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On the evolution of polygyny rates

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How many wives do men want? On the evolution of polygyny rates[*]

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Abstract

Why are some societies monogamous and some polygamous? The question matters for all the reasons that families themselves matter: investment in children, reproductive skew, gender differences in well-being, and more. Most theories of polygyny invoke male heterogeneity as a key driving factor. I argue that such heterogeneity itself depends on males' willingness to fight against each other to acquire more wives. I derive the preferences of *ex ante* identical males over polygyny rates by focusing on evolutionary fundamentals. Preferences are shown to depend on exogenously given factors such as the cost of producing food and care, and the technology of defense. I find that males never prefer intermediate numbers of wives: depending on the ecology, a male either always benefits from having more wives, or he prefers both strict monogamy and high polygyny rates to intermediate polygyny rates. Hence, depending on the ecology evolution should lead either to monogamy, or to a significant amount of polygyny.

Keywords: economics of the family, monogamy, polygyny, reproductive success, evolution

JEL codes: D13, C73

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1 Introduction

Some of the oldest known written records, such as the Code of Hammurabi, show that humans have devised rules and customs to govern mating practices for at least 4000 years. Marital norms have varied over space and time,[1] but in broad terms polygyny has been legal at some time in most societies, monogamy has gained in prevalence over time, and polyandry has been virtually non-existent.[2] Although rules imposing monogamous marriage on all males appeared already in ancient Greece and Rome (Scheidel, 2008), polygynous marriage is still legal in about one fourth of all countries (Zeitzen, 2008). Marital patterns may affect economic outcomes through several channels, such as fertility decisions, investment in children, reproductive skew, gender differences in well-being, the inter-generational transmission of material wealth and human capital, intra-family transfers, and more.[3] It is therefore important to understand why some societies are monogamous while others are polygynous. This paper contributes to this understanding. In particular, it may help understand why two of the most common norms today are strict monogamy and an arguably significant amount of polygyny with a right to marry up to four wives, the former being common in countries with a largely Christian past and the latter in countries with a largely Muslim past.

Whether marital norms are driven by the wishes of the rulers (Lagerlöf, 2010) and/or of the ruled (De La Croix and Mariani, 2015), grassroots individual preferences may influence how marital norms develop, because norms that adhere to underlying preferences are not likely to be contested.[4] The theoretical literature on male and female preferences in this context has given particular attention to the effects of heterogeneity among males. The

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[1] Recent analyses of distributions of Y-chromosome haplotypes provide hard evidence to this effect; see Balaresque et al. (2015), as well as the references therein.

[2] Of the 1231 societies covered by Murdock's ethnographic atlas, 4 were polyandrous, 186 monogamous, and the rest were classified as polygynous (588 of which had frequent polygyny and 453 occasional polygyny). See Gray (1998).

[3] For empirical and theoretical research on how family structure affects economic outcomes, see, *inter alia*, Becker (1991), Lundberg and Pollak (2008), Tertilt (2005, 2006), Cox and Fafchamps (2008), Edlund and Lagerlöf (2006), Doepke and Tertilt (2009), Edlund and Kopczuk (2009), Edlund and Machado (2015), Grossbard (2015), and Chiappori, Salanié, and Weiss (2016).

[4] This is reminiscent of Alexander (1987), who draws a distinction between ecologically imposed and socially imposed norms.

argument is that if females can choose whom they marry and if they rank males according to some attributes such as wealth or intelligence, females should accept to share high-quality males with other females, while low-quality males should attract at most one woman (Becker, 1974, 1991, Grossbard, 1976, Low, 1990, Bergstrom, 1994, Lagerlöf, 2005).[5] It follows that one should see low polygyny rates in societies where resource heterogeneity among men is low (Orians, 1969, Kanazawa and Still, 1999).

That heterogeneity in the opposite sex matters in this context is a well-established fact.[6] However, while the theoretical analyses cited above all take such heterogeneity as given, the attributes of a male at the moment he enters the marriage market in fact depends on his behaviors in the years preceding that moment. Importantly, his desire to accumulate resources such as wealth or human capital during these years may depend on how strongly he cares about achieving greater success in the marriage market. Specifically, a male's eagerness to compete with other males for the purpose of attracting more wives should depend on his preferred number of wives. Following this logic, if all males would prefer to have only one wife, one should expect little such competition, no or low heterogeneity, and monogamy; by contrast, if all males would prefer to have many wives, one should expect males to compete against each other in some way that generates heterogeneity, and polygyny for the winners of this race. In this paper I rely on this logic to push the theory of mating systems one step further by analyzing how natural selection shapes male *preferences* over polygyny rates in the first place, and the consequences of these preferences for the males' willingness to fight to get mores wives.

Following evolutionary logic, I posit that the ultimate goal of individuals is to maximize reproductive success. This allows to find the number of wives a man would like to have if he could freely choose, and how this depends on exogenously given factors of the environment,

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[5] While the focus of this paper is on male heterogeneity, this is not to say that female heterogeneity does not matter (Gould, Moav, and Simhon, 2008). Relatedly, in the anthropology and biology literatures it has been shown that the polygyny rate in a population may depend on the females' willingness to "trade" faithfulness for a lower polygyny rate (Kokko and Morrell, 2005, Fortunato and Archetti, 2010, Gavrilets, 2012).

[6] Most human societies that allow polygyny have positive bride prices, so that wealthier men can acquire more wives than poor ones (Gaulin and Boster, 1990). See also (Boserup, 1970) and Betzig (1993). In other species, there is clear experimental evidence to this effect, starting with Bateman's famous experiments on *drosophilia melanogaster* (Bateman, 1948). In evolutionary biology *Bateman's principle* states that the variability in reproductive success should be larger among males than in females in most species.

or the ecology, in which the population at hand evolves.[7] Given these preferences over polygyny rates, the willingness of males to fight for more wives can be deduced and used to determine the set of polygyny rates that are sustainable in the sense that males who have achieved this polygyny rate would not want to fight further to acquire even more females. The key contribution of the analysis is to show how male preferences over polygyny rates and thus the sustainable polygyny rate(s) may ultimately depend on the ecology in which the population evolves,[8] absent any *a priori* heterogeneity among males.

