1$ for all $k \geq 1$. Finally, note that since the absolute value of s_1 increases while s decreases as k increases (*ceteris paribus*), this equation further implies that $n^*(k)$ is strictly decreasing in k.

6.2 Proof of Proposition 3

The proof is in two steps. I begin by proving that even if a man could impose his most preferred female-symmetric child production allocation on his wives, his reproductive success would not necessarily be increasing in the number of wives. I will then show that this result, together with Proposition 1, implies that his reproductive success would not necessarily be increasing in the number of wives under the assumption that women do have agency over their fertility and time allocation decisions. This modeling strategy simplifies the calculations (note that it also delivers one robustness check as a by-product).

Step 1: Analysis for a (hypothetical) man who would be able to choose his preferred child production allocation.

Consider a man who has k wives, and who chooses n and (x,y) to maximize

$$\widehat{M}(n,x,y) = k\cdot n\cdot a(n)\cdot\left[\frac{Y-y}{(kn)^{\alpha}}\right]^{\lambda}\cdot\left(\frac{X-\phi x}{n^{\beta}}\right)^{\sigma}\cdot\left(\frac{\rho\gamma x}{n}+\frac{\theta y}{kn}\right)^{\tau}. \tag{42}$$

Then:

Lemma 1. *For any ecology ω, any number of wives $k \geq 1$, and any amount of labor resources $Y \geq 0$, there exists a unique female-symmetric child production allocation $(\widehat{n},\widehat{x},\widehat{y})$ that maximizes male reproductive success $\widehat{M}(x,y,n)$. Furthermore, $\widehat{x} = x^*$ and $\widehat{y} = y^*$.*

Proof: To begin, note that the man's maximization problem can be treated as two separate maximization problems, one for n and one for (x,y), since the objective function in (42) can be written

$$\widehat{M}(n,x,y) = k^{1-\alpha\lambda} \cdot g(n) \cdot \tilde{s}(x,y), \tag{43}$$

where $g(n)$ was defined in (23) and $\tilde{s}(x,y)$ in (24). Specifically, choosing $(n,x,y) \in [1,+\infty) \times [0,X] \times [0,Y]$ to maximize $\widehat{M}(n,x,y)$ boils down to choosing $n \in [1,+\infty)$ to maximize $g(n)$ and $(x,y) \in [0,X] \times [0,Y]$ to maximize $\tilde{s}(x,y)$. In view of the system of equations (26), which defines the unique Nash equilibrium time allocations x^* and y^* in game Γ, this observation clearly implies that the solution entails setting $x = x^*$ and $y = y^*$.

Turning now to the choice of n, it is immediate from the end of the proof of Proposition 1 (see (40)) that

$$\widehat{n} = \max\left\{1, \frac{1-\lambda\alpha-\sigma\beta-\tau}{(2-\lambda\alpha-\sigma\beta-\tau)\,b}\right\}.$$

Q.E.D.

Writing the man's preferred fertility and time allocations $\widehat{n}$, $\widehat{x}$, and $\widehat{y}$ as functions of the number of wives $k \geq 1$ and the man's time budget Y, the reproductive success that the man achieves can be written as a function of k and Y:

$$\widetilde{M}(k,Y) \equiv \widehat{M}(\widehat{n}(k,Y), \widehat{x}(k,Y), \widehat{y}(k,Y)). \tag{44}$$

The following lemma shows how $\widetilde{M}$ varies with k, holding Y constant. In this proposition,

$$\hat{\alpha} \equiv \frac{\rho\gamma x^*(1,Y) + (1-\tau)\theta y^*(1,Y)}{\lambda\left[\rho\gamma x^*(1,Y) + \theta y^*(1,Y)\right]}, \tag{45}$$

and (for $\alpha\lambda < 1-\tau$) $\hat{k}$ is implictly defined by the equation

$$(1-\lambda\alpha) \cdot \rho\gamma x^*\left(\hat{k},Y\right) = (\tau - 1 + \lambda\alpha) \cdot \frac{\theta y^*\left(\hat{k},Y\right)}{\hat{k}}. \tag{46}$$

Lemma 2. *Consider a man who, for any given number of wives $k \geq 1$ and labor resources Y can choose (n,x,y) so as to achieve reproductive success $\widetilde{M}(k,Y)$ (see (44)). For such a man:*
(i) if $y^*(1,Y) = 0$ or if $\alpha \leq \hat{\alpha}$, then $\frac{\partial \widetilde{M}(k,Y)}{\partial k} \geq 0$ for all $k \geq 1$ (the inequality being strict if $y^*(1,k) = 0$ and $\alpha\lambda \neq 1$, or if $\alpha < \hat{\alpha}$);
(ii) if $y^*(1,k) > 0$ and $\alpha > \hat{\alpha}$, then there exists $\hat{k} > 1$ such that $\frac{\partial \widetilde{M}(k,Y)}{\partial k} < 0$ for all $k \in \left(1,\hat{k}\right)$ and $\frac{\partial \widetilde{M}(k,Y)}{\partial k} > 0$ for all $k > \hat{k}$;
(iii) if $\alpha\lambda = 1$, then $\widetilde{M}(1,Y) \geq \widetilde{M}(k,Y)$ for all $k > 1$, the inequality being strict if and only if $y^*(1,Y) > 0$.

Proof: Using the notation introduced in the proof of Lemma 1, and letting

$$s^*(x^*(k,Y), y^*(k,Y), k) = [Y - y^*(k,Y)]^{\lambda} \cdot [X - \phi x^*(k,Y)]^{\sigma} \cdot \left[\rho\gamma x^*(k,Y) + \frac{\theta y^*(k,Y)}{k}\right]^{\tau}, \tag{47}$$

one obtains the following expression for male reproductive success as a function of k (see equation (44)):

$$\begin{aligned}\widetilde{M}(k,Y) &= M\left(x^*(k,Y), y^*(k,Y), \widehat{n}, k\right) \\ &= g(\widehat{n}) \cdot k^{1-\lambda\alpha} \cdot s^*\left(x^*(k,Y), y^*(k,Y), k\right),\end{aligned} \tag{48}$$

where $\widehat{n}$ is defined in (40). Upon dividing the expression in (48) by the strictly positive constant $g(\widehat{n})$, one obtains that $\frac{\partial \widetilde{M}(k,Y)}{\partial k}$ has the same sign as

$$\begin{aligned}&(1-\lambda\alpha) \cdot k^{-\lambda\alpha} \cdot s^*\left(x^*(k,Y), y^*(k,Y), k\right) \\ &+k^{1-\lambda\alpha} \cdot \frac{ds^*\left(x^*(k,Y), y^*(k,Y), k\right)}{dk},\end{aligned} \tag{49}$$

where

$$\begin{aligned}\frac{ds^*\left(x^*(k,Y), y^*(k,Y), k\right)}{dk} &= \left.\frac{\partial \tilde{s}(x,y)}{\partial x}\right|_{(x,y)=(x^*(k,Y),y^*(k,Y))} \cdot \frac{\partial x^*(k,Y)}{\partial k} \\ &+ \left.\frac{\partial \tilde{s}(x,y)}{\partial y}\right|_{(x,y)=(x^*(k,Y),y^*(k,Y))} \cdot \frac{\partial y^*(k,Y)}{\partial k} \\ &+ \frac{\partial s^*\left(x^*(k,Y), y^*(k,Y), k\right)}{\partial k}\end{aligned} \tag{50}$$

(for the definition of $\tilde{s}(x,y)$, see (24)). From the proof of Proposition 1, one obtains

$$\left.\frac{\partial \tilde{s}(x,y)}{\partial x}\right|_{(x,y)=(x^*(k,Y),y^*(k,Y))} \cdot \frac{\partial x^*(k,Y)}{\partial k} = \left.\frac{\partial \tilde{s}(x,y)}{\partial y}\right|_{(x,y)=(x^*(k,Y),y^*(k,Y))} \cdot \frac{\partial y^*(k,Y)}{\partial k} = 0,$$

so that (50) reduces to

$$\begin{aligned}\frac{ds^*\left(x^*(k,Y), y^*(k,Y), k\right)}{dk} &= \frac{\partial s^*\left(x^*(k,Y), y^*(k,Y), k\right)}{\partial k} \\ = -\frac{\tau\theta y^*(k,Y)}{k^2} \cdot \left[Y - y^*(k,Y)\right]^{\lambda} &\cdot \left[X - \phi x^*(k,Y)\right]^{\sigma} \cdot \left[\rho\gamma x^*(k,Y) + \frac{\theta y^*(k,Y)}{k}\right]^{\tau-1} \\ = -\frac{\tau\theta y^*(k,Y)}{k^2} &\cdot \left[\rho\gamma x^*(k,Y) + \frac{\theta y^*(k,Y)}{k}\right]^{-1} \cdot s^*\left(x^*(k,Y), y^*(k,Y), k\right).\end{aligned} \tag{51}$$

Plugging this into (49), dividing by $s^*\left(x^*(k,Y), y^*(k,Y), k\right) \cdot k^{-\lambda\alpha} \cdot \left[\rho\gamma x^*(k,Y) + \frac{\theta y^*(k,Y)}{k}\right]^{-1}$ (which is strictly positive), and rearranging the terms, one obtains that $\frac{\partial \widetilde{M}(k,Y)}{\partial k}$ has the same sign as

$$A(k) \equiv (1-\lambda\alpha) \cdot \left[\rho\gamma x^*(k,Y) + \frac{\theta y^*(k,Y)}{k}\right] - \tau \cdot \frac{\theta y^*(k,Y)}{k}. \tag{52}$$

Note the following:

Remark 1. For any $(\alpha, \lambda) \in [0,1] \times (0,1]$, $1 - \lambda\alpha \geq 0$. Furthermore, $1 - \lambda\alpha = 0$ if and only if $\alpha = \lambda = 1$, in which case $A(k)$ has the same sign as $-y^*(k,Y)$. The remaining remarks all pertain to the case $\alpha \cdot \lambda \neq 1$.

