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1 Introduction

A near-canonical claim among observers of highly-divided societies is that the degree to which disagreements or cleavages are "crosscutting" constitutes a critical stabilizing feature of those political systems. When adversaries on one dimension are allies on another, political polarization and disaffection are reduced and stability enhanced, other things being equal. By contrast, when enmity persists across dimensions, disaffection and instability are likely to follow.

Two mechanisms are hypothesized to underlie this relationship. The first is sociological: by creating sympathies for differing arguments and opinions (Coser 1956; Dahrendorf 1959; Lipset 1960; Lipset and Rokkan 1967) and improving communication across divisions (Deutsch ????), crosscutting cleavages improve the likelihood of conciliation. Lipset writes,

> Where a man belongs to a variety of groups that all predispose him toward the same political choice, he is ... much less likely to be tolerant of other opinions. The available evidence suggests that the chances for stable democracy are enhanced to the extent that groups and individuals have a number of crosscutting, politically relevant affiliations. To the degree that a significant proportion of the population is pulled among conflicting forces, its members have an interest in reducing the intensity of political conflict (1960, 88-89).

The second mechanism, which is the primary focus of our analysis in this paper, is political: crosscutting cleavages prevent the emergence of *permanently excluded* groups (Truman 1951; Dahl 1971; Przeworski 1991) by creating the potential for *power sharing*. The nature of this power-sharing is determined by the constitutional structure of the polity: for example, power-sharing may be contemporaneous (if different societal groups are given veto or agenda control in specific policy domains) or it may be inter-temporal (if a group excluded today has the opportunity to influence policy tomorrow).

The claim that crosscutting cleavages have beneficial political consequences may, however, be subject to a number of challenges (e.g., Dahl 1966; Rabushka and Shepsle 1972; Lijphart 1977). First, a majority coalition of factions may indefinitely exclude a minority from the policy making process even in the presence of crosscutting cleavages. Second, issue dimensions over which cleavages are crosscutting may differ in their degree of salience (Rabushka and Shepsle 1972), further limiting the influence of a minority faction if it is only permitted to participate in policy making on irrelevant issues. And third, permanent losers may endure their status as such if the outside option they would exercise by engaging in extra-constitutional politics is sufficiently costly.

The theoretical literature following the initial, largely positive statements about of the salutary effects of crosscutting cleavages suggest that the relative salience or importance of issues may moderate those effects. In their 1972 book, Rabushka and Shepsle argue that a necessary condition for those effects is that the second dimension of conflict be politically salient: for example, if the norms of a society dictate that disagreement with respect to that dimension be adjudicated in the private sphere, then the fact that it crosscuts the primary dimension of political conflict is immaterial. By contrast, Dahl (1966) argues that "unifying effects cannot occur if all the cleavages are felt with equal intensity. Conciliation is encouraged by crosscutting cleavages only if some cleavages are less significant than others" (368). Below, we explicitly incorporate issue salience into our models; this permits us to adjudicate between these two competing claims.

To date, none of the above arguments has been subject to formal theoretical analysis, which is the principal aim of the current inquiry.[1] The model we introduce below explicitly considers both the effect of crosscuttingness on the political disaffection of outgroups in a society, as well as the mediating effects of issue salience. Moreover, by explicitly modeling how issue salience can affect factions' political attachments, our model does not fall prey to the critique, made by Chandra and Boulet (2005), that classic theories of permanent exclusion are based on "primordialist" assumptions that group attachments are permanent and fixed.

In light of these objections, several questions emerge. First, what preference profiles give rise to crosscutting cleavages? And second, does crosscutting reduce the potential for political conflict? If so, under what conditions? In this paper, we take a step toward answering these questions by articulating a very simple model of political conflict under different decision making structures. Our

[1] Critically, while the groundbreaking analysis of Rae and Taylor (1970) introduces measures of crosscuttingness, the authors simply articulate the hope (p. 112) that those measures will ultimately be employed in future theories of democratic stability.

aim is to examine the extent to which, and the conditions under which, received wisdom regarding crosscutting cleavages does and does not hold.

In our model, three groups or "factions" compete to set policy in a two-dimensional space; these factions can coalesce by making binding agreements with one another to make policy at the centroid of their ideal points. We define several intuitive measures of crosscuttingness, each of which quantifies the extent to which adversaries on one dimension might be allies on another. We then consider policy making and political conflict under two alternative governance structures: in the first, policy making on the two dimensions is *bundled*: a single coalition sets policy on both dimensions. Because governing coalitions will invariably exclude one faction, the relevant possible effect of crosscutting cleavages in the distribution of factions' preferences is through their effects on the probability of *realignments* producing changes in the composition of the governing coalition. Thus, the bundled governance structure is one in which power-sharing, if it occurs, is inter-temporal.

Under the second governance structure, policy making on the two dimensions is *unbundled*: the coalition that forms to make policy on one dimension may differ from the coalition that forms on the other dimension. In the unbundled governance structure, crosscutting cleavages are instantiated in dimension-specific coalitions, thus giving the opportunity for all three factions to participate in the creation of policy. Hence, unbundled governance corresponds to contemporaneous power-sharing.

Our model of unbundled policy-making underscores a close association between the changes in crosscutting cleavages and political realignments. According to the "classic" account of the dynamics of realignment (cf., Poole and Rosenthal 2006), a new issue gains in prominence relative to an old one, producing a reshuffling of electoral coalitions such that former allies become adversaries and vice versa.[2] While, holding fixed the preference profile, crosscutting cleavages are a necessary (though not sufficient) condition for realignment, realignments brought about by changes in relative issue salience are instances of changes in relative crosscuttingness of the underlying profiles of individual preferences. In this sense, the analysis of the consequences of susceptibility to political realignments provides a particularly potent test of the traditional intuitions about the salutary effects of crosscuttingness.

[2]Earlier influential works on realignment in U.S. politics include Key 1966; Burnham 1975; Sundquist 1983.

Our results suggest that the effects of crosscutting cleavages on political welfare of societies are far more subtle than previously documented. In the bundled policy model, changes in the preference profile that increase the degree to which cleavages are crosscutting can either increase or decrease the disaffection of the excluded faction, even holding constant the average extent of disagreement between factions – unless that extent of latent disagreement is sufficiently small to begin with. While changes in the relative salience of the policy issues can make a political system more susceptible to realignment, we show that the effects of such changes on disaffection are ambiguous: they sometimes increase and sometimes decrease disaffection.

In the unbundled policy model, the dimension-specific coalitions that form in equilibrium are unaffected by the relative salience of the issues. As in the bundled policy-making model, changes in the preference profile that increase crosscuttingness can either increase or decrease the disaffection of the faction most unhappy with the equilibrium policy.

Unbundling the policy-making is, in effect, a way of institutionalizing crosscutting cleavages, and our final set of results considers the consequences of doing so. We find that, consistent with some of the prior scholarship, unbundling can decrease the disaffection of the most dissatisfied faction. But we also show that under certain conditions, it can have the opposite effect, and, moreover, the cost of unbundling may be the selection of Pareto inefficient policies – something that can never happen in the bundled policy making environment.

In what follows, we first review the theory of crosscutting cleavages in plural societies and describe the concept informally. Then, we articulate common primitives for both models and our measures of disagreement and disaffection. We then describe the bundled policy making model, the unbundled policy making model, and comparisons between the two. Finally, we relate our findings to three well-known historical examples of political conflict in multiple dimensions.

2 Are Crosscutting Cleavages Salutary?

2.1 Some Motivating Examples

Whigs and Tories in Great Britain, 1688-1742. According to Stasavage (2003), the existence of crosscutting cleavages in parliament enhanced Great Britains ability to credibly commit to a moderate policy – namely, the maintenance of a land tax to service public debt – that permitted the country to borrow at low interest rates and fight wars in the first half of the 18th century. In particular, British politics during the period was characterized by conflict over taxation and finance between landed and financial or "moneyed" interests (the latter of whom contained most government creditors), and conflict over other issues, most prominently religious toleration. A diversity of opinion within the landed class over religious toleration made cleavages crosscutting: religiously tolerant landed elites were closer to government creditors on religious issues, and closer to the less-than-tolerant landed elites on economic ones. The Whig coalition in parliament, Stasavage argues, was sustained by a series of compromises between anti-conformist landed and moneyed interests over religious toleration, monarchical succession, and the land tax (122).

The Second Party System in the United States and its Collapse, 1828-1854. The period of American history commencing with the election of Andrew Jackson in 1828 and concluding with the collapse of the Whigs and the emergence of the Republican party in the mid-1850s can be described as one characterized by waxing and waning crosscutting cleavages. At the beginning of the period, the national policy agenda was dominated by economic issues: most prominently, protectionism and the continued existence of a national bank. The emergence of abolitionism in the 1830s made slavery an increasingly prominent issue on the national agenda. While abolitionist sentiment was largely rejected by both the Northern and Southern wings of the Democratic party, the Whig party contained pro- and anti-slavery contingencies (Sundquist 1983). Southern Whigs could coalesce with Democrats on the slavery issue, as they did to impose a series of gag rules preventing consideration of abolitionist petitions in the House of Representatives (Holmes 1988). The relative calm of this partisan alignment could not survive the increasing prominence of the slavery issue, which by the 33rd Congress (1853-1854) had displaced disagreement on economic

Figure 1: The Canonical Intuition: Crosscutting Reduces Polarization

	$M=0$	$R=1$
$I=1$		B,C
$D=0$	A	

	$M=0$	$R=1$
$I=1$	C'	B
$D=0$	A	

issues as the primary dimension of US politics (Poole and Rosenthal 2006, ch. 5). As the Whig coalition collapsed and the unequivocally abolitionist Republican party displaced it, cleavages no longer crosscut. The Civil War would begin seven years later.