In the model, males first go through a fighting stage, in which they are sequentially matched into pairs. In each pair where at least one male fights, one of the males is wiped out from the marriage market. The fighting stage ends when all the remaining males stop fighting. The amount of fighting thus determines the number of males who can marry in the end: the more fighting there is, the smaller is the number of such lucky males.[9] Following the fighting stage, a number of households are formed, each household consisting of one lucky male and his spouse(s). In each household, decisions are taken with respect to fertility as well as the division of labor between the male and the female(s), the goal being to maximize the expected number of children that survive to sexual maturity, i.e., the reproductive success.[10] The male provides protection, the female(s) provide(s) care, and both the male and the female(s) may produce food; the labor division specifies how each individual allocates time between these tasks.

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[7] The baseline model relies on a sexist approach, which consists in assuming that females are completely passive. I relax this assumption when studying the robustness of the results.

[8] The quest for insights as to how ecological factors affect the evolution of mating systems is common in the biology literature (see, e.g., Bateman, 1948, Orians, 1969, Emlen and Oring, 1977, Clutton-Brock and Vincent, 1991, as well as Kokko and Jennions, 2008, and the references therein), as well as in the anthropology literature (see., e.g, Kaplan, Hooper, and Gurven, 2009, Nettle et al., 2013, and Moya, Snopkowski, and Sear, 2016). To the best of my knowledge, however, in this literature parental care is a one-dimensional variable (while in my model parents provide three goods to their offspring) and focus is often on the effects of spatial and temporal availability of mates (features that are disregarded here).

[9] If there is polygyny, some males end up with a higher reproductive success than others. On average, however, males and females have the same reproductive success.

[10] While mating success of offspring also matters, it is not modeled here. Moreover, I rule out female heterogeneity because the focus is on male preferences over the number of wives of a given quality, and I also rule out unfaithfulness by assumption, an approach which is reasonable for human societies where grand-parents may monitor the behaviors of their sons or daughters in law.

The exact way in which fertility and labor division impacts reproductive success in turn depends on *the ecology*, which in the model is a vector of nine exogenously given parameters. Some of the ecological parameters pertain to the relative importance of food, protection, and care in determining child survival, thus allowing to compare, for instance, environments where shelter is more important than in others, perhaps because of harsh weather conditions or the prevalence of predators. Other ecological parameters measure the marginal returns to time spent on different tasks: the returns to gathering food in the savannah may be quite different from the returns to tending crops. Finally, two of the ecological parameters measure the extent to which protection and care are public goods (food being a private good); for example, protecting a given number of children against predators may require more of the father's time in nomadic pastoralist societies than in sedentary agricultural societies, since in the latter a durable wall can be built for this purpose.

The analysis reveals whether or not a (lucky) male would benefit, in terms of reproductive success, from having more wives. I find that males do not necessarily want to simply maximize the number of wives. Depending on the ecology, the reproductive success of a lucky male may be increasing, decreasing, or non-monotonic in the polygyny rate. This is because males face a *polygyny quantity-quality trade-off*: while a greater number of wives implies a direct gain for the male in the form of a greater number of children, it also entails an indirect loss in the form of a decrease in the probability that each child survives to sexual maturity.[11]

While the presence of this trade-off itself is not surprising *per se*, the analysis unveils a striking result: males never prefer intermediate numbers of wives. Depending on the severity of the quantity-quality trade-off, one of two scenarios arises: either a male either always benefits from having more wives, or he prefers both monogamy and high polygyny rates to intermediate polygyny rates. This finding has deep implications for the males' willingness to fight for more wives, since it leads to a bang-bang result: one should either expect to see a relatively high polygyny rate for successful males, or else monogamy. Indeed, in ecologies where successful males always benefit from having more wives, fighting never ceases.[12] By

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[11] The only exception is when the male produces no food and protection is a pure public good: his average contribution to child success is then not diluted by adding more wives.

[12] Unless the act of fighting itself has a negative impact on reproductive success, for instance because it weakens the male for life, or if the fighting technology is such that the biological time constraint becomes relevant.

contrast, in ecologies where successful males prefer both monogamy and high polygyny rates to intermediate polygyny rates, in a society with monogamy males would not want to fight to get one more wife, and hence monogamy would be sustainable. In sum, this result shows that in certain environments monogamy is not a mystery that needs an explanation, for in these environments it arises naturally, as a consequence of fundamental evolutionary forces.

The analysis further reveals a novel insight, namely, that the qualitative nature of male preferences over polygyny rate is intimately linked with his involvement in food production. In ecologies where the male never engages in food production, he always benefits from having more wives. Indeed, since each wife can provide food and care to her own children, in such ecologies the first-order effect of adding one more wife always outweighs the second-order effect of diluting male protection among more children. Thus, male involvement in food production is necessary for male reproductive success to decline in the number of wives. Furthermore, I find that if a male engages in food production, this must happen for low polygyny rates: indeed, because food is a fully private good, a male is better off shifting his time towards producing more protection as the number of children he has to raise grows. For high enough polygyny rates, the male devotes all his time to protection, and a further increase in the polygyny rate then enhances reproductive success. This explains why the polygyny quantity-quality trade-off is always more severe for low than for high polygyny rates, and hence why males either prefer both monogamy and high polygyny rates to intermediate polygyny rates, or always want more wives.

The approach used in this paper rests on the idea, delineated by Bergstrom (1996) and Robson (2001, 2002), that economists may obtain valuable insights about human motivation by including evolutionary forces in their models.[13] It is closely related to the growing literature on preference evolution (see, e.g., Frank, 1987, Güth and Yaari, 1992, Dekel, Ely, and Yilankaya, 2007, Heifetz, Shannon, and Spiegel, 2007, Rayo and Becker, 2007, Robson and Samuelson, 2011, Alger and Weibull, 2010, 2013). To the best of my knowledge, this is the first attempt to derive preferences over polygyny rates from first principles, however. As mentioned above, the economics literature on polygyny has hitherto instead focused on explaining how heterogeneity among males and/or among females impinge on marriage market equilibria (Becker, 1974, Grossbard, 1976, Bergstrom, 1994, Francesconi, Ghiglino, and Perry, 2016), on the dynamic feedback loops between polygyny rates and heterogeneity

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[13] Note that the theoretical models in this literature, including the one proposed here, are silent as to whether traits are genetically or culturally determined.

(Lagerlöf, 2005, Gould, Moav, and Simhon, 2008, De La Croix and Mariani, 2015), on the effects of a mismatch in the ages at which males and females marry (Tertilt, 2006).