Remark 2. The term $\rho\gamma x^*(k,Y)+\frac{\theta y^*(k,Y)}{k}$, which is the total amount of food produced for each brood of $\widehat{n}$ children, writes

$$\rho\gamma x^*(k,Y)+\frac{\theta y^*(k,Y)}{k}=\begin{cases}\frac{X\rho\gamma\tau}{\phi(\sigma+\tau)} \text{ if } k\geq\frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda}\cdot\frac{Y}{X}\\ \frac{X\tau\rho\gamma k+Y\theta\phi}{k\phi(\lambda+\sigma+\tau)} \text{ if } k\in\left[\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}\cdot\frac{Y}{X},\frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda}\cdot\frac{Y}{X}\right]\\ \frac{Y\theta\tau}{k(\lambda+\tau)} \text{ if } k\leq\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}\cdot\frac{Y}{X}\end{cases}\tag{53}$$

if $\frac{\tau}{\sigma+\tau}\leq\phi$, and

$$\rho\gamma x^*(k,Y)+\frac{\theta y^*(k,Y)}{k}=\begin{cases}X\rho\gamma \text{ if } k\geq\frac{\theta\tau}{\rho\gamma\lambda}\cdot\frac{Y}{X}\\ \frac{X\tau\rho\gamma k+Y\theta\tau}{k(\lambda+\tau)} \text{ if } k\in\left[\frac{\theta\phi\sigma}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}\cdot\frac{Y}{X},\frac{\theta\tau}{\rho\gamma\lambda}\cdot\frac{Y}{X}\right]\\ \frac{X\tau\rho\gamma k+Y\theta\phi}{k\phi(\lambda+\sigma+\tau)} \text{ if } k\in\left[\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}\cdot\frac{Y}{X},\frac{\theta\phi\sigma}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}\cdot\frac{Y}{X}\right]\\ \frac{Y\theta\tau}{k(\lambda+\tau)} \text{ if } k\leq\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}\cdot\frac{Y}{X}\end{cases}\tag{54}$$

if $\frac{\tau}{\sigma+\tau}>\phi$. In either case, this term is always strictly positive. Furthermore, it is strictly decreasing in k whenever $y^*(k,Y)>0$ and constant in k whenever $y^*(k,Y)=0$. For further use below, let $\tilde{k}\geq 0$ denote the threshold value such that $y^*(k,Y)>0$ iff $k<\tilde{k}$, and $y^*(k,Y)=0$ iff $k\geq\tilde{k}$ (note that $\tilde{k}$ may be smaller than 1):

$$\tilde{k}\equiv\begin{cases}\frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda}\cdot\frac{Y}{X}\text{if } \frac{\tau}{\sigma+\tau}\leq\phi\\ \frac{\theta\tau}{\rho\gamma\lambda}\cdot\frac{Y}{X} \text{ if}\frac{\tau}{\sigma+\tau}>\phi.\end{cases}\tag{55}$$

Remark 3. $\frac{y^*(k,Y)}{k}$ is strictly decreasing in k for any $k<\tilde{k}$ and constant in k for any $k\geq\tilde{k}$.

Remark 4. Suppose that $\tilde{k}>1$. Then $A(k)$ changes sign at most once for $k\in[1,\tilde{k})$. Indeed, suppose that there exists some $\hat{k}\in[1,\tilde{k})$ such that $A\left(\hat{k}\right)=0$, i.e.,

$$(1-\lambda\alpha)\cdot\rho\gamma x^*\left(\hat{k},Y\right)=(\tau-1+\lambda\alpha)\cdot\frac{\theta y^*\left(\hat{k},Y\right)}{\hat{k}}.\tag{56}$$

(Note that $\hat{k}$ must indeed be strictly smaller that $\tilde{k}$ since $A(k)>0$.for any $k\geq\tilde{k}$.) Note that since $\rho\gamma x^*(k,Y)+\frac{\theta y^*(k,Y)}{k}>0$ for all k, and since $(1-\lambda\alpha)\cdot\rho\gamma>0$, both the left-hand side and the right-hand side of this expression must be strictly positive. Then, since $x^*(k,Y)$ is increasing in k and $\frac{\theta y^*\left(\hat{k},Y\right)}{\hat{k}}$ is decreasing in k, it must be that $(1-\lambda\alpha)\cdot\rho\gamma x^*\left(\hat{k},Y\right)>(\tau-1+\lambda\alpha)\cdot\frac{\theta y^*\left(\hat{k},Y\right)}{\hat{k}}$ for any $k>\hat{k}$.

Remarks 1 - 4 together imply:

A. If $\alpha\lambda=1$ and $y^*(1)=0$, then $\frac{\partial\widetilde{M}(k,Y)}{\partial k}=0$ for all $k\geq 1$.

B. If $\alpha\lambda=1$ and $y^*(1)>0$, then $\frac{\partial\widetilde{M}(k,Y)}{\partial k}<0$ for all $k\in[1,\tilde{k})$ and $\frac{\partial\widetilde{M}(k,Y)}{\partial k}=0$ for all $k\geq\tilde{k}$.

C. If $\alpha\lambda<1$ and $y^*(1)=0$, then $\frac{\partial\widetilde{M}(k,Y)}{\partial k}>0$ for all $k\geq 1$.

D. If $\alpha\lambda < 1$ and $y^*(1) > 0$, then $\frac{\partial \widetilde{M}(k,Y)}{\partial k} > 0$ for all $k \geq \tilde{k} > 1$. Moreover, a sufficient condition for $\widetilde{M}$ to be non-monotonic in k is that $\left.\frac{\partial \widetilde{M}(k,Y)}{\partial k}\right|_{k=1} < 0$, which is true if and only if

$$(1-\lambda\alpha)\left[\rho\gamma x^*(1,Y) + \theta y^*(1,Y)\right] < \tau\theta y^*(1,Y). \tag{57}$$

Because $x^*(1,Y)$ and $y^*(1,Y)$ do not depend on α, and since $\rho\gamma x^*(1,Y) + \theta y^*(1,Y) > 0$, the left-hand side can be viewed as an affine and strictly decreasing function of α, which takes the value 0 for $\alpha = 1/\lambda$ and the value $\rho\gamma x^*(1,Y) + \theta y^*(1,Y) > 0$ for $\alpha = 0$. Hence, (57) is equivalent to

$$\alpha > \frac{\rho\gamma x^*(1,Y) + (1-\tau)\theta y^*(1,Y)}{\lambda\left[\rho\gamma x^*(1,Y) + \theta y^*(1,Y)\right]} \equiv \hat{\alpha}. \tag{58}$$

Because $A(k)$ changes sign at most once (see Remark 4), the condition $\alpha > \hat{\alpha}$ is also necessary for $\widetilde{M}$ to be non-monotonic in k. Furthermore, if $\alpha > \hat{\alpha}$, there exists $\hat{k} \in \left(1, \tilde{k}\right)$ such that $\frac{\partial \widetilde{M}(k,Y)}{\partial k} < 0$ for all $k \in [1, \hat{k})$, $\frac{\partial \widetilde{M}(k,Y)}{\partial k} < 0$ if $k = \hat{k}$, and $\frac{\partial \widetilde{M}(k,Y)}{\partial k} > 0$ for all $k > \hat{k}$. Finally, if $\alpha \leq \hat{\alpha}$, $\frac{\partial \widetilde{M}(k,Y)}{\partial k} \geq 0$ for all $k \geq 1$, with a strict inequality for all $k \geq 1$ if and only if $\alpha < \hat{\alpha}$.

Q.E.D.

Step 2: Analysis of the implications of the analysis in Step 1 for the model in the text.

Returning to the case where the unique Nash equilibrium of the non-cooperative game Γ is played in each household, consider the equilibrium male reproductive success $M^*(k,y)$, which can be written:

$$M^*(k,Y) = g(n^*(k)) \cdot k^{1-\lambda\alpha} \cdot s^*(x^*(k,Y), y^*(k,Y), k), \tag{59}$$

where $s^*(\cdot)$ was defined in (47). Hence,

$$\begin{aligned} \frac{dM^*(k,y)}{dk} = {} & g'(n^*(k)) \cdot \frac{dn^*(k)}{dk} \cdot k^{1-\lambda\alpha} \cdot s^*(x^*(k,Y), y^*(k,Y), k) \\ & + g(n^*(k)) \cdot (1-\lambda\alpha) k^{-\lambda\alpha} \cdot s^*(x^*(k,Y), y^*(k,Y), k) \\ & + g(n^*(k)) \cdot k^{1-\lambda\alpha} \cdot \frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial x^*(k)} \cdot \frac{\partial x^*(k,Y)}{\partial k} \\ & + g(n^*(k)) \cdot k^{1-\lambda\alpha} \cdot \frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial y^*(k)} \cdot \frac{\partial y^*(k,Y)}{\partial k} \\ & + g(n^*(k)) \cdot k^{1-\lambda\alpha} \cdot \frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial k}, \end{aligned} \tag{60}$$

an expression which, due to the fact that $\frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial x^*(k)} \cdot \frac{\partial x^*(k,Y)}{\partial k} = \frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial y^*(k)} \cdot \frac{\partial y^*(k,Y)}{\partial k} = 0$, boils down to

$$\begin{aligned} \frac{dM^*(k,Y)}{dk} = {} & g'(n^*(k)) \cdot \frac{dn^*(k)}{dk} \cdot k^{1-\lambda\alpha} \cdot s^*(x^*(k,Y), y^*(k,Y), k) \\ & + g(n^*(k)) \cdot (1-\lambda\alpha) k^{-\lambda\alpha} \cdot s^*(x^*(k,Y), y^*(k,Y), k) \\ & + g(n^*(k)) \cdot k^{1-\lambda\alpha} \cdot \frac{\partial s^*(x^*(k,Y), y^*(k,Y), k)}{\partial k}. \end{aligned} \tag{61}$$

Recalling that

$$\begin{aligned}\frac{d\widetilde{M}(k,Y)}{dk} &= g(\widehat{n})\cdot(1-\lambda\alpha)k^{-\lambda\alpha}\cdot s^*(x^*(k,Y),y^*(k,Y),k) \\ &\quad +g(\widehat{n})\cdot k^{1-\lambda\alpha}\cdot\frac{\partial s^*(x^*(k,Y),y^*(k,Y),k)}{\partial k},\end{aligned} \tag{62}$$

and noting that $M^*(1,Y)=\widetilde{M}(1,Y)$, the following two cases arise.