2.2 A Heuristic Model

Why might we observe the emergence of crosscutting cleavages sometimes enhance, and sometimes undermine, political stability? To begin to answer this question, imagine a society characterized by two binary cleavages or divisions, which we will refer to as middle (M) versus right (R), and intermediate (I) versus down (D). These divisions could be economic, ethnic, religious, or cultural. The society is composed of three equally-sized factions, A, B, and C. In one version of the society, faction A occupies the MD position, and both B and C occupy the RI position. In the second version, factions A and B are unchanged, but faction C (relabeled C') now occupies the MI position. Figure 1 displays these two variants graphically.

Rae and Taylor (1970) define "crosscuttingness" as the proportion of all pairs of individuals whose two members are in the same group on one cleavage but in different groups on the other cleavage. In the first version of the society, crosscuttingness is zero. In the second, crosscuttingness is 2/3: of the three pairs (AB, AC', and BC'), two (AC' and BC') are allies on one cleavage and adversaries on the other. (The remaining pair, AB, are adversaries on both.)

The widely accepted view, which, henceforth, we refer to as "the canonical intuition," is that societies with crosscutting cleavages experience less polarization and greater stability. The expressly political rationale for it (as distinct from the sociological one noted in the Introduction) is that crosscutting cleavages enhance the likelihood that "out" factions can forge governing coalitions, thus increasing their stake in the current regime. By contrast, when a faction's outsider status is a permanent feature of the polity, it raises the incentives for that faction to engage in obstruction and/or extraconstitutional means to achieve its political ends (Przeworski 1991). This rationale is closely associated with pluralist political thought (e.g., Truman 1951; Dahl 1956). Dahl (1971, 115), for example, argues that political conflict is more likely to be moderate when "no ethnic, religious, or regional subculture is 'indefinitely' denied the opportunity to participate in the government."

By adding some fairly straightforward assumptions, we can capture the canonical intuition. In particular, suppose (a) the four cells in the 2×2 matrices above correspond to ideal policy positions on the unit square in the xy-plane, with $MD = (0, 0)$, $RD = (1, 0)$, $MI = (0, 1)$, and $RI = (1, 1)$; (b) the utility to a faction of a particular policy is decreasing in the linear distance between its ideal point and the policy; and (c) the two closest factions can form a minimum winning coalition and set policy at the centroid of their ideal points. Other things being equal, the stability of the polity is reduced to the extent that the excluded faction finds the implemented policy intolerably far from its own preferred policy.

In the first version of the society, B and C coalesce and set policy at the RI position (1,1), to the exclusion of faction A, which must suffer the loss associated with a policy $\sqrt{2}$ away from its ideal. In the second version of the society, either B and C' or A and C' coalesce. If the former, B and C' set policy at $(\frac{1}{2}, 1)$, and the excluded faction A suffers a smaller loss, as policy is now only $\sqrt{1.25}$ away from it. If A and C' set policy at $(0, \frac{1}{2})$, B suffers an equivalent loss.

While the intuition conveyed by the above examples may seem straightforward, we next show that it relies on an implicit, yet clearly implausible, assumption: namely, that disagreement on either dimension is "all or nothing." To see the effect of relaxing this assumption, consider the next example, which provides a variation on the 2x2 example above. Suppose that divisions between factions are no longer binary, but may differ by degree. In addition to the M and R positions, a faction can also occupy a left (L) position on the horizontal cleavage, which we will assume is at -2 on the horizontal axis; likewise, in addition to the I and D positions, a faction can occupy either an elevated (E) or up (U) position. We will assume the elevated position is at 2 on the vertical axis, and that up is at 4. In the first version of this society, A and B continue to occupy MD and RI as before, but C occupies LE. In the second, A and B are unchanged but C (relabeled C') occupies

Figure 2: Introducing "Distance" Makes the Effect of Crosscuttingness on Polarization Ambiguous

	$L=-2$	$M=0$	$R=1$
$U=4$			
$E=2$	C		
$I=1$			B
$D=0$		A	

$L=-2$	$M=0$	$R=1$
C'		B
	A	

$L=-2$	$M=0$	$R=1$
	C''	
		B
	A	

LI. In the third, C (relabeled C'') occupies the MU. These three preference configurations are depicted in Figure 2.

In all three configurations, given the assumptions described above, A and B coalesce to set policy at $(\frac{1}{2}, \frac{1}{2})$, and C is the outlier faction. In the first configuration, depicted in the left panel, no pair of factions share a common position on either dimension, but clearly, A and B are more closely aligned on the horizontal dimension than either is to C; and on the vertical dimension, A and C are further from each other than either is to B. Because of the absence of commonality, crosscuttingness, per the Rae and Taylor measure, is zero. The loss suffered by $C = (-2, 2)$ from the equilibrium policy is equal to $\sqrt{8.5}$. A change from C to $C'' = (-2, 1)$ (as depicted in the center panel) increases crosscuttingness to 1/3, while decreasing the outlier faction's loss to $\sqrt{6.5}$. Thus, based on this comparison, the canonical intuition appears to hold. But consider a change from the first configuration to the third (depicted in the right panel). A change in the outlier faction from C to $C'' = (0, 4)$ increases crosscuttingness from zero to 1/3. Now, however, it *increases* the outlier faction's loss, from $\sqrt{8.5}$ to $\sqrt{12.5}$.

Why does the comparison of the left and middle panels reproduce the canonical intuition, whereas the comparison of the left and right panels does not? Clearly, the answer is related to distance: in the shift from the left to the right variations, the upward shift from C to C'', which does not contribute to the increase in crosscuttingness, swamps the rightward shift, which does. This is the case even though the horizontal and vertical shifts are, in distance terms, the same.

3 A Model of Crosscutting Cleavages

3.1 Basic Primitives

Three equally-sized factions, labeled A, B, and C, compete to set policy in a two-dimensional policy space, with the dimensions labeled x and y. The three factions' ideal points are given by $A = (x_A, y_A)$, $B = (x_B, y_B)$, and $C = (x_C, y_C)$. We normalize the locations of A and B to $A = (0, 0)$ and $B = (1, 1)$, and permit C to be any point in $\mathbb{R}^2$. We assume throughout that all factions share the functional form of the utility function, which is a version of the standard Euclidean measure of the distance between two points but explicitly weighted by the relative "importance" of the corresponding dimensions. In particular, let $\omega \in [0, 1]$ be the *salience* parameter, which describes the weight of the second, y-dimension relative to the x-dimension; the x-dimension is given the corresponding weight of $1 - \omega$. Given two factions I and J, the *ω-weighted Euclidean distance* between them, D^{ω}_{IJ}, is given by

$$D^{\omega}_{IJ} = [(1 - \omega)(x_I - x_J)^2 + \omega(y_I - y_J)^2]^{\frac{1}{2}}.$$

A faction's utility, then, is simply a decreasing, linear function of the ω-weighted Euclidean distance between its ideal point and the implemented policy vector. Given these assumptions, the conjunction of the location of C's ideal point and the salience parameter ω offers a complete description of the preference profile in the polity.

A political system is characterized by the two elements: (1) a preference set $(\rho, u(\cdot, \omega))$, which is given by a pair of a profile ρ of factions' ideal points and the corresponding vector of utility functions $u(\cdot, \omega)$, which reflects the underlying value of the salience parameter ω; and (2) an institutional framework $\mathcal{I}$, which determines the rules governing coalition formation among the three factions and how policy is made. Because the functional form of utility function is common and held fixed, we will suppress the dependence on it in what follows, except for its corresponding value of ω. As a shorthand, we will refer to a generic political system as $(\rho, \omega, \mathcal{I})$.

We will consider two institutional environments for policy-making. The first, which we will refer to as the *bundled policy-making* environment, is an environment in which a single governing coalition consisting of a majority of factions sets policy on both policy dimensions simultaneously. We abstract away from details of intra-coalitional bargaining and assume, instead, that the policy is set at the centroid of its participating factions' coordinates, leaving as the salient choice for the factions which other factions to coalesce with in anticipation of policy selection.

For factions I, J, and K, we will speak of the coalition IJ as corresponding to the situation where factions I and J form a governing coalition, and likewise of a coalition IJK in the case of a grand coalition consisting of all the factions.

Apart from the bundled policy-making environment, we also consider another commonly discussed institutional setting, in which institutional policy is made separately on each policy dimension. We will refer to it as the *unbundled policy-making* environment. In that environment, we will, likewise, speak of a x-coalition IJ_x as the governing coalition of I and J formed with respect to the policy-making on dimension x, and similarly for y-coalitions for the policy-making on dimension y.

3.2 Measures of Crosscuttingness

We now move to defining precisely the notion of "crosscutting cleavages" for a two-dimensional polity. (Extension to an arbitrary n-dimensional polity is cumbersome but obvious.) First, call the profile of ideal points projected onto a given dimension z, ρ_z, the "cleavage" on that dimension.[3] According to this terminology, the "cleavage structure" of the polity is simply the profile of two-dimensional ideal points, ρ. Our measures of crosscutting cleavages capture fundamental differences between ρ_x and ρ_y.