The paper is organized as follows. In the next section I describe and analyze the baseline model. In Section 3 I lift some of the simplifying assumptions of the baseline model to check the robustness of the results. While the baseline model focuses on male preferences, I discuss female preferences over polygyny rates as well as efficiency in Section 4. A conclusion is provided in Section 5. All the mathematical proofs are in the Appendix.

2 Analysis

I model a population in which each individual lives for at most two periods; in the first period as a non-productive and non-reproductive child, and in the second period as a productive and reproductive adult. In each generation the sex ratio is assumed to be balanced at birth, as evolutionary theory would predict (Fisher, 1930), and for simplicity also at the beginning of the adult period. The model focuses on behaviors in the adult period, which has two stages: first, a mate matching stage, and then a childbearing and child-rearing stage. The baseline model uses a sexist approach, by assuming that males act while females simply obey the orders of the males. Analytical convenience is only one of the three reasons for why this modeling choice is valuable. It may also accurately capture some distant evolutionary past,[14] and it allows to highlight the sources of disagreements between males and females, if any, by determining the extent to which females would favor a different outcome than that obtained in the baseline model with passive females; these matters will be analyzed and discussed in Sections 3 and 4.

The broad lines of the baseline model are as follows. The mate matching stage consists in fighting between males. A key feature of the model is that the males are assumed to be identical prior to fighting, and that heterogeneity between males arises only as a result of fighting. At the end of the mate matching stage, some males have earned the right to mate; any males who have not earned such a right get nothing (one interpretation being that they were killed or disabled in the fight). At the beginning of the childbearing and child-rearing stage, there is thus a number of households, each composed of one male and his (identical) spouse(s). The male imposes his preferred fertility and household labor division

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[14] In this distant past females may have been completely dependent on males. In such a context, if males could choose females, it may have been in their interest to choose obedient ones.

on his spouse(s), his goal being to maximize his own reproductive success,[15] defined as the expected number of his offspring that survive to sexual maturity.[16]

I next proceed to describing and solving the baseline model, starting with a description of the childbearing and child-rearing stage, and then analysis of the male-fighting stage.

2.1 Childbearing and child-rearing

Consider a male who enters the childbearing and child-rearing stage with k wives.[17] He chooses the number of children per female and the division of labor within the household, so as to maximize the expected number of children that survive to sexual maturity. The probability that a child survives to sexual maturity depends on the quantities of food, protection, and care that it receives.[18] Production technologies are given and fixed; females are assumed to have identical production abilities, and likewise for men. The amounts of food, protection, and care that are produced within a household are determined by the division of labor; labor is measured in units of time and the lifetime time budget of each adult is $T \in (0, 1]$. Adults do not consume anything of what they produce,[19] and interactions between households, divorce, and unfaithfulness are ruled out by assumption.[20]

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[15] This assumption is in line with the literature on the evolution of preferences in situations lacking strategic interaction, such as the child-rearing stage in my model (Robson and Samuelson, 2011).

[16] In reality mating success of offspring who have survived to sexual maturity also matters for an adult's reproductive success. I can disregard this, however, since the model relies on the simplifying assumption that all the men who enter the adult stage have an equal chance to mate, and likewise for all the women who enter the adult stage.

[17] The term "wife" is used for convenience only, since marriage has no function *per se*. The key assumption is that both males and females engage in parental care, which is a reasonable assumption for humans and some other species (see Alger and Cox, 2013 for a review of the biology literature on parental care).

[18] The terms protection and care should be interpreted broadly. Thus, protection may include shelter construction and maintenance, active protection against predators, as well as the transmission of human capital pertaining to such activities. Likewise, care may include the production and mending of clothes, storytelling, as well as the transmission of knowledge about social rules, plants, and animals.

[19] Alternatively, the time budget T can be interpreted as the time available to an adult once (s)he has produced and consumed the amounts of food (s)he needs to survive.

[20] It would clearly be highly desirable to endogenize the degree of unfaithfulness, but this has to be left for future research. For the time being, one interpretation of the benchmark model considered here is that

While returns to labor on any given day are often decreasing, the scale of the analysis at hand—the entire time spent in adulthood—calls for the opposite assumption, namely, non-decreasing returns to labor in any given activity. As an illustration, consider a hunter-gatherer society, in which there is food that can be hunted and food that can be gathered. The skills required for successful hunting and gathering are by no means trivial, and it seems reasonable to think that there are gains to specialization. Furthermore, hunting may require similar skills as those required for protection against predators. There may also be economies of scope between gathering and caring, especially if it is possible to engage in both activities at the same time; for instance, an adult who is gathering food may bring a sleeping child on her back, or let the child play next to her. Because breast-feeding ties a mother to her baby and berries do not run away from crying babies, in such a society it would be natural to assume, then, that females specialize in caring and gathering, while males specialize in protecting and hunting.

More generally, there are four productive activities that any adult can engage in. The male specializes in two of them while females specialize in the other two activities. Let $y \in [0, T]$ denote the time that the male devotes to one of the activities, so that he spends $T - y$ on the other activity, where $T \in (0, 1]$ is his time budget. Likewise, let $x_j \in [0, T]$ denote the time that female $j = 1, ..., k$ devotes to one of the two activities, the rest of her time being spent on the other activity. The vector $(\boldsymbol{x}, y)$, where $\boldsymbol{x} = (x_1, ..., x_k)$, is the *household labor division.* I focus on household labor divisions in which all the females adopt the same time allocation, x. Such *female-symmetric* labor divisions are denoted (x, y).

Any labor division (x, y) gives rise to a certain basket of food, protection, and care. The goods produced by the male are divided equally between the $n \cdot k$ children, while the goods of a female are divided equally between her own n children; the case where the goods a female produces are divided equally between all the $n \cdot k$ children in the household will be discussed in Section 3. This in turn gives rise to a certain probability that each child survives to sexual maturity. Formally, then, one can write the survival probability of a child as a function of the labor division and the household stucture; I write s for this survival function. Letting M denote the function that to each household structure (n, k) *cum* household labor division (x, y) associates the male's reproductive success, I let it take the form

$$M(x, y, n, k) = k \cdot n \cdot a(n) \cdot s(x, y, n, k), \tag{1}$$

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there is strong social control (for instance, although they are not explicitly modeled here, there may be grand-parents who monitor how their children behave in the adult stage, or the females monitor each other).

where the function a, which is strictly decreasing, captures the physical toll that childbearing inflicts on a female. For simplicity, I will allow n to take any real value above 1.