Case 1: $1-\lambda\alpha-\sigma\beta-\tau\leq 0$. Then $n^*(k)=\widehat{n}=1$ for all $k\geq 1$, so that $g'(n^*(k))=0$ for all k. Comparison of (61) to (62), together with the fact that $M^*(1,Y)=\widetilde{M}(1,Y)$, then immediately implies that $\frac{dM^*(k,Y)}{dk}=\frac{d\widetilde{M}(k,Y)}{dk}$ for all $k\geq 1$.

Case 2: $1-\lambda\alpha-\sigma\beta-\tau>0$. Then, for all $k>1$, $n^*(k)<\widehat{n}=n^*(1)$, and strict concavity of g (see the end of the proof of Proposition 1) implies $g'(n^*(k))>0$. Furthermore, recall that $g(n^*(k))<g(\widehat{n})$ (since g is strictly concave and $\widehat{n}\in\arg\max_{n\in[1,+\infty)}g(n)$) and that $\frac{dn^*(k)}{dk}<0$ (see the end of the proof of Proposition 1). Comparison of (61) to (62) then allows me to conclude that $\frac{dM^*(k,Y)}{dk}<\frac{d\widetilde{M}(k,Y)}{dk}$ for all $k\geq 1$.

The statement in the proposition follows from these results together with Lemma 2.

6.3 Proof of Proposition 5

Claim *(i)*: $\zeta=0$ is evolutionarily stable if and only if the ecology ω is such that $M^*(1,Y)>M^*(2,Y)$. Suppose that $\zeta=0$ is the resident strategy and that a share $\varepsilon>0$ of the population carries the mutant strategy $\zeta'\neq 0$. The average reproductive success of a resident (who settles down with his teenage girlfriend when matched with another resident and loses with certainty when matched with a mutant who *Competes*) is then

$$(1-\varepsilon)\cdot M^*(1,Y)+\varepsilon\cdot[\zeta'\cdot 0+(1-\zeta')\cdot M^*(1,Y)], \tag{63}$$

while that of a mutant (who wins with certainty against a resident when he plays *Compete* and with probability 1/2 when matched with another mutant who plays *Compete*) is

$$\begin{aligned}&\zeta'\cdot\left[[1-\varepsilon+\varepsilon(1-\zeta')]\cdot M^*(2,Y)+\varepsilon\zeta'\cdot\frac{M^*(2,Y)}{2}\right] \\ &+(1-\zeta')\cdot[(1-\varepsilon+\varepsilon(1-\zeta'))\cdot M^*(1,Y)+\varepsilon\zeta'\cdot 0].\end{aligned} \tag{64}$$

Strategy $\zeta=0$ is evolutionarily stable against $\zeta'=1$ iff there exists some $\overline{\varepsilon}>0$ such that for all $\varepsilon\in(0,\overline{\varepsilon})$ the expression in (63) is strictly larger than the expression in (64). Clearly, by virtue of the continuity of the expressions in ε, a sufficient condition is that when evaluated at $\varepsilon=0$ the expression in (63) is strictly larger than that in (64), a condition which reduces to $M^*(1,Y)>M^*(2,Y)$.

I will now show that it is also a necessary condition. Suppose, to the contrary, that $M^*(1,Y)\leq M^*(2,Y)$. Then, continuity in ε of the expressions in (63) and (64) implies that for $\zeta=0$ to be evolutionarily stable against $\zeta'\in(0,1]$, when evaluated at $\varepsilon=0$ the expression in (63) must equal that in (64), i.e, $M^*(1,Y) = M^*(2,Y)$, and, moreover, the difference between the expression in (63) and that in (64) must be strictly increasing in ε. This difference equals:

$$(1-\varepsilon\zeta')\cdot M^*(1,Y) - \zeta'\cdot(1-\varepsilon\zeta')\cdot M^*(2,Y) - \varepsilon(\zeta')^2\cdot\frac{M^*(2,Y)}{2} - (1-\zeta')\cdot(1-\varepsilon\zeta')\cdot M^*(1,Y),$$

an expression which reduces to $-\varepsilon(\zeta')^2\cdot\frac{M^*(2,Y)}{2}$ once $M^*(1,Y)$ is replaced by $M^*(2,Y)$. In sum, for $\zeta = 0$ to be evolutionarily stable against $\zeta' \in (0,1]$ when $M^*(1,Y) = M^*(2,Y)$, $-\frac{M^*(2,Y)}{2}$ must be strictly positive. Since this is false, this concludes the proof of the claim.

Claim *(ii)*: there exists no mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable. Suppose, to the contrary, that there exists some mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable, and note first that ζ must then be such that the two pure strategies *Compete* and *Surrender* yield the same reproductive success, i.e.:

$$(1-\zeta)\cdot M^*(1,Y) + \zeta\cdot 0 = (1-\zeta)\cdot M^*(2,Y) + \zeta\cdot\frac{M^*(2,Y)}{2}. \tag{65}$$

Let Z denote the value of this reproductive success.

Assume now that ζ is the resident strategy, and consider the mutant strategy $\zeta' = 1$, represented in a share ε of the population. Then, residents get, on average, reproductive success equal to

$$(1-\varepsilon)\cdot Z + \varepsilon\cdot\zeta\cdot\frac{M^*(2,Y)}{2} \tag{66}$$

while mutants achieve an average reproductive success of

$$(1-\varepsilon)\cdot Z + \varepsilon\cdot\frac{M^*(2,Y)}{2}. \tag{67}$$

Since $M^*(2,Y) > 0$, the expression in (67) strictly exceeds that in (66) for any $\zeta \in (0,1)$ and any $\varepsilon \in (0,1)$. Hence, there does not exist any $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0,\bar{\varepsilon})$ the resident strategy ζ achieves a strictly higher reproductive success than the mutant strategy $\zeta' = 1$.

Claim *(iii)*: $\zeta = 1$ is evolutionarily stable in any ecology $\omega \in \Omega$. Suppose that $\zeta = 1$ is the resident strategy and that a share $\varepsilon > 0$ of the population carries the mutant strategy $\zeta' = 0$. The average reproductive success of a resident (who wins with probability 1/2 when matched with another individual playing *Compete* and with certainty when matched with an individual playing *Surrender*) is then

$$(1-\varepsilon)\cdot\frac{M^*(2,Y)}{2} + \varepsilon\cdot\left[\zeta'\cdot\frac{M^*(2,Y)}{2} + (1-\zeta')\cdot M^*(2,Y)\right], \tag{68}$$

while that of a mutant is

$$\zeta'\cdot[(1-\varepsilon)+\varepsilon\cdot\zeta']\cdot\frac{M^*(2,Y)}{2} + (1-\zeta')\cdot\varepsilon(1-\zeta')\cdot M^*(1,Y). \tag{69}$$

By continuity of these expressions in ε, a sufficient condition for $\zeta = 1$ to be evolutionarily stable against $\zeta' \in [0,1)$ is that, when evaluated at $\varepsilon = 0$, the expression in (68) be strictly greater than that in (69). This condition writes

$$\frac{M^*(2,Y)}{2} > \zeta'\cdot\frac{M^*(2,Y)}{2}, \tag{70}$$

which is true for any $\zeta' \in [0,1)$.