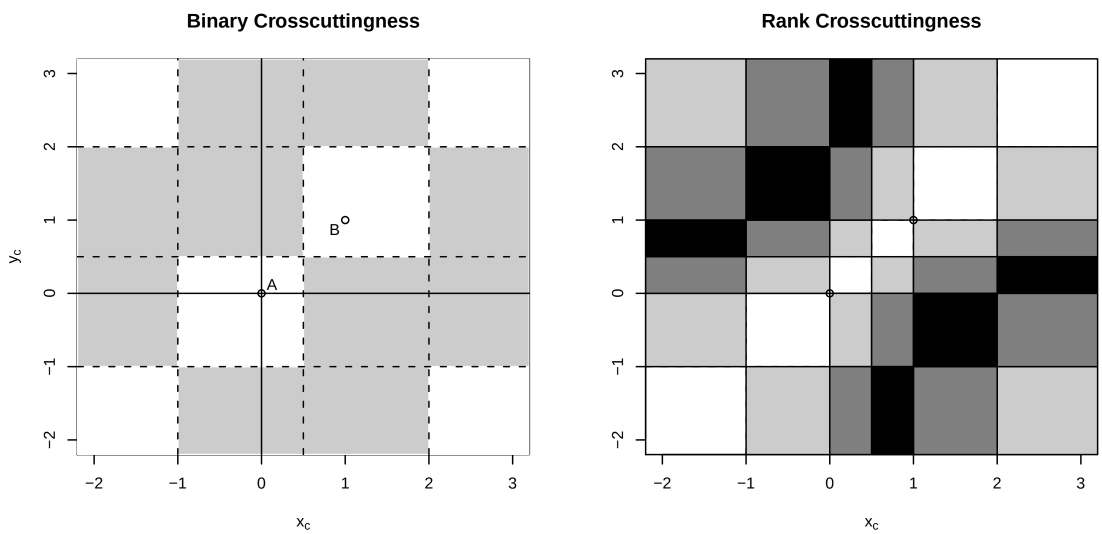
The first of these measures is binary:

Definition 1 (Binary Crosscuttingness)**.** *A profile of ideal points ρ is **binary crosscutting** if the subset of factions that generates the two closest points in ρ_x differs from the subset of factions that generates the two closest points in ρ_y.*

For example, suppose, as assumed above, that $A = (0, 0)$, $B = (1, 1)$, and that, further, $C = (\frac{3}{4}, -\frac{1}{2})$. Then cleavages are crosscutting: B and C are the closest factions on the x-dimension,

[3] Generally, and in this model in particular, we assume that cleavages are "non-zero," by which we mean that the factions' ideal points do not collapse to the same point on that dimension.

Figure 3: Crosscutting Cleavages as a Function of the Location of Faction C



Darker shading indicates greater crosscuttingness under the corresponding measure.

whereas, A and C are the closest on the y-dimension. If, instead, $C = (\frac{3}{4}, \frac{3}{4})$, then B and C are the closest factions on both dimensions, and cleavages are not crosscutting. The left panel in Figure 3 displays locations of C for which cleavages may be said to be binary crosscutting in our model.

Informally, if cleavages are binary crosscutting, no faction finds itself an "outlier" on both dimensions. For convenience, below we use below a (binary) index, denoted $Q^b \in \{0, 1\}$, to indicate whether the cleavages are binary crosscutting, with $Q^b(\rho) = 0$ indicating that ρ is not binary crosscutting, and $Q^b(\rho) = 1$ indicating that ρ is.

Our second measure of crosscuttingness generalizes this intuition, giving rise to finer variations in the extent of crosscuttingness. To define it, construct binary rankings of distances between ideal point projections onto dimension x, and separately, between projections onto dimension y. Denote by $(\overline{IJ})_x$ the distance between the projections on x of arbitrary points I and J. Given ideal points I, J, K, and L, and dimensions x, y we will say that $(\overline{IJ}, \overline{KL})$ ranking is *reversed across issue dimensions* x, y if and only if $(\overline{IJ})_x$, is smaller (larger) than $(\overline{KL})_x$ but $(\overline{KL})_y$ is smaller (larger) than $(\overline{IJ})_y$. We, then, have the following definitions:

Definition 2 (Rank Crosscuttingness)**.** *A profile ρ's* ***rank crosscuttingness index*** *$Q^r(\rho)$ is the* *total number of rankings reversed across issue dimensions for the points in ρ. A profile of ideal points ρ has* ***higher rank crosscuttingness*** *than the profile ρ' if and only if $Q^r(\rho) > Q^r(\rho')$.*

To take the two above examples, if $C = (\frac{3}{4}, -\frac{1}{2})$, then $(\overline{BC})_x < (\overline{AC})_x < (\overline{AB})_x$, and $(\overline{AC})_y < (\overline{AB})_y < (\overline{BC})_y$. $(\overline{BC}, \overline{AC})$ and $(\overline{BC}, \overline{AB})$ are reversed across issue dimension, while $(\overline{AC}, \overline{AB})$ is not; therefore $Q^r = 2$. If, by contrast, $C = (\frac{3}{4}, \frac{3}{4})$, then there are no reversals across issue dimensions and $Q^r = 0$. Note that in our model $Q^r(\rho) \in \{0, 1, 2, 3\}$. The right panel in Figure 3 displays areas of C that give rise to different Q^r values, given the fixed positions of A and B.

The comparison of that panel to the one illustrating binary crosscuttingness suggests clear similarities. These are not coincidental: binary crosscuttingness is also an index that counts ranking reversals across issue dimensions, but it is concerned with a possibility of a single reversal, in effect, asking whether there is at least one reversal across issue dimensions for the rankings that include shortest distances between ideal point projections. For example, there are no such reversals in the white square around point B in the binary crosscuttingness panel, but for locations of C in that square, there exist rankings reversed across issues dimensions that do not change the ideal points that give rise to the shortest distances between corresponding projections - in particular, the $(\overline{BC}, \overline{AC})$ is reversed in the lightest-gray rectangles nested in that square in the rank crosscuttingness panel.

STATE THAT THESE MEASURES ARE SPATIAL GENERALIZATIONS OF TAYLOR RAE – B/C XC=0 EXCEPT IN KNIFE'S EDGE CASES.

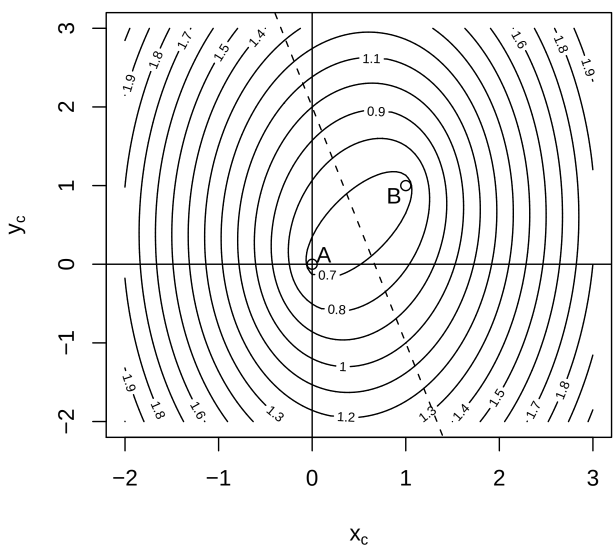
3.3 Measures of Disagreement and Estrangement

Next, we introduce a measure of the *potential* for political conflict brought about by the extent of disagreement within a polity (given the relative salience of the issue dimensions), independent of the polity's political institutions.

Definition 3. *The* ***mean latent disagreement*** *(MLD) in a political system, denoted $\overline{D}^{\omega}$, is the average ω-weighted Euclidean distance between the ideal points of all pairs of factions.*

For example, suppose $\omega = 0.25$ and $C = (3, 2)$. Then given our assumptions about the locations of A and B, MLD would be $\overline{D}^{.25} = \frac{1}{3}(D^{.25}_{AB} + D^{.25}_{AC} + D^{.25}_{BC}) \approx 1.86$. The level curves for mean latent

Figure 4: Level curves of Mean Latent Disagreement for different values of C and Axis of Symmetry at $\omega = 0.25$



disagreement given $\omega = 0.25$ are depicted in Figure 4. Also depicted in the figure is the *axis of symmetry*, defined as the set of points for which C is equidistant from A and B ($D^{\omega}_{AC} = D^{\omega}_{BC}$). Holding ω fixed, for any value of C above the axis of symmetry there exists a value C', below that induces an equivalent profile of ideal points (ignoring labels) up to a rotation about the axis.

Mean latent disagreement provides a spatial generalization of the familiar ethnolinguistic fractionalization (ELF) index from comparative politics. ELF may be interpreted as the probability that any two randomly drawn individuals belong to different ethnolinguistic categories. A common observation regarding ELF is that it does not incorporate information about the extent of policy disagreement among those categories. For example, if A, B, and C were ethnic groups, ELF in our model would be equal to $1/3$ irrespective of their locations in the policy space or the salience of the policy dimensions. By contrast, mean latent disagreement does take this information into account.

As the name implies, however, $\overline{D}^{\omega}$ is a measure of latent, rather than actual, conflict. The next measure captures the maximal estrangement of the various factions from the equilibrium policies produced by the political system, and consequently, the potential for realized conflict in the political system. As will become clear below, this measure depends both on the underlying policy-making institutions as well as on the preference profile of the citizens.