The male chooses $(x,y,n) \in [0,1]^2 \times [1,+\infty)$ in order to maximize $M(x,y,n,k)$ (recall that k is given when this choice is made). For simplicity, I assume that for all parameter values there exists a unique solution to this problem, and that there always exists some $(x,y,n) \in [0,1]^2 \times [1,+\infty)$ for which $M(x,y,n,k) > 0$. Writing $(x^*(k), y^*(k), n^*(k))$ for the unique solution, and $\widetilde{M}(k)$ for the reproductive success thus achieved,

$$\widetilde{M}(k) = M(x^*(k), y^*(k), n^*(k), k). \tag{2}$$

I will provide a detailed analysis of the characterstics of $\widetilde{M}(k)$ after having shown the role that $\widetilde{M}(k)$ plays in the male fighting stage in the next subsection.

2.2 Male fighting

Turning now to the male-fighting stage, which precedes the childbearing and child-rearing stage, to fix ideas consider to begin the simplest possible approach. Suppose that when entering the adult stage (from the teenage years) each male has one girlfriend, and that males are then randomly matched into pairs to play a simultaneous-move game with two pure strategies, *Fight* and *Peace*, and payoffs as shown in Figure 1.

	Peace	Fight
Peace	$\widetilde{M}(1), \widetilde{M}(1)$	$0, \widetilde{M}(2)$
Fight	$\widetilde{M}(2), 0$	$\frac{1}{2}\widetilde{M}(2), \frac{1}{2}\widetilde{M}(2)$

Figure 1. Payoffs in the one-shot Fighting game

In this game, that I call the *One-round fighting game*, if both play *Peace*, then each gets to marry his teenage sweetheart and each achieves reproductive success $\widetilde{M}(1)$ (see (2)). If at least one male plays *Fight*, then one of them gets to marry both girlfriends while the other one becomes mateless and remains so forever. The probability that a male who plays *Fight* wins, is 1 if the other plays *Peace* and 1/2 if the other plays *Fight*. The lucky male achieves reproductive success $\widetilde{M}(2)$.

Applying a standard evolutionary game theory approach, suppose that each male is programmed to play a certain strategy in this game, a strategy that he inherited from his father, and suppose that the payoffs in Figure 1 represent the payoffs in the evolutionary game. Further allow for mixed strategies, and write $\zeta \in [0,1]$ for the probability of playing *Fight*. Now ponder the following thought experiment: suppose that a given strategy ζ is used by almost everyone in the population, except for a small fraction $\varepsilon > 0$ of individuals who use another strategy ζ': is there any "resident" strategy ζ that would outperform every possible "mutant" strategy $\zeta' \in [0,1]$, $\zeta' \neq \zeta$, in the sense that those who carry the resident strategy would get a strictly higher reproductive success on average than those who carry the mutant strategy? More specifically: what is the set of Evolutionarily Stable Strategies (ESS) of the One-round fighting game?

Applying standard results (Weibull, 1995), and recalling that $\widetilde{M}(k) > 0$ for $k = 1, 2$, one immediately obtains:

Proposition 1 *In the One-round fighting game, there is either one or two evolutionarily stable strategies:* $\zeta = 1$ *(*Fight*) is always evolutionarily stable, and* $\zeta = 0$ *(*Peace*) is evolutionarily stable if and only if* $\widetilde{M}(1) > \widetilde{M}(2)$*.*

This result has implications for the kind of marital patterns that could prevail in a population in which all males would play the same evolutionarily stable strategy. First, it says that bigyny can always be sustained: this is because if other males are expected to *Fight*, then it never pays off to play *Peace* (note that this would be true even under the less stark assumption that playing *Peace* against someone playing *Fight* would give a positive probability of winning, as long as this probability would be below 1/2). Second, it says that monogyny can be sustained if only if a male would lose from trying to steal a peaceful male's girlfriend, i.e., if $\widetilde{M}(1) > \widetilde{M}(2)$.

It may seem obvious that a male would always prefer having two wives rather than one, since this allows him to double the number of children. However, this intuition misses the fact that when the number of children doubles, the survival probability of each child typically decreases due to the resulting decline in paternal resources flowing to each child. But this intuition in turn misses the fact that if a male has two wives rather than one, he can adjust the number of children per female downward to mitigate the decline in survival probability per child. Overall, then, it is not clear whether a male would necessarily prefer to have two rather than one wife. Before analyzing these trade-offs in greater detail in the next subsection, I propose a generalization of the One-round fighting game.

In this generalized game there may be up to $R \geq 2$ rounds, where $R \in \mathbb{N}$. In each round, all the males who did not yet lose a fight are matched pairwise, and each male plays either

Peace or *Fight*, exactly as above. However, by contrast to the one-round game, now there are fewer and fewer males around as the number of rounds of fighting increases. Hence, the stakes are different in each round, since the number of females per male increases with every round. Formally, let a (pure) strategy be a number $r \in \{0, 1, ..., R\}$, which specifies the number of rounds in which the male will play *Fight*, following which he will play *Peace.* I maintain the assumptions introduced in the one-round game that (a) in a pair where both play *Peace*, both survive and settle down to form a household (thereby exiting the fighting game); (b) in a pair where one plays *Peace* and the other *Fight*, the former loses his females to the latter; (c) in a pair where both play *Fight*, one of the males loses his females to the other, with equal probability for both. Each male is programmed to play a specific strategy. In a population where all males play strategy r, each male achieves expected reproductive success

$$\frac{1}{2^r} \cdot \widetilde{M}(2^r).$$

For each round of fighting, the number of females per male is doubled, and there is a probability 1/2 of winning the fight.

The following proposition identifies the set of *locally* evolutionarily stable strategies, i.e., strategies r that are evolutionarily stable against the two mutant strategies $r' = r - 1$ and $r' = r + 1$.[21]

Proposition 2 *In the R-round fighting game, $r = R$ is locally evolutionarily stable, and $r \in \{0, 1, ..., R-1\}$ is locally evolutionarily stable if and only if $\widetilde{M}(2^r) > \widetilde{M}(2^{r+1})$.*

This proposition shows that the two main qualitative features of the set of evolutionarily stable strategies in the One-round fighting game carry over to the R-round fighting game. First, any level of polygyny such that each lucky male would prefer not to further double the number of wives by way of fighting against a peaceful rival is locally stable. In particular, monogyny is locally stable under the same condition as in the One-shot game (too see this, note that when $r = 0$ the condition $\widetilde{M}(2^r) > \widetilde{M}(2^{r+1})$ boils down to $\widetilde{M}(1) > \widetilde{M}(2)$). Second, it never pays off for a male to stop fighting before the other males do: "never stop fighting" ($r = R$) is always locally stable.[22]

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[21] Note that I restrict attention to pure strategies; this is done for simplicity.