6.4 Proof of Proposition 6

Claim *(i)*: $\zeta = 0$ is evolutionarily stable if and only if

- **either the ecology ω is such that $M^*(1,Y) > M^*(2,Y)$ and $\varphi < Y + A$, where $A > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y+A)$;**

- **or the ecology ω is such that $M^*(1,Y) < M^*(2,Y)$ and $\varphi < Y - B$, where $B > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y-B)$.**

Suppose that $\zeta = 0$ is the resident strategy and that a share $\varepsilon > 0$ of the population carries the mutant strategy $\zeta' \neq 0$. The average reproductive success of a resident (who settles down with his teenage girlfriend when matched with another resident and loses with certainty when matched with a mutant who *Competes*) is then

$$(1-\varepsilon)\cdot M^*(1,Y) + \varepsilon \cdot [(1-\zeta')\cdot M^*(1,Y) + \zeta' \cdot 0], \tag{71}$$

while that of a mutant (who wins with certainty against a resident when he plays *Compete* and with probability 1/2 when matched with another mutant who plays *Compete*) is

$$\begin{aligned} &\zeta' \cdot \left[[1-\varepsilon+\varepsilon(1-\zeta')]\cdot M^*(2,\varphi) + \varepsilon\zeta' \cdot \frac{M^*(2,\varphi)}{2} \right] \\ &+(1-\zeta')\cdot[(1-\varepsilon)\cdot M^*(1,Y) + \varepsilon\cdot[(1-\zeta')\cdot M^*(1,Y)+\zeta'\cdot 0]]. \end{aligned} \tag{72}$$

Strategy $\zeta = 0$ is evolutionarily stable against $\zeta' \in (0,1]$ if and only if there exists some $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0,\overline{\varepsilon})$ the difference between the expression in (71) and the expression in (72) is strictly positive. Clearly, by virtue of the continuity of this difference in ε, a necessary condition is that it be non-negative when evaluated at $\varepsilon = 0$, a condition which reduces to

$$M^*(1,Y) \geq M^*(2,\varphi). \tag{73}$$

Continuity further implies that this condition is sufficient if it holds strictly:

$$M^*(1,Y) > M^*(2,\varphi). \tag{74}$$

Since M^* is continuous and strictly increasing in its second argument, condition (74) is equivalent to the following pair of statements:

- either the ecology ω is such that $M^*(1,Y) > M^*(2,Y)$ and $\varphi < Y + A$, where $A > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y+A)$;

- or the ecology ω is such that $M^*(1,Y) < M^*(2,Y)$ and $\varphi < Y - B$, where $B > 0$ is the amount of male resources such that $M^*(1,Y) = M^*(2,Y-B)$.

I will now show that the condition $M^*(1,Y) > M^*(2,\varphi)$ is also necessary for $\zeta = 0$ to be evolutionarily stable. Suppose, to the contrary, that $M^*(1,Y) \leq M^*(2,\varphi)$. Then, the necessary condition (73) implies that $M^*(1,Y) = M^*(2,\varphi)$. Furthermore, the difference between the expression in (71) and that in (72) must be strictly increasing in ε. This difference equals:

$$(1-\varepsilon\zeta')\cdot M^*(1,Y) - \zeta'\cdot(1-\varepsilon\zeta')\cdot M^*(2,\varphi) - \varepsilon(\zeta')^2\cdot\frac{M^*(2,\varphi)}{2} - (1-\zeta')\cdot(1-\varepsilon\zeta')\cdot M^*(1,Y),$$

an expression which reduces to $-\varepsilon(\zeta')^2 \cdot \frac{M^*(2,\varphi)}{2}$ once $M^*(1,Y)$ is replaced by $M^*(2,\varphi)$. In sum, for $\zeta = 0$ to be evolutionarily stable against $\zeta' \in (0,1]$ when $M^*(1,Y) = M^*(2,\varphi)$, $-\frac{M^*(2,\varphi)}{2}$ must be strictly positive. Since this is false, this concludes the proof of the claim.

Claim *(ii)*: there exists no mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable. Suppose, to the contrary, that there exists some mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable. Then, ζ is such that the two pure strategies *Compete* and *Surrender* yield the same reproductive success, i.e.:

$$(1-\zeta)\cdot M^*(1,Y) + \zeta \cdot 0 = (1-\zeta)\cdot M^*(2,\varphi) + \zeta \cdot \frac{M^*(2,\varphi)}{2}. \tag{75}$$

Let Z denote the value of this reproductive success. Assume now that ζ is the resident strategy, and consider the mutant strategy $\zeta' = 1$, represented in a share ε of the population. Then, residents get, on average, reproductive success equal to

$$(1-\varepsilon)\cdot Z + \varepsilon \cdot \zeta \cdot \frac{M^*(2,\varphi)}{2} \tag{76}$$

while mutants achieve an average reproductive success of

$$(1-\varepsilon)\cdot Z + \varepsilon \cdot \frac{M^*(2,\varphi)}{2}. \tag{77}$$

Since $M^*(2,\varphi) > 0$, the expression in (77) strictly exceeds that in (76) for any $\zeta \in (0,1)$ and any $\varepsilon \in (0,1)$. Hence, there does not exist any $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0,\bar{\varepsilon})$ the resident strategy ζ achieves a strictly higher reproductive success than the mutant strategy $\zeta' = 1$.

Claim *(iii)*: $\zeta = 1$ is evolutionarily stable in any ecology $\omega \in \Omega$. Suppose that $\zeta = 1$ is the resident strategy and that a share $\varepsilon > 0$ of the population carries the mutant strategy $\zeta' = 0$. The average reproductive success of a resident (who wins with probability 1/2 when matched with another individual playing *Compete* and with certainty when matched with an individual playing *Surrender*) is then

$$(1-\varepsilon)\cdot \frac{M^*(2,\varphi)}{2} + \varepsilon \cdot [\zeta' \cdot \frac{M^*(2,\varphi)}{2} + (1-\zeta')\cdot M^*(2,\varphi)], \tag{78}$$

while that of a mutant is

$$\zeta' \cdot [(1-\varepsilon) + \varepsilon \cdot \zeta'] \cdot \frac{M^*(2,\varphi)}{2} + (1-\zeta')\cdot \varepsilon(1-\zeta')\cdot M^*(1,Y). \tag{79}$$

By continuity of these expressions in ε, a sufficient condition for $\zeta = 1$ to be evolutionarily stable against $\zeta' \in [0,1)$ is that, when evaluated at $\varepsilon = 0$, the expression in (78) be strictly greater than that in (79). This condition writes

$$\frac{M^*(2,\varphi)}{2} > \zeta' \cdot \frac{M^*(2,\varphi)}{2}, \tag{80}$$

which is true for any $\zeta' \in [0,1)$.

6.5 Proof of Proposition 7

Claim *(i)*: $\zeta = 0$ is evolutionarily stable if and only if the ecology ω is such that $M^*(1,Y) > M^*(2,Y)$ and $\varphi - Y$ is not too large. Suppose that $\zeta = 0$ is the resident strategy and that a share $\varepsilon > 0$ of the population carries the mutant strategy $\zeta' \neq 0$. The average reproductive success of a resident (who settles down with his teenage girlfriend when matched with another resident and loses with certainty when matched with a mutant who *Competes*) is then

$$\Pr[0|0,\varepsilon] \cdot M^*(1,Y) + \Pr[\zeta'|0,\varepsilon] \cdot [\zeta' \cdot 0 + (1-\zeta') \cdot M^*(1,Y)], \tag{81}$$

while that of a mutant (who wins with certainty against a resident when he plays *Compete* and with probability 1/2 when matched with another mutant who plays *Compete*) is

$$\begin{aligned} &\zeta' \cdot \left[[\Pr[0|\zeta',\varepsilon] + \Pr[\zeta'|\zeta',\varepsilon](1-\zeta')] \cdot M^*(2,\varphi) + \Pr[\zeta'|\zeta',\varepsilon] \cdot \zeta' \cdot \frac{M^*(2,\varphi)}{2} \right] \\ &+(1-\zeta') \cdot [(\Pr[0|\zeta',\varepsilon] + \Pr[\zeta'|\zeta',\varepsilon](1-\zeta')) \cdot M^*(1,Y) + \Pr[\zeta'|\zeta',\varepsilon]\zeta' \cdot 0]. \end{aligned} \tag{82}$$

Strategy $\zeta = 0$ is evolutionarily stable against $\zeta' \in (0,1]$ if and only if there exists some $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon})$ the expression in (81) is strictly larger than the expression in (82). Clearly, by virtue of the continuity of the expressions in ε (recall that the conditional probabilities are continuous in ε), a sufficient condition is that when evaluated at $\varepsilon = 0$ the expression in (81) is strictly larger than that in (82), a condition which reduces to

$$M^*(1,Y) > M^*(2,\varphi) - [(1-\zeta')\sigma \cdot M^*(1,Y) + \sigma\zeta'/2 \cdot M^*(2,\varphi)] \tag{83}$$

where $\sigma = \lim_{\varepsilon \to 0} \Pr[\zeta'|\zeta',\varepsilon]$. Note now that for any $\sigma \in (0,1]$ and any $\zeta' \in (0,1]$ the term inside the square brackets in inequality (83) is strictly positive, and compare this inequality to inequality (73) in the Proof of Proposition 6. Recalling the definitions of A and B in that proof, I can thus conclude that, for any $\zeta' \in (0,1]$:

- if the ecology ω is such that $M^*(1,Y) > M^*(2,Y)$, then there exists some $A'(\zeta') > A > 0$ such that strategy $\zeta = 0$ is evolutionarily stable against ζ' if $\varphi < Y + A'(\zeta')$;
- if the ecology ω is such that $M^*(1,Y) < M^*(2,Y)$, then there exists some $B'(\zeta')$, where $B > B'(\zeta') > 0$, such that strategy $\zeta = 0$ is evolutionarily stable against ζ' if $\varphi < Y - B'(\zeta')$.