Definition 4. ***Maximal Estrangement*** *in a political system,* $\Pi(\omega, \rho; \mathcal{I})$ *is the largest* ω*-weighted Euclidean distance between a faction's ideal point and the equilibrium policy vector.*

If political stability is a measure of political welfare of societies, and actions that affect that stability are a function of agents' (dis-)satisfaction with the policy status quo chosen under a given set of political institutions, then maximal estrangement must be a key relevant statistic of political welfare. Higher maximal estrangement means that the agents who are least satisfied with the outcome of the political system (i.e., agents with highest estrangement) may be more enticed to actions that subvert it. We suppress the microfounded model that would generate this type of result.

3.4 The Bundled Policy-Making Model

3.4.1 Equilibrium

We begin with a characterization of the equilibrium of the bundled policy making game, and then move to the analysis of the effects of crosscutting cleavages under the corresponding bundled policy-making institutions.

The following proposition summarizes the equilibrium coalitions and outcomes:

Proposition 1. *In the unique Nash equilibrium of the bundled policy-making model, the governing coalitions and policy vector* (x, y) *are, respectively:*

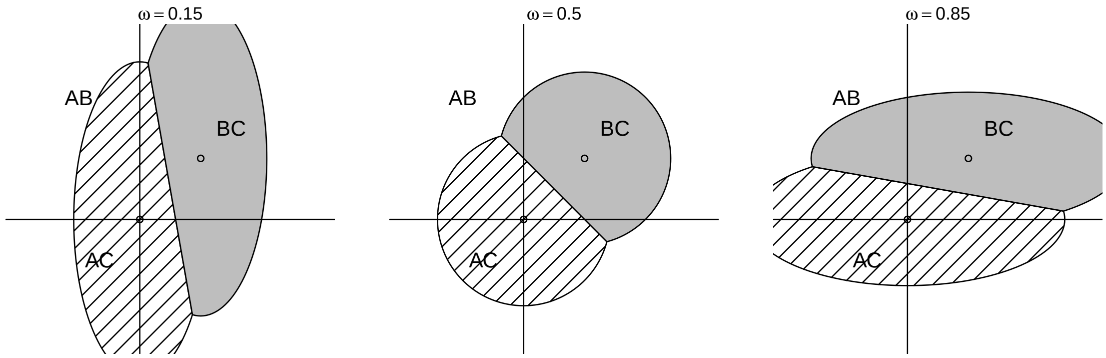
(1) AB *and* $(\frac{1}{2}, \frac{1}{2})$ *if and only if* $D^{\omega}_{AB} \leq D^{\omega}_{AC}$ *and* $D^{\omega}_{AB} \leq D^{\omega}_{BC}$*;*

(2) AC *and* $(\frac{x_C}{2}, \frac{y_C}{2})$ *if and only if* $D^{\omega}_{AC} \leq D^{\omega}_{AB}$ *and* $D^{\omega}_{AC} \leq D^{\omega}_{BC}$*;*

(3) BC *and* $(\frac{1+x_C}{2}, \frac{1+y_C}{2})$ *if and only if* $D^{\omega}_{BC} \leq D^{\omega}_{AB}$ *and* $D^{\omega}_{BC} \leq D^{\omega}_{AC}$*.*

In equilibrium, the two closest factions will form a coalition and set policy at the midpoint of the line segment connecting their ideal points. The equilibrium coalition that sets policy is minimum

Figure 5: Equilibrium Governing Coalitions for Three Different Values of ω



winning because in a "grand" coalition of all three factions, two of the three factions could always be made better off by excluding the third.

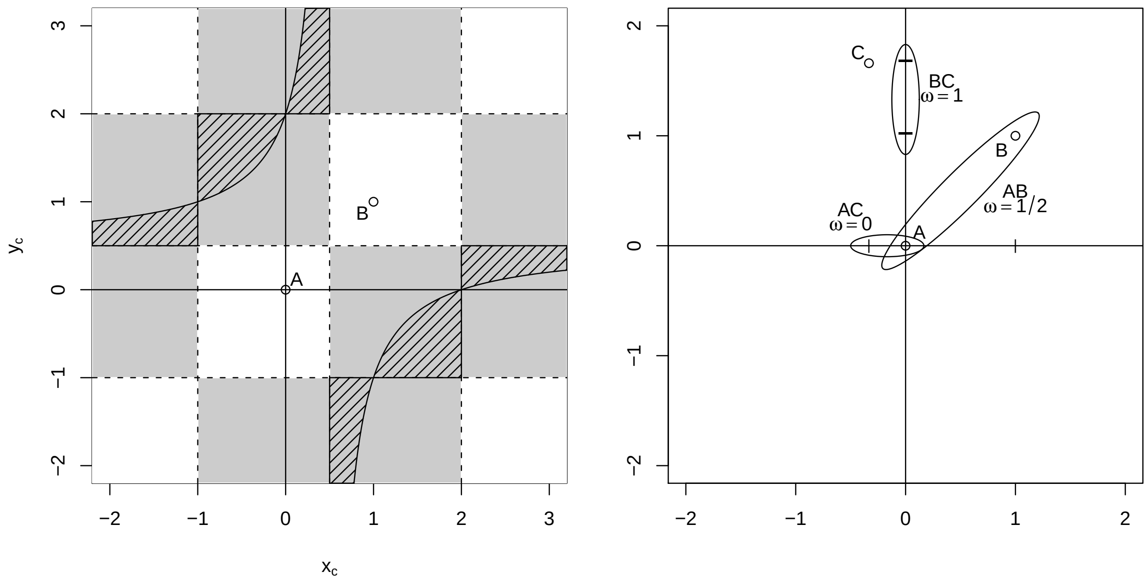
It is noteworthy that the equilibrium coalition that forms will depend not only on the locations of the three factions, but also on the relative salience of the two dimensions, as summarized by the parameter ω. This dependence is encapsulated in the following remark:

Remark 1. *(1) In the bundled policy-making environment, the equilibrium governing coalition depends on the relative salience of the two policy dimensions. (2) For a fixed set of ideal points, different values of the relative salience of the two policy dimensions can give rise to one, two, or three distinct governing coalitions under conditions displayed in the left panel of Figure 6.*

Figure 5 displays the first part of the remark graphically. First, consider the middle panel, which corresponds to the situation in which the policy dimensions are equally weighted. The gray region corresponds to locations of faction C in which B and C will form a coalition against A; likewise, the striped region contains locations of C that yield coalitions of A and C against B. Finally, the white area corresponds to situations in which C is sufficiently far from A and B that A and B will form a coalition against C.

Note, however, that the sets of preference profiles giving rise to these coalitions vary as a function of ω. In the left panel of Figure 5, the x-dimension is significantly more salient than the y-dimension. This results in a vertical stretch of the AC and BC regions. In the limit, as ω

Figure 6: Coalitions at Intermediate Levels of Relative Salience May Differ from Coalitions That Would Form on the x- and y-Dimensions.



In the left panel, white regions correspond to values of C for which the same governing coalition will form irrespective of the value of ω; gray regions to values for which two governing coalitions are possible depending on ω, and striped regions to values for which three are possible. The right panel gives an example of the third category, in which $C = (-\frac{1}{3}, \frac{5}{3})$.

approaches zero (from the right), the corresponding ellipses converge to vertical bands – the value of x_C is all that matters, while y_C is irrelevant. Likewise, the right panel of the figure corresponds to a situation in which the y-dimension is significantly more salient than the x. Here, the stretch is horizontal; as ω approaches one, the ellipses would converge to horizontal bands.

The second part of the remark is depicted graphically in the left panel of Figure 6. The white regions in the figure correspond to locations of C in which only one coalition is possible irrespective of the value of ω. For example, if C is within the same white region as A, A and C will coalesce against B no matter what the relative salience of the two policy dimensions. Note that these regions correspond to areas of C-space in which cleavages are not binary crosscutting, as described above and in Figure 3.

The solid gray regions in the figure correspond to locations in which two coalitions are possible. In these situations, for low values of ω the equilibrium coalition profile is the same as that which would form were politics restricted to the x-dimension, whereas for high values of ω the coalition profile is that which would occur were relevant disagreement only on the y-dimension.

Surprisingly, however, areas of the parameter space in which two and only two coalitions are possible are a proper subset of the areas in which cleavages are binary crosscutting. This is because values of C exist such that any of the three possible governing coalitions may form in equilibrium: these values are denoted by the striped regions in Figure 6. In those regions, intermediate values of ω give rise to a different coalition profile than would occur if politics were restricted to either the $x-$ or the $y-$ dimension.

The right panel of Figure 6 provides a useful example. Suppose $C = (-\frac{1}{3}, \frac{5}{3})$. For low values of ω, A and C coalesce against B; whereas for high values, B and C coalesce against A. For intermediate values of ω, A and B coalesce against C.

3.4.2 Estrangement and Crosscuttingness in the Bundled Policy Environment

CHAIN RULE ARGUMENT

We now turn to our analysis of the relationship between crosscuttingness and political estrangement. We begin by considering the effect of vertical or horizontal changes in the location of C. Because our measures of crosscuttingness, Q^b or Q^r, are discrete, we are interested in changes in C that would yield discrete changes in either or both of those measures. If the change in C that produces an increase in crosscuttingness also produces a decrease in maximal estrangement, this would be consistent with the canonical intuition. However, a finding that a change in C produced an increase in both crosscuttingness *and* maximal estrangement would be inconsistent with the canonical intuition.

The following lemma establishes conditions under which a horizontal or vertical change in C increases or decreases maximal estrangement.

Lemma 1. *An increase in x_C (y_C) yields an increase in maximal estrangement if and only if*

1. *the governing coalition is AB and $x_C > \frac{1}{2}$ ($y_C > \frac{1}{2}$); or*

2. *the governing coalition is AC and $x_C > 2$ ($y_C > 2$); or*

3. the governing coalition is BC and $x_C > -1$ ($y_C > -1$).