[22] Note that if one were instead to look for strategies that are globally stable, i.e., stable against all alternative strategies, the condition $\widetilde{M}(2^r) > \widetilde{M}(2^{r+1})$ would still be necessary for a strategy $r < R$ to be stable.

The set of locally evolutionarily stable strategies in the fighting game determines the set of polygyny rates that are *sustainable*:

Definition 1 *A polygyny rate k is sustainable if and only if $k = 2^r$ for some locally evolutionarily stable r.*

Ultimately, then, the set of polygyny rates that are sustainable in a particular society depends on males' preferences over polygyny rates, as captured by the characteristics of the male reproductive success function $\widetilde{M}$. These are analyzed in detail in the next subsection.

2.3 Male preferences over polygyny rates

Acquiring more wives provides a male with the ability to sire a larger number of children, but it may also reduce the probability that each child survives to sexual maturity. Can the latter effect be strong enough to outweigh the former effect and thus make the male want to restrict the number of wives? And if so, will this tend to happen for small or large polygyny rates? To address these questions, I study the shape of the function $\widetilde{M}$ (see (2)), assuming for simplicity that it is a continuous function of k; below I will relate the findings to the results in the fighting game, stated for integer values of k.

Thus, assume that M is continuously differentiable in (x, y, n), for any $(x, y, n) \in [0,1]^2 \times [1, +\infty)$. Then, from the envelope theorem:

$$\begin{aligned} \frac{d\widetilde{M}(k)}{dk} &= \frac{\partial M\left(x^*(k), y^*(k), n^*(k), k\right)}{\partial k} \\ &= n \cdot a(n) \cdot \left[s(x,y,n,k) + k \cdot \frac{\partial s(x,y,n,k)}{\partial k} \right]_{|(x,y,n,k)=(x^*(k),y^*(k),n^*(k),k)} . \end{aligned} \tag{3}$$

An increase in the number of wives affects male reproductive success positively by increasing the number of children; this first-order effect is captured by the term $s(x,y,n,k) > 0$. However, such an increase also affects reproductive success negatively if the survival probability drops, something which would affect the children of all wives, even the inframarginal ones; this second-order effect is captured by the second term inside the square brackets. Borrowing terminology from price theory, I define the *child survival elasticity* as:

$$\begin{aligned} \epsilon_s &= \frac{\partial s(x,y,n,k)}{\partial k} \cdot \frac{k}{s(x,y,n,k)} \\ &= \frac{\partial s(x,y,n,k)}{s(x,y,n,k)} \Bigg/ \frac{\partial k}{k} . \end{aligned} \tag{4}$$

As is shown in the second line, this elasticity measures the percent change in survival probability relative to the percent change in the number of wives. Since

$$\frac{d\widetilde{M}(k)}{dk} = \left[n \cdot a(n) \cdot s(x,y,n,k) \cdot (1+\epsilon_s)\right]_{|(x,y,n,k)=(x^*(k),y^*(k),n^*(k),k)},$$

I conclude:

Remark 1 *The sign of* $\frac{d\widetilde{M}(k)}{dk}$ *is determined by the sign of* $1+\epsilon_s$*:*
(i) $\frac{d\widetilde{M}(k)}{dk} > 0$ *iff child survival is inelastic (*$\epsilon_s > -1$*);*
(ii) $\frac{d\widetilde{M}(k)}{dk} = 0$ *iff child survival is unit elastic (*$\epsilon_s = -1$*);*
(iii) $\frac{d\widetilde{M}(k)}{dk} < 0$ *iff child survival is elastic (*$\epsilon_s < -1$*).*

Hence, male preferences for polygyny rates hinge on whether child survival is elastic or not. From the expression in the second line of (4), it is clear that the denominator, $\frac{\partial k}{k}$, becomes smaller as k increases, a feature that contributes to rendering child survival more elastic as k increases. But if the numerator moves in the opposite direction, the net effect may be that child survival becomes less elastic as k increases. In general, it is not clear what the net effect would be. However, the child survival elasticity may be expected to depend on a number of factors in the environment in which the population evolves: how easy is it to find food? how strong is the need to protect the children from predators? how much education does a child need to learn how to survive? etc. In order to get a grip on how factors in the environment may affect the child survival elasticity, and therefore also male preferences over polygyny rates, I study a specific child survival function, which is general yet analytically tractable.

To make the description more concrete, I use a specific setting from our evolutionary past, namely, a hunter-gatherer setting (other settings can easily be imagined). The four productive activities that the adults can engage in are hunting game, gathering berries, caring, and protecting. Letting G_i and H_i denote the amounts of gathered and hunted food that child i receives, and P_i and C_i the amounts of protection and care that (s)he receives, I assume that the probability that the child (whether a boy or a girl) survives to sexual maturity is

$$a(n) \cdot S(C_i, G_i, H_i, P_i),$$

where

$$S(C_i, G_i, H_i, P_i) = P_i^{\lambda} \cdot C_i^{\sigma} \cdot (\rho G_i + H_i)^{\tau}, \tag{5}$$

for $\lambda, \sigma, \tau \in (0,1]$ and $\rho > 0$. This functional form captures two realistic features. First, food, protection, and care are all essential goods: food is useless unless some protection and some care is provided, and *vice versa*. Second, gathered food and hunted food are perfectly substitutable, except for the fact that gathered food may be more important than hunted food (if $\rho > 1$), equally important (if $\rho = 1$), or less important (if $\rho < 1$), as this may depend on the nutritional attributes of the gathered food in the location occupied by the population at hand. The parameters λ, σ, and τ measure how protection, care, and total food intake, respectively, impact survival probability. Again, these parameter values would typically depend on the local environment: protection and care is relatively more important if there are many predators around and if there are many dangers that children need to learn to avoid. I assume that $\rho\gamma + \theta \leq 1$; as will be seen below, this ensures that S always takes a value between 0 and 1. Furthermore, below it will be assumed that $a(n) = \max\{0, 1 - bn\}$ for $b \in (0,1)$, where $1/b$ can be thought of as the maximum number of children a female can have before she dies with certainty (in which case the children die since the mother's inputs are essential).[23]