Claim *(ii)*: there exists no mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable. Suppose, to the contrary, that there exists some mixed strategy $\zeta \in (0,1)$ which is evolutionarily stable. Then, ζ must be such that the two pure strategies *Compete* and *Surrender* yield the same reproductive success, i.e.:

$$(1-\zeta) \cdot M^*(1,Y) + \zeta \cdot 0 = (1-\zeta) \cdot M^*(2,\varphi) + \zeta \cdot \frac{M^*(2,\varphi)}{2}. \tag{84}$$

Assume now that ζ is the resident strategy, and consider the mutant strategy $\zeta' = 0$, represented in a share ε of the population. Then, residents get, on average, reproductive success equal to

$$\begin{aligned} &(1-\zeta)[\Pr[\zeta|\zeta,\varepsilon](1-\zeta) + \Pr[\zeta'|\zeta,\varepsilon]] \cdot M^*(1,Y) + (1-\zeta)\Pr[\zeta|\zeta,\varepsilon]\zeta \cdot 0 \\ &+\zeta[\Pr[\zeta|\zeta,\varepsilon](1-\zeta) + \Pr[\zeta'|\zeta,\varepsilon]] \cdot M^*(2,\varphi) + \zeta\Pr[\zeta|\zeta,\varepsilon]\zeta \cdot \frac{M^*(2,\varphi)}{2}, \end{aligned} \tag{85}$$

while mutants achieve an average reproductive success of

$$[\Pr[\zeta|\zeta',\varepsilon](1-\zeta)+\Pr[\zeta'|\zeta',\varepsilon]]\cdot M^*(1,Y)+\Pr[\zeta|\zeta',\varepsilon]\zeta\cdot 0. \tag{86}$$

A necessary condition for ζ to be evolutionarily stable against $\zeta'=0$ is that, when evaluated at $\varepsilon=0$, the expression in (85) be at least as large as that in (86), i.e.:

$$(1-\zeta)^2\cdot M^*(1,Y)+\zeta(1-\zeta/2)\cdot M^*(2,\varphi)\geq[(1-\sigma)(1-\zeta)+\sigma]\cdot M^*(1,Y). \tag{87}$$

Using (84), this reduces to

$$0\geq\zeta\sigma\cdot M^*(1,Y). \tag{88}$$

Given that $\zeta\cdot M^*(1,Y)>0$, this inequality is false for any $\sigma>0$.

Claim *(iii)*: $\zeta=1$ is evolutionarily stable if $\sigma<\tilde{\sigma}$ but it is not evolutionarily stable if $\sigma>\tilde{\sigma}$. Suppose that $\zeta=1$ is the resident strategy and that a share $\varepsilon>0$ of the population carries the mutant strategy $\zeta'=0$. The average reproductive success of a resident (who wins with probability 1/2 when matched with another individual playing *Compete* and with certainty when matched with an individual playing *Surrender*) is then

$$\Pr[\zeta|\zeta,\varepsilon]\cdot\frac{M^*(2,\varphi)}{2}+\Pr[\zeta'|\zeta,\varepsilon]\cdot\left[\zeta'\cdot\frac{M^*(2,\varphi)}{2}+(1-\zeta')\cdot M^*(2,\varphi)\right], \tag{89}$$

while that of a mutant is

$$\zeta'\cdot[\Pr[\zeta|\zeta',\varepsilon]+\Pr[\zeta'|\zeta',\varepsilon]\cdot\zeta']\cdot\frac{M^*(2,\varphi)}{2}+(1-\zeta')\cdot\Pr[\zeta'|\zeta',\varepsilon](1-\zeta')\cdot M^*(1,Y). \tag{90}$$

By continuity of these expressions in ε, a necessary condition for $\zeta=1$ to be evolutionarily stable against $\zeta'\in[0,1)$ is that, when evaluated at $\varepsilon=0$, the expression in (89) be at least as large as that in (90). This condition writes

$$\frac{M^*(2,\varphi)}{2}\geq\sigma\left[\zeta'\cdot\frac{M^*(2,\varphi)}{2}+(1-\zeta')\cdot M^*(1,Y)\right]. \tag{91}$$

Since $\frac{M^*(2,\varphi)}{2}\leq M^*(1,Y)$ (recall Proposition 4), the right-hand side of this inequality attains its maximum for $\zeta'=0$. Hence, a necessary and sufficient condition for $\zeta=1$ to be evolutionarily stable against any $\zeta'\in[0,1)$ is that it be evolutionarily stable against $\zeta'=0$. Replacing ζ' by 0 in the preceding inequality, I conclude that a necessary condition for $\zeta=1$ to be evolutionarily stable against any $\zeta'\in[0,1)$ is:

$$\sigma\leq\frac{M^*(2,\varphi)}{2\cdot M^*(1,Y)}. \tag{92}$$

By continuity of the expressions in (89) and (90) in ε (recall that the conditional probabilities are continuous in ε), this condition is also sufficient if it holds as a strict inequality.

7 Appendix B: The R-round competition game

Consider a generalized game, in which there may be up to $R \geq 1$ competition rounds, where $R \in \mathbb{N}$. As above, suppose that when entering the adult stage (from the teenage years) each man has one girlfriend. In each round, all the men who did not yet lose a competition are matched pairwise, and in each such matched pair each man plays either *Surrender* or *Compete*, exactly as above. However, by contrast to the one-round game, now the number of women at stake increases with every round. Formally, let a man's (pure) strategy be a number $r \in \{0, 1, ..., R\}$, which specifies the number of rounds in which he plays *Compete*, following which he plays *Surrender*; below I will refer to a man's r as his *degree of competitiveness*.

I maintain the assumptions introduced in the one-round game that (a) in a pair where both play *Surrender*, each man survives and settles down to form his household (thereby exiting the competition game); (b) in a pair where one plays *Surrender* and the other plays *Compete*, the former loses his women to the latter; (c) in a pair where both play *Compete*, one of the men doubles his number of women by winning over the other, each man having probability 1/2 of winning. Hence, in a population where all men play strategy r each man achieves expected reproductive success

$$\frac{1}{2^r} \cdot M^*(2^r, Y).$$

For each competition round, the number of women per man is doubled, and there is a probability 1/2 of winning.

The following proposition identifies the set of evolutionarily stable degrees of competitiveness.[38]

Proposition 8. *In the R-round competition game:*
(i) $r = R$ *is evolutionarily stable in any ecology* $\omega \in \Omega$*;*
(ii) $r \in \{0, 1, ..., R-1\}$ *is evolutionarily stable if and only if the ecology* ω *is such that:*

$$M^*(2^r, Y) > M^*\left(2^{r+1}, Y\right). \tag{93}$$

The two main qualitative features of the *One-round fighting* game carry over to the *R-round competition* game: (i) it never pays off for a man to stop fighting before the other men do: full competitiveness ($r = R$) is always an evolutionarily stable strategy; (ii) a strategy which consists in fighting less than R rounds may also be evolutionarily stable. As revealed by condition (93), for $r < R$ to be evolutionarily stable, a man who has won the first r rounds must not benefit from further doubling the number of wives by way of fighting against a peaceful rival in round $r+1$, i.e., $M^*(2^r, Y) > M^*\left(2^{r+1}, Y\right)$ is a necessary condition. As shown in the proof of the proposition, this condition is also sufficient by virtue of the non-increasing returns to wives (see Proposition 4).

Together with the results on how male reproductive success varies with the number of wives for a given labor resource amount Y reported in Proposition 3 (see also this proposition for the definitions of Ω_1, Ω_2, and Ω_3), Proposition 8 implies:

---

[38] For simplicity I restrict attention to pure strategies.

Corollary 1. *For any given* $Y \in (0,1]$*:*
(i) full competitiveness ($r = R$) *is evolutionarily stable for any ecology* $\omega \in \Omega$*;*
(ii) for any ecology $\omega \in \Omega_1$ *full competitiveness* ($r = R$) *is the unique evolutionarily stable degree of competitiveness;*
(iii) for any ecology $\omega \in \Omega_2 \cup \Omega_3$*, lack of competitiveness* ($r = 0$) *is also evolutionarily stable if* $M^*(1, Y) > M^*(2, Y)$*; moreover, there may exist some* $r^{\max} \in \{2, ..., R-1\}$ *such that any* $r \in \{2, ..., r^{\max}\}$ *is also evolutionarily stable.*

This corollary makes a central point: when the man's productive resources (Y) do not depend on the outcome of the male-male competition, the set of degrees of competitiveness that are evolutionarily stable in a population are fully determined by the ecology in which the population evolves. This is because the ecology then fully determines men's benefits from outcompeting other men in view of acquiring reproductive resources (women).

Turning finally to analysis of the R-round generalized competition game, the following proposition obtains:

Proposition 9. *In the* R*-round competition game:*
(i) $r = R$ *is evolutionarily stable for all* $\omega \in \Omega$ *and all* $\varphi : \{0, 1, ..., R\} \to \mathbb{R}_{++}$*;*
(ii) $r \in \{0, 1, ..., R-1\}$ *is evolutionarily stable if and only if, for any* $r' \in \{r+1, r+2, ..., R\}$*: either*

$$M^*\left(2^r, \varphi(r)\right) > \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, \varphi(r')\right), \tag{94}$$

or

$$M^*\left(2^r, \varphi(r)\right) = \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, \varphi(r')\right) \text{ and } \frac{1}{2^{r'-r}} \cdot M^*\left(2^{r'}, \varphi(r')\right) > M^*\left(2^r, \varphi(r)\right). \tag{95}$$

The two main qualitative features of the R-round competition game with constant productive resources (see Proposition 8) carry over to the generalized R-round competition game: (1) it never pays off for a man to stop fighting before the other men do: full competitiveness ($r = R$) is always an evolutionarily stable strategy; (2) a strategy which consists in fighting less than R rounds may also be evolutionarily stable. However, comparison of part (ii) of Proposition 9 with that of Proposition 8 reveals that the conditions for a limited degree of competitiveness to be evolutionarily stable are quite different. This is due to the lack of restrictive assumptions on the resource-allocation function φ. When productive resources are constant (see Proposition 8), the non-increasing returns to competing for reproductive resources imply that a limited degree of competitiveness is evolutionarily stable as long as a man would not want to further double the number of wives (see (93)). When the competition also determines productive resources, such non-decreasing returns are not guaranteed and more conditions are required. Thus, inequality (94) says that $r < R$ is evolutionarily stable if (1) a man who has won the first r rounds does not benefit from further doubling the number of wives by way of fighting against a peaceful rival in round $r+1$, i.e., $M^*\left(2^r, \varphi(r)\right) \geq M^*\left(2^{r+1}, \varphi(r+1)\right)$, and (2) does not benefit either from mutating towards a strategy which consists in fighting for some number $r' > r+1$ of rounds. The term $M^*\left(2^{r'}, \varphi(r')\right)$ in the right-hand side of (94) is the reproductive success that a lucky such mutant would achieve, while the "discount factor" $1/\left[2^{r'-(r+1)}\right]$ shows that such a mutant strategy entails a risk of ending up unlucky, since beyond round $r+1$ a mutant would be certain to face another mutant and thus halve his chance of winning in each round.