Proof. See Appendix. □

Intuitively, if C is the outlier faction, then a horizontal (vertical) increase in the direction of the equilibrium policy of $(\frac{1}{2}, \frac{1}{2})$ will reduce maximal estrangement, while an increase away from the equilibrium policy will increase estrangement. By contrast, if either A or B is the outlier faction, than holding the governing coalition fixed, a change in C moves the equilibrium policy closer to or farther away from the outlier, leading to a corresponding decrease or increase in the estrangement of that faction.

Our next result relates the discrete changes in crosscuttingness (binary and rank) brought about by vertical or horizontal changes in C to changes in maximal estrangement brought about by the same change in C.

Proposition 2. *Under bundled policy-making, if C is the outlier faction then a horizontal or vertical change in C that increases either binary or rank crosscuttingness strictly decreases maximal estrangement. If A or B is the outlier faction, then a horizontal or vertical change in C that increases either binary or rank crosscuttingness can either increase or decrease maximal estrangement.*

Proof. See Appendix. □

To understand the logic behind the first part of the result, suppose that y_C is sufficiently high that C is the outlier faction for any value of x_C. From the definitions of binary and rank-crosscuttingness above, increases in x_C weakly increase both kinds of crosscuttingness for $x_C < 1/2$ – precisely the region in which maximal estrangement is decreasing in x_C. Likewise, increases in x_C weakly decrease both kinds of crosscuttingness for $x_c > 1/2$ – when maximal estrangement is increasing in x_C. An analogous intuition holds for horizontal changes in x_C when $y_C < -1$, and for vertical changes in y_C when $x_C < -1$ and $x_C > 2$.

As noted above, however, matters change when A or B is the outlier faction, because a change in C brings about a change in the equilibrium policy. Depending on the initial location of C, such a change can either increase or decrease maximal estrangement. For example, suppose that $\omega = \frac{1}{2}$

and $C = (2-\varepsilon, 1\frac{1}{2})$. Here, A is the outlier faction. An increase in x_C to $2+\varepsilon$ increases both binary and rank crosscuttingness. It also pulls the equilibrium policy to the right, thereby increasing A's estrangement. By contrast, suppose $C = (\frac{1}{2}+\varepsilon, 1)$. A decrease in x_C to $\frac{1}{2}-\varepsilon$ increases both kinds of crosscuttingness while decreasing estrangement.

Our interpretation of this result is two-fold. First, it underscores the point made by our motivating example – that the relationship between crosscutting cleavages and polities' welfare is not as straightforward as we may have been accustomed to thinking. Second, we interpret this result as a kind of baseline relative to which we may seek to refine the analysis of the effects of crosscutting cleavages. In particular, the ambiguity of the result highlights an important fact about the relationship between crosscuttingness and estrangement: namely, that change in the preference profile that brings about a change in the former can also bring about change in the latter via two mechanisms.

The first mechanism is associated with change in the *relative* distances between factions. It is these relative changes that underlie our construction of both the binary and rank crosscuttingness indices. The second mechanism is associated with changes in *absolute* distances. In other words, a vertical or horizontal shift in C does not generically hold mean latent disagreement fixed. In the examples offered above, the contemplated shifts that resulted in decreases in estrangement also simultaneously decreased mean latent disagreement.

To be sure, in the standard economic comparative statics analysis, *ceteris paribus* clauses hold fixed changes that are *not* consequences of the change in the variable that is the focus of comparative statics. In contrast, here, the changes in $\bar{D}^{\omega}$ are a direct consequence of a change in C that alters relative crosscuttingness.

Nonetheless, we wish to rule out the possibility that the relationships documented in Proposition 2 arise as artifacts of changes in the polity's latent degree of conflict. To do so, we next consider changes in x_C *along level curves of mean latent disagreement,* and, in so doing, get a stronger result on the *direct* effect of crosscuttingness. In other words, we examine counterfactual changes in the preference profile that vary the extent of crosscuttingness while holding mean latent disagreement fixed.

The following lemma articulates how changes in the location of C along a level curve of mean latent disagreement affect the degree of maximal estrangement.

Lemma 2. *Under bundled policy-making, a change in C that decreases the distance between C and the axis of symmetry while holding mean latent disagreement fixed strictly decreases maximal estangement.*

Proof. See Appendix. □

When C is the outlier faction, the posited change moves C closer to the equilibrium policy of $(\frac{1}{2}, \frac{1}{2})$. Otherwise, the posited change moves the equilibrium policy closer to the outlier faction.

With this Lemma in hand, we are now in a position to relate changes in crosscuttingness to changes in maximal estrangement, holding mean latent disagreement fixed.

Proposition 3. *Under bundled policy-making, holding constant mean latent disagreement, (a) if mean latent disagreement is sufficiently low, a change in C that increases binary crosscuttingness strictly decreases maximal estrangement; (b) if mean latent disagreement is sufficiently high, a change in C that increases binary or rank crosscuttingness can either increase or decrease maximal estrangement.*

WHY IS THERE AN ASYMMETRY BETWEEN (a) AND (b) PARTS?

Proof. See Appendix. □

To understand part (a) of the result, concerning binary crosscuttingness, note that for low values of mean latent disagreement, a move in the direction of the axis of symmetry, which from Lemma 2 produces a decrease in maximal estrangement, also corresponds to an increase in binary crosscuttingness – for example, a shift from the white region containing B in the left panel of Figure 3 to the gray region to either of the adjacent gray regions. By contrast, suppose mean latent disagreement is high, and $C = (-1 + \varepsilon, 2\frac{1}{2})$. A change in C toward the axis of symmetry holding mean latent disagreement fixed to $C' = (-1 - \varepsilon, 2\frac{1}{2} - \delta)$, would simultaneously *decrease* binary crosscuttingness while decreasing estrangement.

Part (b) of the result emerges because changes in C along a MLD curve in the direction of the axis of symmetry yield generically non-monotonic changes in rank crosscuttingness. For example, suppose mean latent disagreement is high. Then a change in C toward the axis of symmetry can produce a transition from $Q^r = 2$ to $Q^r = 3$ to $Q^r = 2$ to $Q^r = 1$, as is evident from the right panel of Figure 3 for $y_C > 2$ and $x_C < 1$. In this sense, the result in part (b) parallels part (a). However, when considering rank crosscuttingness, the ambiguity persists when mean latent disagreement is *low*. For example, a change in C toward the axis of symmetry along a low $\overline{D}^{\omega}$ level curve in the neighborhood of B can transition from $Q^r = 1$ to $Q^r = 2$ and back to $Q^r = 1$, while C on an even still lower level curve can pass through $Q^r = 1$ to $Q^r = 0$ and back to $Q^r = 1$.

Interestingly, for at least some values of ω, the canonical intuition concerning the negative relationship between crosscuttingness and estrangement holds for a *band* of different values of mean latent disagreement. The following corollary provides a result for the special case of $\omega = \frac{1}{2}$.

Corollary 1. *Suppose $\omega = \frac{1}{2}$. Then a change in C that holds constant mean latent disagreement and that increases rank crosscuttingness strictly decreases maximal estrangement if and only if $\overline{D}^{\omega} \in (1, \frac{4}{3})$.*

3.4.3 Estrangement, Realignment, and Political Salience

Our next set of results concerns the effect of changes in the relative salience of the two policy dimensions. Note that in our model, the fact that cleavages are or are not crosscutting does not have uniformly remedial effects on estrangement except in unusual circumstances. Note, however, that this may be a consequence of the fact that when governance is conducted in a bundled policy making system, one faction is invariably excluded. If the political rationale for crosscutting cleavages is that they create the potential for all factions to participate in the political process (i.e., through a system of power-sharing), bundled policy-making does not permit this. In the bundled environment, the more relevant notion of power-sharing is intertemporal: a faction may be out of power today, but is just one shock away from being brought to power tomorrow. In this sense, the channel through which crosscuttingness manifests its relevant effect on estrangement is through increases in the susceptibility of the political system to *realignment*.

In our model, the relative salience of policy dimensions does not have an effect on crosscuttingness. However, changes in salience can affect the likelihood of realignment (e.g., Burnham 1975; Sundquist 1983). Perhaps susceptibility to realignment, by increasing the stake of an outlier faction, reduces estrangement and (ultimately) improves political stability.

Suppose C is located in an area of C-space in which cleavages are binary crosscutting, so realignment is feasible. A change in salience could, in principle, bring about one or (via Remark 1) two realignments. Let $\omega^*(C)$ represent a critical value of ω that yields a transition from one coalition profile to another. (Because values of C exist that give rise to three possible governing coalitions depending on ω, $\omega^*(C)$ need not be unique.) Changes in ω in the direction of ω^* represent increases in the susceptibility of the political system to realignment; when $\omega = \omega^*$, that susceptibility is at its maximum. Of course, an increase in susceptibility can occur given a decrease in ω given $\omega > \omega^*$, or an increase in ω given $\omega < \omega^*$.