The basket (C_i, G_i, H_i, P_i) that a child receives depends not only on the total amounts of hunted and gathered food, protection, and care produced by the adults in the household, but also on whether these are private or public goods. Let y denote the time that the male devotes to hunting, $T - y$ being the time he devotes to protecting the children, and let x_j denote the time that female $j = 1, ..., k$ devotes to gathering, and $T - \phi x_j$ the time that she spends on caring, where the parameter $\phi \in (0,1)$ measures the extent to which a female may gather food while providing care. While food is a private good, care and protection may be public goods; for instance, a wall around the village protects all the children equally well, and a class about poisonous plants may benefit several children simultaneously. Letting an index $i = 1, 2, ..K$ refer to the i'th child in the household, where $K \equiv kn$ is the total number of children, child i receives the amount

$$H_i(y) = \frac{\theta y}{kn} \tag{6}$$

of hunted food, where $\theta > 0$ is the marginal return to male effort devoted to hunting, and the amount

$$G_i(x_i) = \frac{\gamma x_i}{n}, \tag{7}$$

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[23] Here the physical toll is modeled as a scaling factor: *ceteris paribus*, the more children a woman has, the smaller is the survival probability of each of her children. Alternatively, one could let the physical toll have an impact on female productivity. This is left for future research.

of gathered food, where $\gamma > 0$ is the marginal return to female effort devoted to gathering.[24] By contrast to food, protection may be a public good. Formally, let

$$P_i\left(y\right) = \frac{T-y}{\left(kn\right)^{\alpha}} \tag{8}$$

be the amount of protection that child i receives, where $\alpha \in [0,1]$ measures the extent to which protection is a public good. In the extreme case where $\alpha = 0$, protection is a pure public good, and each child receives the full benefit of the total amount of protection produced by the father: $P_i\left(y\right) = T - y$. At the other extreme, if $\alpha = 1$, protection is a fully private good, and each child receives $1/K$ of the total amount produced: $P_i\left(y\right) = \left(T-y\right)/K$. By a slight abuse of language, henceforth α will be referred to as the degree of publicness of protection, where a lower α means a greater degree of publicness. Likewise, the amount of care that child i receives is

$$C_i\left(x_i\right) = \frac{T-\phi x_i}{n^{\beta}}, \tag{9}$$

where $\beta \in [0,1]$ measures the extent to which care is a public good. If $\beta = 0$, care is a pure public good, while if $\beta = 1$, it is a private good. Henceforth β will be referred to as the degree of publicness of care (where a smaller value of β implies a greater degree of publicness).

This completes the description of the setup. In the extremely long run, everything in life, including the features and the prevalence of animals and plants eaten by humans, is endogenous. However, the speeds at which different elements of a human society evolve, differ. In the model, I assume that the production technology, the degrees of publicness of protection and care, and the child survival probability function, are exogenous and fixed, and I refer to the associated set of parameters as the ecology. Formally, then, *the ecology* is the vector $\boldsymbol{\xi} = (\alpha, \beta, \gamma, \sigma, \theta, \lambda, \rho, \tau, \phi)$. The ecology determines how parental time allocations are transformed into offspring success. Below it will be seen how the ecology affects the set of sustainable polygyny rates.

Fixing the number of wives k and the ecology $\boldsymbol{\xi}$, and replacing H_i, G_i, P_i, and C_i in $S\left(C_i, G_i, H_i, P_i\right)$ with the expressions in (6), (7), (8), and (9), the child survival probability s as a function of the female-symmetric[25] household labor division (x, y), the fertility rate

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[24] Constant returns to effort are perfectly compatible with specialization, if, for instance, the marginal return to male effort devoted to gathering and to caring is strictly lower than that of a female, and the marginal return to female effort devoted to hunting and to protecting is strictly lower than that of the male.

[25] Thanks to the constant returns to time devoted to gathering and to care, this focus entails no loss of generality.

n, and the number of wives k writes

$$s\left(x,y,n,k\right)=\left(\frac{T-y}{\left(kn\right)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{T-\phi x}{n^{\beta}}\right)^{\sigma}\cdot\left(\frac{\rho\gamma x}{n}+\frac{\theta y}{kn}\right)^{\tau}, \tag{10}$$

so that the male's reproductive success (see (1)) writes

$$M\left(x,y,n,k\right)=k\cdot n\cdot a\left(n\right)\cdot\left(\frac{T-y}{\left(kn\right)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{T-\phi x}{n^{\beta}}\right)^{\sigma}\cdot\left(\frac{\rho\gamma x}{n}+\frac{\theta y}{kn}\right)^{\tau}. \tag{11}$$

For any given number of wives k, the male chooses $(x,y,n)\in[0,T]^2\times\mathbb{N}$ in order to maximize this. The solution is as follows.

Proposition 3 *For any ecology* $\boldsymbol{\xi}$ *and any number of wives* k*, there exists a unique fertility rate* n^* *and a unique female-symmetric household labor division* $\left(x^*\left(k\right),y^*\left(k\right)\right)$ *that maximizes* $M\left(x,y,n,k\right)$*.*

The exact expressions for $x^*\left(k\right)$, $y^*\left(k\right)$, and n^* are as follows. First, the preferred fertility rate does not depend on k:

$$n^*=\max\left\{1,\frac{1-\lambda\alpha-\sigma\beta-\tau}{2-\lambda\alpha-\sigma\beta-\tau}\cdot\frac{1}{b}\right\}. \tag{12}$$

For any given polygyny rate $k\geq1$, an increase in fertility means that each child gets less food, and less (or as much) protection and care. The ensuing decline in average child success eventually outweighs the direct, positive impact of fertility on male reproductive success. This trade-off may be so severe that having one child per female maximizes male reproductive success. When the preferred number of children exceeds one, it is a fraction $\frac{1-\lambda\alpha-\sigma\beta-\tau}{2-\lambda\alpha-\sigma\beta-\tau}$ of the maximum number of children that a female can get $(1/b)$, and it varies in intuitive ways with the ecology. First, if protection and/or care becomes a more private good, i.e., α and/or β increases, there is less protection and/or care per child, and the preferred fertility rate declines. The parameters σ, λ, and τ have the same qualitative effect, because an increase of any of these parameter values is as if the corresponding good became more private in nature. Finally, the preferred fertility rate does not depend on the relative effect of food produced by the male compared to that produced by the females $(\theta/\left(\rho\gamma\right))$, nor on the economies of scope in female production (ϕ), because while these parameters affect the males' preferred household labor division, they do not affect how food and care is being shared among the household's children.