The expressions in (95) show that second-order effects may matter when competition determines both productive and reproductive resources: if residents and mutants are equally well off when matched with a resident (this is the equality in (95)), for a limited degree of competitiveness $r < R$ to be evolutionarily stable, the loss incurred by a mutant from being matched with another mutant (the left-hand side of the inequality in (95)) must exceed the loss incurred by a resident from being matched with a mutant (the right-hand side of the inequality in (95)).

7.1 Proof of Proposition 8

Claim 1: any strategy $r \in \{1, ..., R\}$ is evolutionarily stable against any strategy $r' \leq r-1$. To prove this, consider first the case where $r' = r-1$. If the resident strategy is r and the mutant strategy is $r' = r-1$, all men play *Compete* for the first $r-2$ rounds. Assuming that prior to the first competition round there is a share ε of mutants, there is still a share $1-\varepsilon$ of residents and a share ε of mutants at the beginning of round $r-1$. In round $r-1$, each remaining resident is matched with a mutant with probability ε, and he then wins for sure; with probability $1-\varepsilon$ he is matched with another resident, in which case he wins with probability $1/2$. Whether matched with a mutant or a resident in this round, if he wins he will play *Surrender* in round r and he is certain that his opponent will also play *Surrender*, since there are only residents around in round r. Conditional on making it to round r, he thus gets reproductive success $M^*(2^r, Y)$ for sure. In sum, the expected reproductive success of a resident who is still in the game at the beginning of round $r-1$ is

$$\frac{1-\varepsilon}{2} \cdot M^*(2^r, Y) + \varepsilon \cdot M^*(2^r, Y). \tag{96}$$

Turning now to a mutant who is still around at the beginning of round $r-1$, his expected reproductive success is

$$(1-\varepsilon) \cdot 0 + \varepsilon \cdot M^*\left(2^{r-1}, Y\right). \tag{97}$$

The mutant, who plays *Surrender* in round r, gets zero when matched with a resident, and $M^*\left(2^{r-1}, Y\right)$ if matched with another mutant. Strategy r is evolutionarily stable against $r' = r-1$ iff there exists some $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon})$ the expression in (96) is strictly larger than the expression in (97). This is true by virtue of the continuity of the expressions in ε and the fact that when evaluated for $\varepsilon = 0$, the expression in (96) is strictly larger than that in (97); indeed, $M^*(2^r, Y) > 0$. Next, note that any mutant strategy $r' < r-1$ would fare even worse than $r' = r-1$ in a population where the resident strategy is r: indeed, such mutants would all be eliminated with certainty in or before period r'. This proves **Claim 1**. Note that Claim 1 implies that the degree of competitiveness R is evolutionarily stable.

Claim 2: any strategy $r \in \{0, 1, ..., R-1\}$ is evolutionarily stable against any mutant strategy $r' \geq r+1$ if and only if

$$M^*(2^r, Y) > M^*\left(2^{r+1}, Y\right). \tag{98}$$

To prove this, consider some resident strategy $r \in \{0, 1, ..., R-1\}$, and suppose that prior to the first competition round some mutant strategy $r' \geq r+1$ is present in a share $\varepsilon > 0$ of the population. Since residents and mutants alike compete in the first r rounds, at the beginning of round $r+1$ there is still a share $1-\varepsilon$ of residents and a share ε of mutants. Consider now round $r+1$. In this round, residents play *Surrender* while mutants play *Compete*. Thus, the expected reproductive success of a resident who is still in the game at the beginning of round $r+1$ is

$$(1-\varepsilon) \cdot M^*(2^r, Y) + \varepsilon \cdot 0. \tag{99}$$

With probability $1-\varepsilon$ the said resident is matched with another resident, in which case both play *Surrender*, both survive and each gets to keep the 2^r women that he had at the beginning of round $r+1$; otherwise he is matched with a mutant, in which case he gets 0 since he plays *Surrender* while the mutant plays *Compete*. Now consider a mutant who is still in the game at the beginning of round $r+1$. In this round, a mutant plays *Compete*, and he wins with probability $1-\varepsilon/2$ (i.e., with certainty whenever matched with a resident and with probability one half when matched with another mutant). If $r' > r+1$, the mutants who remain after round $r+1$ face an equal probability of winning and losing in each round between $r+2$ and r'. Hence, at the beginning of round $r+1$, the expected reproductive success of a mutant is

$$\frac{1}{2^{r'-r-1}}\left(1-\frac{\varepsilon}{2}\right) \cdot M^*\left(2^{r'}, Y\right). \tag{100}$$

Strategy r is evolutionarily stable against r' iff there exists some $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$ the expression in (99) is strictly larger than the expression in (100). Since the expressions are continuous in ε, a sufficient condition for this to be true is that the strict inequality hold for $\varepsilon = 0$:

$$M^*(2^r, Y) > \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, Y\right). \tag{101}$$

Recalling further from Proposition 4 that there are non-increasing returns to reproductive resources ($M^*(2k, Y) \leq 2M^*(k, Y)$), a sufficient condition for (101) to hold for $r' \geq r+1$ is that

$$M^*(2^r, Y) > M^*\left(2^{r+1}, Y\right). \tag{102}$$

In the event that the strict inequality in (101) is not satisfied, for r to be evolutionarily stable against $r' \geq r+1$ it is necessary that (i) for $\varepsilon = 0$ the expressions in (99) and (100) be equal,

$$M^*(2^r, Y) = \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, Y\right), \tag{103}$$

and (ii) that the difference between the average reproductive success of residents, in (99), and that of mutants, in (100), be strictly increasing in ε. The latter condition is equivalent to:

$$\frac{1}{2^{r'-r}} \cdot M^*\left(2^{r'}, Y\right) > M^*(2^r, Y). \tag{104}$$

But this inequality is violated by virtue of the non-increasing returns to reproductive resources (see above). In sum, inequality (102) is necessary and sufficient for $r \in \{0, 1, ..., R-1\}$ to be evolutionarily stable against any mutant strategy $r' \geq r+1$.

7.2 Proof of Proposition 9

Claim 1: any strategy $r \in \{1,...,R\}$ is evolutionarily stable against any mutant strategy $r' < r$. (The proof of this claim is similar to that of Claim 1 in the proof of Proposition 8; to avoid redundance, the text has been shortened.) Consider some resident strategy $r \in \{1,...,R\}$ and suppose first that the mutant strategy is $r' = r-1$. Then, all men play *Compete* for $r-2$ rounds, and at the beginning of round $r-1$ there is still a share $1-\varepsilon$ of residents and a share ε of mutants. The expected reproductive success of a resident who is still in the game at the beginning of round $r-1$ is thus

$$\frac{1-\varepsilon}{2} \cdot M^*(2^r, \varphi(r)) + \varepsilon \cdot M^*(2^r, \varphi(r)), \tag{105}$$

while that of a mutant who is still around at the beginning of round $r-1$ is

$$(1-\varepsilon) \cdot 0 + \varepsilon \cdot M^*\left(2^{r-1}, \varphi(r-1)\right). \tag{106}$$

Strategy r is evolutionarily stable against $r' = r-1$ iff there exists some $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon})$ the expression in (105) is strictly larger than the expression in (106). This is true by virtue of the continuity of the expressions in ε and the fact that when evaluated for $\varepsilon = 0$, the expression in (105) is strictly larger than that in (106); indeed, $M^*(2^r, \varphi(r)) > 0$. Next, note that any mutant strategy $r' < r-1$ would fare even worse than $r' = r-1$ in a population where the resident strategy is r: indeed, such mutants would all be eliminated with certainty in or before period r'. This proves **Claim 1**. Note that Claim 1 implies that the degree of competitiveness R is evolutionarily stable.