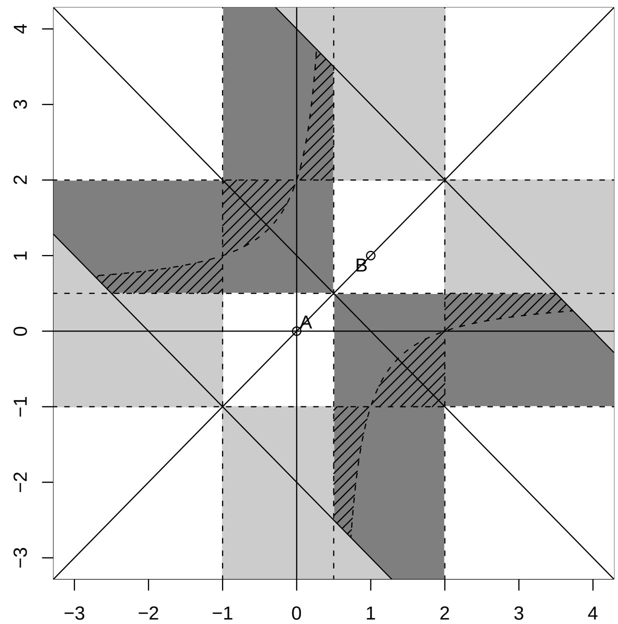
This yields three possibilities: it could be that estrangement is *locally minimized* at ω^*, which would be indicated by $\frac{\partial \hat{S}}{\partial \omega}|\omega < \omega^* < 0$ and $\frac{\partial \hat{S}}{\partial \omega}|\omega > \omega^* > 0$; that disaffection is *locally maximized* at ω^* ($\frac{\partial \hat{S}}{\partial \omega}|\omega < \omega^* > 0$ and $\frac{\partial \hat{S}}{\partial \omega}|\omega > \omega^* < 0$; or that increasing susceptibility depends on the status quo alignment: e.g., given the possibility of realignment from AB to AC, increasing susceptibility to realignment increases estrangement if the current coalition is AB, but decreases it if the current coalition is AC. We have the following result:

Proposition 4. *A change in ω that increases the susceptibility of a political system to realignment can lead to an increase or decrease in maximal estrangement. However, maximal estrangement is never locally maximized at the point of transition.*

Proof. See Appendix. □

As the proposition indicates, it is never the case that changing the relative salience of dimensions in such a way as to increase the political system's susceptibility to realignment can increase estrangement *irrespective of the status quo alignment.* That said, the relationship between susceptibility to realignment and maximal estrangement is frequently ambiguous. In particular, values of C exist for which the effect of an increase in susceptibility is contingent on the status quo. The

Figure 7: The Relationship Between Susceptibility to Realignment and Maximal Estrangement



Pale gray regions in the figure correspond to values of C for which an increase in susceptibility to realignment can either increase or decrease estrangement, depending on the status quo alignment. Darker gray regions correspond to values of C for which maximal susceptibility to realignment is associated with local estrangement minima. For values of C in the striped regions, two realignments are possible: maximal susceptibility to one gives rise to a local estrangement minimum, while an increase in susceptibility to the other can either increase or decrease estrangement depending on the status quo alignment.

pale gray regions in Figure 7 correspond to such cases.

In the dark gray regions of the figure, one realignment is possible, and maximal estrangement is locally minimized when the system is maximally susceptible to realignment. As noted in Remark 6, in the striped regions, two realignments are possible. Increases in the susceptibility of the system to one of those realignments can either increase or decrease maximal estrangement depending on the current alignment. And maximal susceptibility to the other locally minimizes estrangement.

Thus, to the extent that susceptibility to realignment can be interpreted as the potential for power-sharing, the intuition that increasing that susceptibility invariably lowers estrangement is partially confirmed and partially disconfirmed. In particular, the validity of the intuition is contingent on the profile of ideal points and/or the current alignment. In some cases, maximal susceptibility to realignment does indeed locally minimize estrangement. For others, however, increasing susceptibility can increase or decrease estrangement, depending on the status quo governing coalition.

3.5 Crosscutting Cleavages with Unbundled Policy Making

A number of scholars have suggested that highly divided societies might decentralize policy making authority to protect the polity against the centrifugal forces of factional conflict. With this in mind, we next consider a model in which the dimensions of political conflict are unbundled, in the sense that conflict on one dimension does not affect conflict on the other. As above, factions do not vote directly on policy; rather, their choices are simply which other faction(s) to join with on a particular policy dimension in order to form a majority coalition that then sets policy.[4]

Again, we do not model intra-coalitional bargaining explicitly. Instead factions choose which other factions to coalesce with on each dimension, with the majority coalition on a dimension (which may consist of either two or three factions) setting policy at the mean of its factions' coordinates on that dimension.

As above, the coalition that sets policy on each dimension will be minimum winning – that is, it will consist of two factions. This is because in a "grand" coalition of all three factions, two of the three factions could always be made better off by excluding the third.

With this in mind, denote by $(IJ)_x$ an outcome corresponding to the formation of a governing coalition between factions I and J on the x dimension. We will refer to $(IJ)_x$ as the *x-coalition*, and corresponding $(IJ)_y$, the outcome on the y-dimension, as the *y-coalition.* The Nash equilibrium of the game is very straightforward: for two factions to form a coalition on a given dimension, they must each prefer the policy that would emerge from coalescing with the other to the one that emerge from coalescing with the third faction. This is equivalent to each coalition member being closer to the other on the relevant dimension than to the third faction. Thus:

[4]For purposes of comparison, we explore a variant of unbundled policy making in which policy on each dimension is determined by majority rule in the Appendix. The primary insight of this exploration is the dependence of estrangement, defined below, on the allocation of agenda-control rights and the location of the status quo.

Proposition 5. *In equilibrium, z-coalition profiles and policy p_z on the z dimension for $z \in \{x, y\}$ are, respectively:*

(1) $(AB)_z$, $p_z = \frac{1}{2}$ if and only if $|z_C| \geq 1$ and $|z_C - 1| \geq 1$

(2) $(AC)_z$, $p_z = \frac{z_C}{2}$ if and only if $|z_C| \leq 1$ and $|z_C| \leq |z_C - 1|$

(3) $(BC)_z$, $p_z = \frac{1+z_C}{2}$ if and only if $|z_C - 1| \leq 1$ and $|z_C - 1| \leq |z_C|$.

Areas of the space of Cs that give rise to different coalition configurations are delineated by the left panel of Figure 3: in other words, unbundled policy making instantiates binary crosscuttingness. For example, if $C = (0, 1)$, the coalition profile on the x-dimension would be $(AC)_x$, while the y-coalition profile would be $(BC)_y$.

Note that the *existence* of binary crosscutting cleavages is independent of the relative salience of the two dimensions as represented by ω (cf. Rabushka and Shepsle 1972; Lijphart 1977). Moreover, we have the following remark:

Remark 2. *Under unbundled policy making, for a given preference profile ρ, z-coalitions that form in equilibrium are independent of the value of ω.*

Indeed, this remark is a direct consequence of the separability of dimensions, and reflects a significant departure from the bundled policy making environment. It may be the case, in other words, that a faction is only included in the coalition setting policy on a relatively unimportant dimension.

3.5.1 Estrangement Under Unbundled Policy Making

We next consider the relationship between crosscuttingness and maximal estrangement under an unbundled governance structure. The first result in this section considers the effects of vertical or horizontal changes in C.

Proposition 6. *In the unbundled policy making environment, a horizontal (vertical) change in C that yields an increase in binary crosscuttingness strictly decreases maximal estrangement.*

Proof to be added, along with result holding mean latent disagreement fixed.

3.6 Institutional Comparisons

Next, we examine how, for a given preference profile and issue salience, institutions affect political conflict. First, as noted above, a common argument in favor of decentralizing policy making is that it will reduce the degree of estrangement by giving a stake to groups that would otherwise be left out of the decision making process. As the next result suggests, however, decentralization is not necessarily a panacea, *even in the presence of crosscutting cleavages.*

Proposition 7. *If cleavages are not binary crosscutting, then maximal estrangement is the same under bundled and unbundled policy making. If cleavages are binary crosscutting, and the outlier faction is the same under both institutions and sufficiently centrist on one dimension and sufficiently extreme on the other, then maximal estrangement is strictly higher under unbundled than under bundled policy making. Otherwise, maximal estrangement is strictly higher under bundled than under unbundled policy making.*

Proof. See Appendix □

The intuition behind the first part of the proposition is immediate: if cleavages are not crosscutting, then the equilibrium policy (and thus the maximally estranged faction) are the same under bundling and unbundling. To understand the second part of the intuition, imagine that $C = (0.48, 3)$ is the maximally estranged faction under both institutions, and that $\omega = \frac{1}{2}$. Clearly, C's estrangement is generated primarily by its vertical distance from A and B. Now note that under both bundling and unbundling, A and B will set policy at $\frac{1}{2}$ on the y-dimension. Under bundled policy making, A and B will set policy at $\frac{1}{2}$ on the x-dimension as well. But under unbundled policy making, A and C will set policy at 0.24. In other words, the policy on the horizontal dimension that would be arrived at via the multidimensional logroll between A and B is better for C than the policy it could get were it to join a governing coalition with A on that dimension. In other words, conditions exist under which an outlier faction would prefer *not* to govern.

The final result concerns the efficiency of policy making under different institutions:

Proposition 8. *Under bundled institutions, equilibrium policy is Pareto efficient. If cleavages are not binary crosscutting, then equilibrium policy under unbundled institutions is Pareto efficient. If* *cleavages are binary crosscutting, then equilibrium policy under unbundled institutions is Pareto efficient if and only if the preference profile exhibits maximal rank crosscuttingness.*

Proof. See Appendix □

Under bundled institutions or unbundled institutions when cleavages are not binary crosscutting, policy lies on the contract curve between two factions and is thus Pareto efficient. Under unbundled institutions when cleavages are binary crosscutting, conditions can emerge in which the equilibrium policy falls outside of the triangle formed by the ideal points of factions A, B, and C. However, when $Q^r = 3$, policies are once again Pareto efficient.