Second, for the household labor division, to simplify the notation let $\mu \equiv \theta / (\rho\gamma)$; this ratio measures the importance of food contributed by the male relative to that contributed by the females. Two cases arise: if $\frac{\tau}{\sigma+\tau} \leq \phi$,

$$(x^*(k), y^*(k)) = \begin{cases} \left(\frac{\tau T}{\phi(\sigma+\tau)}, 0\right) \text{ if } k \geq \frac{\mu\phi(\sigma+\tau)}{\lambda} \\ \left(\frac{(\lambda+\tau)kT-\sigma\phi\mu T}{k\phi(\lambda+\sigma+\tau)}, \frac{\mu\phi(\sigma+\tau)T-\lambda kT}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\mu\phi\sigma}{\lambda+\tau}, \frac{\mu\phi(\sigma+\tau)}{\lambda}\right] \\ \left(0, \frac{\tau T}{\lambda+\tau}\right) \text{ if } k \leq \frac{\mu\phi\sigma}{\lambda+\tau}; \end{cases} \tag{13}$$

and if $\frac{\tau}{\sigma+\tau} > \phi$,

$$(x^*(k), y^*(k)) = \begin{cases} (T, 0) \text{ if } k \geq \frac{\mu\tau}{\lambda} \\ \left(T, \frac{\tau T}{\lambda+\tau} - \frac{k\lambda T}{\mu(\lambda+\tau)}\right) \text{ if } k \in \left[\frac{\mu\sigma\phi}{\lambda+\tau-\phi(\lambda+\sigma+\tau)}, \frac{\mu\tau}{\lambda}\right] \\ \left(\frac{(\lambda+\tau)kT-\sigma\phi\mu T}{k\phi(\lambda+\sigma+\tau)}, \frac{\mu\phi(\sigma+\tau)T-\lambda kT}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\mu\sigma\phi}{\lambda+\tau}, \frac{\mu\sigma\phi}{\lambda+\tau-\phi(\lambda+\sigma+\tau)}\right] \\ \left(0, \frac{\tau T}{\lambda+\tau}\right) \text{ if } k \leq \frac{\mu\sigma\phi}{\lambda+\tau}. \end{cases} \tag{14}$$

A few features are noteworthy. First, the time that the male spends on food production, $y^*(k)$, is weakly decreasing in k. This is because an increase in the number of wives k implies that the male's food output is shared between a larger number of children, and, hence, to an increase in the relative impact of time spent by females on the total amount of food that each child receives (it's as if the females became more productive in food production). As a result, an increase in k makes the male adjust the labor division by reducing male food production and increasing female food production. Eventually, for k large enough, all the food is produced by the females ($y^*(k) = 0$). As will be seen below this feature is linked with the qualitative characteristics of male preferences over polygyny rates. Second, most comparative statics results stated in the proposition are straightforward. Thus, comparing two ecologies, with τ and $\tau' > \tau$, respectively, the marginal benefit from producing food for both males and females is larger in the latter and therefore both the male and the female(s) spend more time on food production in the latter ecology. Likewise, the male spends less time and the female(s) more time on food production in ecologies where protection is more important (a higher λ), while the opposite occurs in ecologies where female care is more important (a higher σ). Similarly, in ecologies with greater economies of scope between the two female activities (a higher ϕ), the cost for females of allocating time away from caring is smaller, and hence, female food production is larger; this in turn entails a smaller marginal effect of male food production on child success, and hence the male devotes less time to food production.

Proposition 3 implies that for any ecology $\boldsymbol{\xi}$ and any number of wives k, the achieved male reproductive success is uniquely determined; in other words, there is a unique value associated to each k (formally, $\widetilde{M}$ is a function rather than a correspondence). I am therefore in a position to turn to the main question, namely, does $\widetilde{M}$ increase or decrease as k increases? In the following proposition, which states the answer to this question,

$$\hat{\alpha} \equiv \frac{\rho\gamma x^*(1) + (1-\tau)\,\theta y^*(1)}{\lambda\left[\rho\gamma x^*(1) + \theta y^*(1)\right]}, \tag{15}$$

and $\hat{k}$ is implictly defined by the equation

$$(1-\lambda\alpha)\cdot\rho\gamma x^*\left(\hat{k}\right) = (\tau - 1 + \lambda\alpha)\cdot\frac{\theta y^*\left(\hat{k}\right)}{\hat{k}}. \tag{16}$$

Proposition 4 *Consider a male who, for any given number of wives $k \geq 1$, chooses (x, y, n) so as to achieve reproductive success $\widetilde{M}(k)$ (see (2)). For such a male:*
(i) if $y^*(1) = 0$ or if $\alpha \leq \hat{\alpha}$, then $\frac{d\widetilde{M}(k)}{dk} \geq 0$ for all $k \geq 1$ (the inequality being strict if $y^*(1) = 0$ and $\alpha\lambda \neq 1$, or if $\alpha < \hat{\alpha}$);
(ii) if $y^*(1) > 0$ and $\alpha > \hat{\alpha}$, then there exists $\hat{k} > 1$ such that $\frac{d\widetilde{M}(k)}{dk} < 0$ for all $k \in \left(1, \hat{k}\right)$ and $\frac{d\widetilde{M}(k)}{dk} > 0$ for all $k > \hat{k}$;
(iii) if $\alpha\lambda = 1$, then $\widetilde{M}(1) \geq \widetilde{M}(k)$ for all $k > 1$, the inequality being strict if and only if $y^*(1) > 0$.

In sum, male reproductive success follows one of three possible patterns: it may be decreasing, increasing, or non-monotonic in the polygyny rate. Specifically, a male would: *(i)* prefer monogamy to any polygyny rate if $\alpha = \lambda = 1$, the preference being strict if $y^*(1) > 0$; *(ii)* strictly prefer monogamy to low polygyny rates, and high polygyny rates to monogamy, if $y^*(1)$ and $\alpha > \hat{\alpha}$; and *(iii)* prefer any polygyny rate $k > 1$ to monogamy otherwise. Interestingly, a male cannot prefer intermediate polygyny rates to both low and high ones.