Claim 2: any strategy $r \in \{1,...,R\}$ is evolutionarily stable against any mutant strategy $r' \geq r+1$ if and only if either

$$M^*(2^r, \varphi(r)) > \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, \varphi(r')\right), \tag{107}$$

or

$$M^*(2^r, \varphi(r)) = \frac{1}{2^{r'-r-1}} \cdot M^*\left(2^{r'}, \varphi(r')\right) \text{ and } \frac{1}{2^{r'-r}} \cdot M^*\left(2^{r'}, \varphi(r')\right) > M^*(2^r, \varphi(r)). \tag{108}$$

To prove this, consider some resident strategy $r \in \{0, 1, ..., R-1\}$, and suppose that prior to the first competition round some mutant strategy $r' \geq r+1$ is present in a share $\varepsilon > 0$ of the population. Since residents and mutants alike compete in the first r rounds, at the beginning of round $r+1$ there is still a share $1-\varepsilon$ of residents and a share ε of mutants. Consider now round $r+1$. In this round, residents play *Surrender* while mutants play *Compete*. Thus, the expected reproductive success of a resident who is still in the game at the beginning of round $r+1$ is

$$(1-\varepsilon) \cdot M^*(2^r, \varphi(r)) + \varepsilon \cdot 0. \tag{109}$$

With probability $1-\varepsilon$ the said resident is matched with another resident, in which case both play *Surrender*, both survive and each gets to keep the 2^r women and labor resources $\varphi(r)$ that he had at the beginning of round $r+1$; otherwise he is matched with a mutant, in which case he gets 0 since he plays *Surrender* while the mutant plays *Compete*. Now consider a mutant who is still in the game at the beginning of round $r+1$. In this round, a mutant plays *Compete*, and he wins with probability

$1-\varepsilon/2$ (i.e., with certainty whenever matched with a resident and with probability one half when matched with another mutant). If $r' > r+1$, the remaining mutants face an equal probability of winning and losing in each round between $r+2$ and r'. Hence, at the beginning of round $r+1$, the expected reproductive success of a mutant is

$$\frac{1}{2^{r'-r-1}}\left(1-\frac{\varepsilon}{2}\right)\cdot M^{*}\left(2^{r'},\varphi\left(r'\right)\right). \tag{110}$$

Strategy r is evolutionarily stable against r' iff there exists some $\overline{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon})$ the expression in (109) is strictly larger than the expression in (110). Since the expressions are continuous in ε, a sufficient condition for this to be true is that the strict inequality hold for $\varepsilon = 0$:

$$M^{*}\left(2^{r},\varphi\left(r\right)\right) > \frac{1}{2^{r'-r-1}}\cdot M^{*}\left(2^{r'},\varphi\left(r'\right)\right), \tag{111}$$

which is condition (94) in the proposition. In the event that the strict inequality in (111) is not satisfied, then, for r to be evolutionarily stable against $r' \geq r+1$ it is necessary that (i) for $\varepsilon = 0$ the expressions in (109) and (110) be equal,

$$M^{*}\left(2^{r},\varphi(r)\right) = \frac{1}{2^{r'-r-1}}\cdot M^{*}\left(2^{r'},\varphi(r')\right), \tag{112}$$

and (ii) that the difference between the average reproductive success of residents, in (109), and that of mutants, in (110), be strictly increasing in ε. The latter condition is equivalent to:

$$\frac{1}{2^{r'-r}}\cdot M^{*}\left(2^{r'},\varphi(r')\right) > M^{*}\left(2^{r},\varphi(r)\right). \tag{113}$$

This condition together with the one in (112) are the ones stated in (95) in the proposition.

References

Alexander, R.D (1987) *The Biology of Moral Systems*. Hawthorne NY: Aldine de Gruyter.

Alger, I., and D. Cox (2013) "The evolution of altruistic preferences: mothers versus fathers," *Review of Economics of the Household*, 11, 421-446.

Alger I., P. Hooper, D. Cox, J. Stieglitz, and H.S. Kaplan (2020). "Paternal provisioning results from ecological change," *Proceedings of the National Academy of Sciences* (forthcoming).

Alger, I., and J.W. Weibull (2010) "Kinship, incentives and evolution," *American Economic Review*, 100, 1725-1758.

Alger, I. and J.W. Weibull (2013) "Homo Moralis—preference evolution under incomplete information and assortative matching," *Econometrica*, 81, 2269-2302.

Alger, I., and J.W. Weibull (2019) "Evolutionary Models of Preference Formation," *Annual Review of Economics*, 11, 329–354.

Alger, I. and J.W. Weibull, and L. Lehmann (2020) "Evolution of preferences in group-structured populations: genes, guns, and culture," *Journal of Economic Theory*, 185, 104951.

Alvergne, A., C. Faurie, and M. Raymond (2009) "Variation in testosterone levels and male reproductive effort: Insight from a polygynous human population,"*Hormones and Behavior*, 56(5), 491–497.

Anderson, K. (2006) "How well does paternity confidence match actual paternity? Evidence from worldwide nonpaternity rates," *Current Anthropology*, 47, 513 -520.

Balaresque, P., and M.A. Jobling (2007) "Human populations: houses for spouses," *Current Biology*, 17(1), R14–R16.

Balaresque, P., N. Poulet, S. Cussat-Blanc, P. Gerard, L. Quintana-Murci, E. Heyer, and M.A. Jobling (2015) "Y-chromosome descent clusters and male differential reproductive success: young lineage expansions dominate Asian pastoral nomadic populations," *European Journal of Human Genetics*, 23, 1413-1422.

Bateman, A.J. (1948) "Intra-sexual selection in *Drosphilia*," *Heredity*, 2, 277-287.

Becker, G. S. (1974) "A theory of marriage: Part II," *Journal of Political Economy*, 82, 11-26.

Becker, G. S. (1991) *A Treatise on the Family*, second ed. Cambridge MA: Harvard University Press.

Bergstrom, T.C. (1994a) "Primogeniture, monogamy and reproductive success in a stratified society," Working paper, University of Michigan.

Bergstrom, T.C. (1994b) "On the economics of polygyny," Working paper, University of Michigan.

Bergstrom, T.C. (1995) "On the evolution of altruistic ethical rules for siblings," *American Economic Review* 85, 58-81.

Bergstrom, T.C. (1996) "Economics in a family way," *Journal of Economic Literature*, 34, 1903-1934.

Bergstrom, T.C. (2003) "The algebra of assortative encounters and the evolution of cooperation," *International Game Theory Review* 5, 211-228.

Bertocchi, G., and A. Dimico (2019) "The long-term determinants of HIV infection among African women: the slave trade, polygyny, and sexual behavior, " *Journal of Development Economics*, 190, 90-105.

Betzig, L. (1986) *Despotism and Differential Reproduction: A Darwinian View of History*. Hawthorne, New York: Aldine de Gruyter.

Betzig, L. (1992). "Roman monogamy, " *Ethology and Sociobiology*, 13(5), 351–383.

Betzig, L. (1993) "Sex, succession, and stratification in the first six civilizations: How powerful men reproduced, passed power on to their sons, and used power to defend their wealth, women, and children." in Ellis, L. (Ed.), *Social Stratification and Socioeconomic Inequality: Vol. 1. A Comparative Biosocial Analysis* (pp. 37-74). Westport, CT: Praeger Publishers/Greenwood Publishing Group.

Boserup, E. (1970) *Woman's Role in Economic Development*. New York NY: St. Martin's Press.

Boserup, Ester (1997), "The economics of polygamy," in Grinker, R.R., and C.B Steiner (Eds.), *Perspectives on Africa: A Reader in Culture, History, and Representation*. Cambridge, MA: Blackwell.

Cavalli-Sforza, L.L., and W.F. Bodmer (1971) *The Genetics of Human Populations.* San Francisco: W.H. Freeman.

Chiappori, P.-A., M.C. Dias, and C. Meghir (2018) "The marriage market, labor supply, and education choice," *Journal of Political Economy*, 126, S26–S72.

Chiappori, P.-A., B. Salanié, and Y. Weiss (2017) "Partner choice, investment in children, and the marital college premium," *American Economic Review*, 107(8), 2109–2167.

Clutton-Brock T.H. (1991) *The Evolution of Parental Care*. Princeton: Princeton University Press.

Clutton-Brock, T.H., and A.C. Vincent (1991) "Sexual selection and the potential reproductive rates of males and females," *Nature*, 351, 58-60.

Cox, D., and M. Fafchamps (2008) "Extended family and kinship networks: economic insights and evolutionary directions." in Schultz, T.P., and J.A. Strauss (Eds.),*Handbook of Development Economics*. Amsterdam: North-Holland.

Dekel, E., J.C. Ely, and O. Yilankaya (2007) "Evolution of preferences," *Review of Economic Studies*, 74, 685-704.

De La Croix, D., and F. Mariani (2015) "From polygyny to serial monogamy: a unified theory of marriage institutions," *Review of Economic Studies*, 82(2), 565–607.

Doepke, M., and M. Tertilt (2009) "Women's liberation: what's in it for men?" *Quarterly Journal of Economics*, 124(4), 1541-1591.

Edlund, L., and N-.P. Lagerlöf (2006) "Individual versus parental consent in marriage: implications for intra-household resource allocation and growth," *American Economic Review*, 96(2), 304-307.

Edlund, L., and W. Kopczuk (2009) "Women, wealth, and mobility," *American Economic Review*, 99(1), 146-178.

Edlund, L., and C. Machado (2015) "How the other half lived: Marriage and emancipation in the age of the Pill," *European Economic Review*, 80, 295-309.

Emlen, S.T., and L.W. Oring (1977) "Ecology, sexual selection, and the evolution of mating systems," *Science*, 197 (4300), 215-223.

Fisher H. (2016) *Anatomy of Love*. New York: Norton.

Fisher R.A. (1930) *The Genetical Theory of Natural Selection*. Oxford: Clarendon Press.

Fortunato, L., and M. Archetti (2010) "Evolution of monogamous marriage by maximization of inclusive fitness," *Journal of Evolutionary Biology*, 23, 149-156.

Francesconi, M., C. Ghiglino, and M. Perry (2016) "An evolutionary theory of monogamy," *Journal of Economic Theory*, 166, 605-628.

Frank, R.H. (1987) "If *Homo Economicus* could choose his own utility function, would he want one with a conscience?" *American Economic Review*, 77, 593-604.

Frank, S.A. (1998) *Foundations of Social Evolution*. Princeton, NJ: Princeton University Press.

Gaulin, S.J.C., and J.S Boster (1990) "Dowry and female competition," *American Anthropologist*, 92, 994–1005.

Gavrilets, S. (2012) "Human origins and the transition from promiscuity to pair bonding," *Proceedings of the National Academy of Sciences*, 109, 9923-9928.

Gettler, L.T., T.W. McDade, A.B. Feranil, and C.W. Kuzawa (2011) "Longitudinal evidence that fatherhood decreases testosterone in human males," *Proceedings of the National Academy of Sciences*, 108(29), 16194–16199.