The next remark follows immediately from Proposition 8

Remark 3. *Under unbundled policy making, Pareto efficiency is non-monotonic in rank crosscuttingness.*

4 Discussion and Extensions

Still to do: robustness of results relating crosscuttingness and maximal estrangement using spatial generalization of Rae and Taylor XC measure.

Add Appendix in which unbundled policy making follows median voter model

5 Conclusion

To come...

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Appendix

We begin with the following preliminary lemmata:

Lemma A 1. *The axis of symmetry defined by* $D^{\omega}_{AC} = D^{\omega}_{BC}$ *is* $y_C = -\frac{1-\omega}{\omega}x_C + \frac{1}{2\omega}$.

Proof. Substituting the expressions for D^{ω}_{AC} and D^{ω}_{BC} yields

$$\left((1-\omega)x_C^2 + \omega y_C^2\right)^{\frac{1}{2}} = \left((1-\omega)\left(x_C - 1\right)^2 + \omega\left(y_C - 1\right)^2\right)^{\frac{1}{2}}.$$

Solving for y_C gives the expression in the proposition. □

Lemma A 2. *Under bundled policy making, the excluded faction is the maximally estranged faction.*

Proof. In the triangle formed by the factions' ideal points, the midpoint of the shortest side is the equilibrium policy under bundled policy making. Suppose C is the excluded faction. Estrangement $\hat{S}$ is the length of the triangle's median from C to that point. The distance from either A or B to the implemented policy is $\frac{D^{\omega}_{AB}}{2}$. From Appolonius' theorem, $\hat{S} = \frac{1}{2}\sqrt{2D^{\omega 2}_{AC} + 2D^{\omega 2}_{BC} - D^{\omega 2}_{AB}}$.[5] $\hat{S} > \frac{D^{\omega}_{AB}}{2}$ if and only if $D^{\omega 2}_{AC} + D^{\omega 2}_{BC} > D^{\omega 2}_{AB}$. By the law of cosines, $D^{\omega 2}_{AB} = D^{\omega 2}_{AC} + D^{\omega 2}_{BC} - 2D^{\omega}_{AB}D^{\omega}_{BC}\cos\gamma$, where γ is the angle associated with vertex C. Substituting for $D^{\omega 2}_{AB}$, the inequality holds if $-2D^{\omega}_{AB}D^{\omega}_{BC}\cos\gamma < 0$. if C is the excluded faction, D^{ω}_{AB} is the shortest side of the triangle, implying γ is acute; therefore $\cos\gamma > 0$ and the inequality holds. Analogous proofs establish the results when A or B is the excluded faction. □

Lemma A 3. *Under unbundled policy making, the maximally estranged faction participates in the governing coalition on no more than one dimension.*

Proof. When cleavages are not binary crosscutting, equilibrium policy is identical under bundling and unbundling, and one faction participates in the governing coalition on neither dimension; by Lemma A2, it is also the most estranged. Suppose cleavages are binary crosscutting, and C is maximally estranged. It is straightforward to demonstrate that either $x_C < -1$ and $y_C \in (-1, 2)$, $x_C > 2$ and $y_C \in (-1, 2)$, $y_C < -1$ and $x_C \in (-1, 2)$, or $y_C > 2$ and $x_C \in (-1, 2)$. Given $A = (0, 0)$

[5] Establishing that Appolonius' theorem holds for ω-weighted Euclidean distance is trivial.

and $B = (1, 1)$, it is immediate that C does not participate in policy making on the x-dimension in the first two cases and on the y-dimension in the second two. Next, suppose A is maximally estranged. It is straightforward to demonstrate that this occurs when $y_C > 1 - x_C$, $y_C < 2$, and $x_C < 2$. For these values of C, A never coalesces with B on either dimension. A coalesces with C on the x-dimension if and only if $y_C > 1 - x_C$, $y_C < 2$, and $x_C < 0$, in which case B coalesces with C on the y-dimension. A coalesces with C on the y-dimension if and only if $y_C > 1 - x_C$, $y_C < 0$, and $x_C < 2$, in which case B coalesces with X on the x-dimension. The proof when B is maximally estranged is symmetric to that when A is. □

Lemma A 4. *(a) Under both bundled and unbundled policy making, if a faction is the dimension-by-dimension median it cannot be the most estranged faction. (b) Suppose cleavages are binary-crosscutting. Under unbundled policy making, if the maximally estranged faction is the median on one dimension and not the other, then the dimension on which the faction participates is the one on which it is the median.*

Proof. (a) Suppose A is the median on both dimensions. Then $x_C, y_C < 0$, in which case B or C is maximally estranged under both bundled and unbundled policy making. Suppose B is the median on both dimensions. Then $x_C, y_C > 1$, in which case A or C is maximally estranged on both dimensions. Suppose C is the median on both dimensions. Then $x_C, y_C \in (0, 1)$, in which case A or B is the maximally estranged faction. (b) Suppose C is maximally estranged and median on x. Then $y_C > 2$ or $y_C < -1$ and $x_C \in (0, 1)$. Then the x-coalition is AC_x or BC_x. Suppose A is maximally estranged and median on x. Then $x_C \in (-1, 0)$ and the x-coalition is AC_x. Suppose B is maximally estranged and median on x. Then $x_C \in (1, 2)$ and x-coalition is BC_x. The proof is substantively identical when the estranged faction is the median on y. □

Lemma A 5. *Estrangement under bundled policy making is everywhere continuous.*

Proof. Holding the excluded faction fixed, the expression for S is everywhere continuous. To rule out discontinuities at realignment frontiers, note that in the triangle formed by the factions' ideal points, estrangement is the length of the triangle's median from the ideal point of the excluded faction to the midpoint of the opposite side. At any point along a realignment frontier, two factions' ideal points are equidistant from the third and thus the triangle is isosceles. In an isosceles triangle, the medians extending from the vertices connecting the unequal side are equal in length. □

Next, we provide proofs for results in the text.

Proof of Lemma 1. From Proposition 1 and the definition of weighted-Euclidean distance, $\hat{S}(AB, bundled) = ((1-\omega)(x_C - \frac{1}{2})^2 + \omega(y_C - \frac{1}{2})^2)^{\frac{1}{2}}$, $\hat{S}(AC, bundled) = ((1-\omega)(1 - \frac{x_C}{2})^2 + \omega(1 - \frac{y_C}{2})^2)^{\frac{1}{2}}$, and $\hat{S}(BC, bundled) = ((1-\omega)(\frac{x_C+1}{2})^2 + \omega(\frac{y_C+1}{2})^2)^{\frac{1}{2}}$. Differentiating each with respect to x_C and y_C and comparing to zero gives the expressions in the Lemma. □

Proof of Proposition 2. If C is the outlier faction, then for any y_C a change in x_C produces an increase (decrease) in both binary and rank crosscuttingness for $x_C < (>)\frac{1}{2}$, as well as a decrease (increase) in maximal estrangement via Lemma 1. Likewise, for any x_C a change in y_C produces an increase (decrease) in both binary and rank crosscuttingness for $yc < (>)\frac{1}{2}$, as well as a decrease (increase) in maximal estrangement. If B is the outlier faction, then for any $\omega \in (0, 1)$, $\exists$ a pair $C = (1 - \varepsilon, y_C)$ that yields governing coalition AC and for which an increase in x_C to $1 + \varepsilon$ yields a decrease in both binary and rank crosscuttingness and a decrease in B's estrangement. Likewise, for any $\omega \in (0, 1)$, $\exists$ a pair $C = (\frac{1}{2} - \varepsilon, y_C)$ that yields governing coalition AC and for which an increase in x_C to $\frac{1}{2} + \varepsilon$ yields an increase in both binary and rank crosscuttingness and a decrease in B's estrangement. The same logic holds for changes in y_C, and in the case where A is the outlier faction. □

Proof of Lemma 2. There are three cases to consider. First, suppose C is the outlier faction. By Appolonius' Theorem, $\hat{S} = \frac{1}{2}\sqrt{2D_{AC}^{\omega 2} + 2D_{BC}^{\omega 2} - 1}$. From the definition of $\overline{D}^{\omega}$, $D_{AC}^{\omega} + D_{BC}^{\omega} = k$, where $k = 3d - 1$ for $\overline{D}^{\omega} = d$. Therefore $D_{AC}^{\omega} = k - D_{BC}^{\omega}$, and $D_{AC}^{\omega 2} = k^2 - 2kD_{BC}^{\omega} + D_{BC}^{\omega 2}$. Substituting into the expression for $\hat{S}$ yields $\hat{S} = \frac{1}{2}\sqrt{2k^2 - 4kD_{BC}^{\omega} + 4D_{BC}^{\omega 2} - 1}$. Differentiating with respect to D_{BC}^{ω} gives $\frac{\partial \hat{S}}{\partial D_{BC}^{\omega}} = \frac{2D_{BC}^{\omega} - k}{\hat{S}}$, which is strictly negative for $D_{BC}^{\omega} < \frac{k}{2} = \frac{3d-1}{2}$. Recall $d = \frac{1}{3}(D_{AC}^{\omega} + D_{BC}^{\omega} + 1)$. Substituting and rearranging yields $D_{BC}^{\omega} < D_{AC}^{\omega}$, which holds for all y_C above the axis of symmetry as defined in Lemma A1. When $D_{BC}^{\omega} < D_{AC}^{\omega}$, an increase in D_{BC}^{ω} corresponds to a move in the direction of the axis of symmetry. When $D_{BC}^{\omega} > D_{AC}^{\omega}$, $\frac{\partial \hat{S}}{\partial D_{BC}^{\omega}} > 0$, and y_C falls below the axis of symmetry. In this case, an increase corresponds to a move away from the axis of symmetry; thus, a move toward that boundary would, again, decrease $\hat{S}$.

2. Suppose A is the outlier faction. From Proposition 1, this can only occur if $D^{\omega}_{BC} < D^{\omega}_{AC}$, which implies y_C lies above the axis of symmetry. Thus, a move in the direction of the axis of symmetry holding MLD fixed implies an increase in D^{ω}_{BC}. From Appolonius' theorem, $\hat{S} = \frac{1}{2}\sqrt{2D^{\omega 2}_{AC} + 2 - D^{\omega 2}_{BC}}$. From above, $D^{\omega 2}_{AC} = k^2 - 2kD^{\omega}_{BC} + D^{\omega 2}_{BC}$. Substituting yields $\hat{S} = \frac{1}{2}\sqrt{2k^2 - 4kD^{\omega}_{BC} + D^{\omega 2}_{BC} + 2}$ Differentiating with respect to D^{ω}_{BC} yields $\frac{\partial \hat{S}}{\partial D^{\omega}_{BC}} = \frac{D^{\omega}_{BC} - 2k}{2\hat{S}}$, which is negative if and only if $D^{\omega}_{BC} < 2k$. Substituting for $k = 3d - 1$ and $d = \frac{1}{3}(D^{\omega}_{AC} + D^{\omega}_{BC} + 1)$, this simplifies to $D^{\omega}_{BC} > -2D^{\omega}_{AC}$, which always holds. An increase in D^{ω}_{BC} corresponds to a move toward the axis of symmetry given y_C lies above the axis, which, as noted above, must be the case when A is the outlier faction.

3. Suppose B is the outlier faction. From Proposition 1, this can only occur if $D^{\omega}_{BC} > D^{\omega}_{AC}$, which implies y_C falls below the axis of symmetry. From Appolonius' Theorem, $\hat{S} = \frac{1}{2}\sqrt{2 + 2D^{\omega 2}_{BC} - D^{\omega 2}_{AC}}$. Substituting for $D^{\omega 2}_{AC}$ as above yields $\hat{S} = \frac{1}{2}\sqrt{2 + D^{\omega 2}_{BC} - k^2 + 2kD^{\omega}_{BC}}$. Differentiating with respect to D^{ω}_{BC} yields $\frac{\partial \hat{S}}{\partial D^{\omega}_{BC}} = \frac{D^{\omega}_{BC} + k}{2\hat{S}}$, which is strictly positive. For y_C below the axis of symmetry, the increase in D^{ω}_{BC} that would increase $\hat{S}$ corresponds to a move away from the axis of symmetry. Therefore a move toward the boundary would correspondingly decrease $\hat{S}$.□

Proof of Proposition 3. Proof relies on Lemma 2 plus the superimposition of mean latent disagreement curves on the binary and rank crosscuttingness figures. □

Proof of Proposition 4. When cleavages are not binary crosscutting, the political system is not susceptible to realignment. When cleavages are crosscutting, there are two possibilities: either the preference profile gives rise to one realignment, or it gives rise to two. In regions of C-space for which one is possible, it is sufficient, for a given value of C, to compare the signs of the derivative under the governing coalition that would form on the x-dimension with that which would form on the y-dimension, in the manner specified in the text. In regions of C-space for which two realignments are possible, the governing coalition that would form for intermediate levels of ω is that which would *not* form if policy were restricted to the x- or y- dimension. In these cases, it is sufficient to compare the signs associated with the transition from the governing coalition that would form on the x-dimension to that which would form on neither the x- nor y-dimensions, and the latter with the coalition that would form on the y-dimension. Differentiating the expression for

$\hat{S}$ with respect to ω, if AB is the governing coalition then $\frac{\partial \hat{S}}{\partial \omega}$ is positive if and only if ($y_C > x_C$ and $y_C > 1 - x_C$) or ($y_C < x_C$ and $y_C < 1 - x_C$); if AC is the governing coalition if and only if ($y_C > x_C$ and $y_C > 4 - x_C$) or ($y_C < x_C$ and $y_C < 4 - x_C$); and if BC is the governing coalition if and only if ($y_C > x_C$ and $y_C > -2 - x_C$) or ($y_C < x_C$ and $y_C < -2 - x_C$). These conditions permit one to (mechanically) identity the possible patterns described in the text. □

Proof of Proposition 7. When cleavages are binary-crosscutting, there are two cases to consider. In the first, the maximally estranged faction under bundled and that under unbundled policy making differ. Suppose the maximally estranged faction under unbundled policy making was excluded under bundled policy making and that the remaining factions set policy. Call the resultant estrangement of that faction $\tilde{S}$. In any triangle, the median to its shortest side is its longest median. Thus $\tilde{S} < \hat{S}(bundled)$. If cleavages are binary-crosscutting, then the maximally estranged faction under unbundled policy making participates in the coalition setting policy on one dimension. Thus, policy on that dimension must be closer to her ideal point then under the scenario that generates estrangement $\tilde{S}$, while policy on the other dimension is unchanged. Therefore $\hat{S}(unbundled) < \tilde{S}$, implying $\hat{S}(unbundled) < \hat{S}(bundled)$.

In the second case, the maximally estranged faction is the same under bundled and unbundled policy making. Suppose that faction is the median on neither dimension. By Lemma A3, that faction participates in policy making on one dimension; thus, policy on that dimension is closer than it would be under bundled policy making, in which case the other two factions would set policy on that dimension. On the remaining dimension policy is the same under both institutions; therefore $\hat{S}(unbundled) < \hat{S}(bundled)$.

Next, suppose the maximally estranged faction is the median on one dimension but not the other. By part (b) of Lemma A4, under unbundled policy making, the faction participates in the dimension on which it is the median. This implies that policy on the other dimension is the same across bundled and unbundled institutions. Suppose, without loss of generality, the dimension on which it is the median is x, and for factions i, j, and k, that $x_i < x_j < x_k$. Then

$S(unbundled) < S(bundled)$ if and only if

$$\min\left\{\left|x_j - \frac{x_j + x_i}{2}\right|, \left|x_j - \frac{x_j + x_k}{2}\right|\right\} < \left|x_j - \frac{x_i + x_k}{2}\right|$$

This inequality holds if and only if x_j is sufficiently far from the midpoint of x_i and x_k.

If the most estranged faction is the median on both dimensions, it cannot be the most estranged faction by part (a) of Lemma A4.

If cleavages are not binary-crosscutting, then policy (and hence the most disaffected faction) is the same, and so estrangement is the same under bundled and unbundled policy making. □

Proof of Proposition 8. To establish the policy's Pareto efficiency it is necessary and sufficient to demonstrate that the equilibrium policy lies in the open interior of the triangle formed by A, B, and C's ideal points. If cleavages are not binary crosscutting, then the equilibrium policy lies on the contract curve between two factions, and is therefore efficient. Suppose cleavages are binary crosscutting. The column vectors $B = (1, 1)$ and $C = (x_C, y_C)$ form a basis for $\mathbb{R}^2$. Therefore, the policy vector $P = (x_P, y_P)$ can be expressed as the linear combination $P = \alpha B + \beta C$. Solving for α and β yield $\alpha = \frac{y_C x_P - y_P x_C}{y_C - x_C}$ and $\beta = \frac{y_P - x_P}{y_C - x_C}$. The necessary and sufficient conditions for P to lie in the open interior of the triangle formed by A, B, and C are $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta \leq 1$.

Suppose $(AB)_x < (AC)_x < (BC)_x$. Then $x_P = \frac{1}{2}$ and $x_C < -1$. Given the ordering, $y_C < x_C$ only if cleavages are not binary crosscutting, which violates the above supposition. Therefore $y_C > x_C$. Binary crosscutting cleavages implies that $(AB)_y$ cannot be the shortest y-distance between factions. Suppose $(AC)_y$ is the shortest y-distance. Then $y_P = \frac{y_C}{2}$. Substituting the expressions for x_P and y_P into the condition $\beta \geq 0$ and simplifying yields $y_C \geq 1$. But $(AC)_y$ is the shortest y-distance if and only if $y_C \in (-1, \frac{1}{2})$, a contradiction. Therefore P is not Pareto efficient. Next, suppose $(BC)_y$ is the shortest y-distance, so $y_P = \frac{y_C+1}{2}$ and $y_C \in (\frac{1}{2}, 1)$. Substituting into the condition $\alpha > 0$ yields $y_C \geq (y_C + 1)x_C$, which holds given the conditions on y_C and x_C. Substituting into the condition $\beta > 0$ and simplifying yields $y_C > 0$, which holds given the condition on y_C. Substituting into the condition $\alpha + \beta \leq 1$ and simplifying yields $y_C \leq 1$, which holds only for $(BC)_y < (AC)_y < (AB)_y$, for which $Q^r = 3$.

A similar logic establishes the result for $(AB)_x < (BC)_x < (AC)_x$, $(AC)_x < (AB)_x < (BC)_x$,

$(AC)_x < (BC)_x < (AB)_x$, $(BC)_x < (AB)_x < (AC)_x$, and $(BC)_x < (AC)_x < (AB)_x$.