Goody, J. (1976) *Production and Reproduction*. Cambridge: Cambridge University Press.

Gould, E.D., O. Moav, and A. Simhon (2008) "The mystery of monogamy," *American Economic Review*, 98, 333-57.

Grafen, A. (1979). "The hawk-dove game played between relatives," *Animal Behaviour*, 27, 905–907.

Grafen, A. (1985) "A geometric view of relatedness," in Dawkins, R. and M. Ridley (Eds.), *Oxford Surveys in Evolutionary Biology*. Oxford University Press, Oxford.

Güth, W., and M. Yaari (1992) "An evolutionary approach to explain reciprocal behavior in a simple strategic game," in U. Witt (Ed.), *Explaining Process and Change – Approaches to Evolutionary Economics*. Ann Arbor MI: University of Michigan Press.

Gray, J.P. (1998) "Ethnographic Atlas Codebook derived from George P. Murdock's Ethnographic Atlas recording the marital composition of 1231 societies from 1960 to 1980." *World Cultures* 10(1), 86-136.

Grossbard, A. (1976) "An economic analysis of polygyny: The case of Maiduguri," *Current Anthro-*

pology 17, 701-707.

Grossbard, S. (2015) *The Marriage Motive: A Price Theory of Marriage. Why Marriage Markets Affect Employment, Consumption and Savings*. New York NY: Springer.

Grueter, C.C., B. Chapais, and D. Zinner (2012) "Evolution of multilevel social systems in nonhuman primates and humans," *International Journal of Primatology* 33, 1002-1037.

Hamilton, W.D. (1964) "The genetical evolution of social behaviour," *Journal of Theoretical Biology* 7, 1-52.

Hartl, D.L., and A.G. Clark (2007) *Principles of Population Genetics* (4th edition). Sunderland, MA: Sinauer and Associates.

Heifetz, A., C. Shannon, and Y. Spiegel (2007) "What to maximize if you must," *Journal of Economic Theory*, 133, 31-57.

Helle, S., J.E. Brommer, J.E. Pettay, V. Lummaa, M. Enbuske, and J. Jokela (2014) "Evolutionary demography of agricultural expansion in preindustrial northern Finland," *Proceedings of the Royal Society B: Biological Sciences*, 281(1794).

Henrich, J., R. Boyd, and P.J. Richerson (2012) "The puzzle of monogamous marriage," *Philosophical Transactions of the Royal SocietyB*, 367, 657–669.

Hrdy, S.B. (1999) *Mother Nature: Maternal Instincts and how they Shape the Human Species*. New York: Ballantine Books.

Jensen, M.K., and A. Rigos (2018) "Evolutionary games and matching rules," *International Journal of Game Theory*, 47(3), 707–735.

Kanazawa, S., and M.C. Still (1999) "Why monogamy?" *Social Forces*, 78, 25-50.

Kaplan, H.S., Hill, K., Hurtado, A. M. & Lancaster, J. B. (2001) "The embodied capital theory of human evolution," in Ellison, P.T. (Ed.) *Reproductive ecology and human evolution* (pp. 293 – 317), Hawthorne, NY: Aldine de Gruyter.

Kaplan, H.S., P. Hooper, and M. Gurven (2009) "The evolutionary and ecological roots of human social organization," *Philosophical Transactions of the Royal Society B*, 364, 3289-3299.

Kaplan, H.S., and J.B. Lancaster (2003) "An evolutionary and ecological analysis of human fertility, mating patterns, and parental investment," in Waechter, K.W., and R. A. Bulatao (Eds.), *Offspring: Human Fertility Behavior in Biodemographic Perspective* (pp. 170–223), Washington D.C.: National Academies Press.

Karmin, M., L. Saag, M. Vicente, M.A. Wilson Sayres, M. Järve, U. Gerst Talas, ..., and T. Kivisild (2015) "A recent bottleneck of Y chromosome diversity coincides with a global change in culture," *Genome Research*, 25, 459-466

Knight, E.L., A. Sarkar, S. Prasad, and P.H. Mehta (2020) "Beyond the challenge hypothesis: The emergence of the dual-hormone hypothesis and recommendations for future research," *Hormones* *and Behavior*, 104657.

Kokko, H., and M. Jennions (2008) "Parental investment, sexual selection and sex ratios," *Journal of Evolutionary Biology*, 21, 919–948.

Kokko, H., and L.J. Morrell (2005) "Mate guarding, male attractiveness, and paternity under social monogamy," *Behavioral Ecology*, 16, 724-731.

Kramer, K.L., and R.D Greaves (2007) "Changing patterns of infant mortality and maternal fertility among Pumé foragers and horticulturalists," *American Anthropologist*, 109(4), 713–726.

Lagerlöf, N.-P. (2005) "Sex, equality, and growth," *Canadian Journal of Economics*, 38, 807–831.

Lagerlöf, N.-P. (2010) "Pacifying monogamy," *Journal of Economic Growth*, 15, 235–262.

Layton, R., S. O'Hara, and A. Bilsborough (2012) "Antiquity and social functions of multilevel social organization among human hunter-gatherers," *International Journal of Primatology* 33, 1215-1245.

Low, B. (1990) "Marriage systems and pathogen stress in human societies," *American Zoologist*, 30, 325–339.

Lundberg, S., and R.A. Pollak (2007) "The American family and family economics," *Journal of Economic Perspectives*, 21(2), 3–26.

Malone, N., A. Fuentes, and F.J. White (2012) "Variation in the social systems of extant hominoids: comparative insight into the social behavior of early hominins," *International Journal of Primatology* 33, 1251-1277.

Moya, C., K. Snopkowski, and R. Sear (2016) "What do men want? Re-examining whether men benefit from higher fertility than is optimal for women," *Philosophical Transactions of the Royal SocietyB*, 371.

Møller, A.P. (2003) "The evolution of monogamy: mating relationships, parental care and sexual selection," in Ulrich, R.H., and C. Boesch (Eds.), *Monogamy: Mating strategies and partnerships in birds, humans and other mammals*. Cambridge: Cambridge University Press.

Murdock, G.P. (1949) *Social Structure*. New York: The Macmillan Company.

Murdock, G.P., Textor, R., Barry III, H., White, D.R., Gray, J.P., and W. Divale (2000) "Ethnographic Atlas," *World Cultures* 10, 24-136.

Nettle, D., M.A. Gibson, D.W. Lawson, and R. Sear (2013) "Human behavioral ecology: current research and future prospects," *Ecology* 24, 1031-1040.

Nitsch, A., V. Lummaa, and C. Faurie (2016) "Sibship effects on dispersal behaviour in a pre-industrial human population," *Journal of Evolutionary Biology*, 29(10), 1986–1998.

Orians, G.H. (1969) "On the evolution of mating systems in birds and mammals," *American Naturalist*, 103, 589–603.

Rayo, L, and G.S. Becker (2007) "Evolutionary efficiency and happiness," *Journal of Political Economy*, 115, 302-337.

Robson, A.J. (2001) "The biological basis of economic behavior," *Journal of Economic Literature*, 39, 11-33.

Robson, A.J. (2002) "Evolution and human nature," *Journal of Economic Perspectives*, 16, 89-106.

Robson, A.J., and L. Samuelson (2011) "The evolutionary optimality of decision and experienced utility," *Theoretical Economics*, 6, 311-339.

Ross, C.T., M. Borgerhoff Mulder, S.-Y. Oh, S. Bowles, B. Beheim, J. Bunce, . . . J. Ziker (2018). "Greater wealth inequality, less polygyny: rethinking the polygyny threshold model," *Journal of The Royal Society Interface*, 15, 1-15.

Rousset, F. (2004) *Genetic Structure and Selection in Subdivided Populations*. Princeton, NJ: Princeton University Press.

Schacht, R., and K.L Kramer, K. L. (2019). "Are We Monogamous? A Review of the Evolution of Pair-Bonding in Humans and Its Contemporary Variation Cross-Culturally," *Frontiers in Ecology and Evolution*, 7(July), 1–10.

Scheidel, W. (2008) "Monogamy and polygyny in Greece, Rome, and world history," Princeton/Stanford Working Papers in Classics.

Seabright, P. (2004) *The Company of Strangers: A Natural History of Economic Life*. Princeton and Oxford: Princeton University Press.

Simmons, L.W., R.C. Firman, and G. Rhodes (2004) "Human sperm competition: Testis size, sperm production and rates of extrapair copulations," *Animal Behaviour*, 68, 297–302.

Tertilt, M. (2005) "Polygyny, fertility, and savings," *Journal of Political Economy*, 113, 1341–71.

Tertilt, M. (2006) "Polygyny, women's rights, and development," *Journal of the European Economic Association*, 4, 523–30.

Van Schaik, C.P., and C.H. Janson (2000) *Infanticide by Males and its Implications*. Cambridge: Cambridge University Press.

Van Schaik, C.P. (2016) *The Primate Origin of Human Behavior*. Hoboken, NJ: Wiley-Blackwell.

Weibull, J.W. (1995) *Evolutionary Game Theory*. Cambridge MA: MIT Press.

Wright, S. (1931): "Evolution in Mendelian populations," *Genetics* 16, 97–159.

Zeitzen, M.K. (2008) *Polygamy: A Cross-Cultural Analysis.* Oxford: Berg Publishers.

Zeng, T.C., Aw, A.J. & Feldman, M.W. (2018) "Cultural hitchhiking and competition between patrilineal kin groups explain the post-Neolithic Y-chromosome bottleneck," *Nature Communications*, 9, 2077.