The proposition reveals that two key forces are at play. First, the degree of publicness of protection, α, is central. The greater is the public nature of protection (i.e., the lower is α), the more likely is the male to benefit from an increase in the number of wives. In the extreme case where protection is a pure public good ($\alpha = 0$), he benefits from an increase in k at all polygyny levels. By continuity, the same result obtains as long as α is sufficiently small ($\alpha \leq \hat{\alpha}$). This result is intuitive: if protection is quite public in nature, the male can feed many mouths while also providing protection to many children.

The second key force, which is novel in the literature, is intimately linked to the division of labor. The proposition indeed reveals that whether the male engages in food production or not matters for the qualitative nature of his preferences over polygyny rates, as long as $\alpha > 0$ so that protection is not a pure public good. Thus, if he devotes all his time to protection for any polygyny rate (a sufficient condition for this is that $y^*(1) = 0$) he always benefits from having more wives, whereas if he engages in food production for certain family structures (i.e., if $y^*(1) > 0$) his reproductive success may exhibit decreasing marginal returns to polygyny. The reason for this is as follows. If the male engages in food production, an increase in k means that each child gets less food and also less protection. The resulting reduction in child survival probability outweighs the first-order gain from increasing k if protection is sufficiently private in nature (i.e., if α is high enough), which is intuitive. Interestingly, if this happens, it happens either for all polygyny rates (case *(iii)* in the proposition) or only for low polygyny rates (case *(ii)*). In other words, a male cannot prefer intermediate polygyny rates to both low and high ones. This feature is intimately linked to the division of labor: indeed, as k becomes larger, the male shifts the labor division in favor of more female food production and less male food production, since his time then becomes relatively more valuable when used protecting the children. But beyond some polygyny level the labor division shift ceases: the male then produces only protection. Adding more wives must then be beneficial as long as the protection is somewhat public in nature ($\alpha < 1$).

Thanks to Proposition 4, I am now in a position to characterize the set of sustainable polygyny rates.

2.4 Sustainable polygyny rates

Here I return to the case where the number of wives k is an integer. To be more precise, since any sustainable polygyny rate equals 2^r for some $r \in \{0, 1, 2, ..., R\}$ (see Definition 1), I restrict attention to polygyny rates $k \in \{1, 2, 4, ..., 2^R\}$. Combining the results on the male fighting game (Proposition 2) with those on male preferences over polygyny rates (Proposition 4), I obtain the following proposition, where $\hat{k}$ is defined in (16):

Proposition 5 *Consider a population in which males play the R-round fighting game, and where, in each household formed following the fighting game, the male imposes his preferred fertility rate as well as his preferred division of labor on his wives. Then the set of sustainable polygyny rates is as follows:*
(i) if $y^*(1) = 0$ *or if* $\alpha \leq \hat{\alpha}$*, the only sustainable polygyny rate is* $k = 2^R$*;*
(ii) if $y^*(1) > 0$ *and* $\alpha > \hat{\alpha}$*, any polygyny rate* $k \in \{1, 2, 4, ..., 2^R\}$ *such that* $k \leq \hat{k}$ *is*

sustainable, and so is $k = 2^R$*;*
(iii) if $\alpha\lambda = 1$*, any polygyny rate* $k \in \left\{1, 2, 4, ..., 2^R\right\}$ *is sustainable.*

This result is striking for two reasons.[26] First, there is a bang-bang effect: either only the highest possible polygyny rate is sustainable (case *(i)*), or the highest rate as well as the *lowest* rates are sustainable (case *(ii)*). This is explained by the fact that whenever male preferences are non-monotonic over polygyny rates, the male prefers small to intermediate rates. Second, whether low polygyny rates are sustainable or not is correlated with the extent to which the male engages in food production: for low rates to be sustainable, it *must* be that the male engages in food production. Male engagement in food production means that some of his time is used producing a good that is fully private, which in turn means that adding more wives and therefore more children lessens the male's contribution to child survival. Put differently, male engagement in food production is a sign that the ecology is harsh enough for the male not to focus his entire time on producing the somewhat public good protection. This in turn is a sign that the inframarginal children may suffer so much if more children are added, that the male's reproductive success declines. Since, as discussed above, male food production declines with the number of wives, this severe trade-off can only appear for low polygyny rates. Hence, in such ecologies monogamy and low polygyny rates (below $\hat{k}$) are sustainable, while intermediate polygyny rates (slightly above $\hat{k}$) are not. If, by contrast, the ecology is generous enough for the male to produce no food, or if the degree of publicness of protection is large enough, male reproductive success never suffers from adding more wives, and each male then always benefits from fighting more than other males, and the only sustainable poygyny rate is the maximal one.

Prior to moving on to the robustness analysis, it is worth noticing that in this model a preference for monogamy over low polygyny rates go hand in hand with low fertility, since both are favored by a high value of α.

3 Robustness

In this section I lift some of the simplifying assumptions used in the baseline model to check the robustness of the results. I thus study three extensions of the baseline model.

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[26] In this discussion I disregard the extreme and unlikely case where protection is a fully private good and the marginal return to protection is constant, i.e., $\alpha\lambda = 1$.

3.1 Decentralization of time allocation decisions

Until now I have assumed that the male imposes his preferred fertility and labor division on his (obedient) wives. Do the results depend on this assumption? Here I investigate this question by giving each woman full decision-making power over her own time allocation.

The interaction within a household unfolds as follows. Given that he has successfully fought to get the right to marry k women, the male first chooses the fertility n per wife. Each female $j = 1, ..., k$ then chooses the time allocation x_j that maximizes her reproductive success (see below), taking the male's and the other females' time allocations as given, and the male chooses the time allocation y that maximizes his reproductive success, taking the females' time allocations as given. Formally, given k and n, the situation at hand is a simultaneous-move game with $k + 1$ players; each female player $j = 1, ..., k$ has strategy x_j and strategy set $[0, T]$, while the male player has strategy y and strategy set $[0, T]$. To complete the description of the game, it is necessary to define each player's payoff as a function of the strategy profile $(\boldsymbol{x}, y)$, where $\boldsymbol{x} = (x_1, ..., x_k)$. Starting with the females, the reproductive success of female j, and hence her payoff in the game, writes

$$F^D(x_j, y) = n \cdot a(n) \cdot s^D(x_j, y), \tag{17}$$

where

$$s^D(x_j, y) = \left(\frac{T - y}{(kn)^\alpha}\right)^\lambda \cdot \left(\frac{T - \phi x_j}{n^\beta}\right)^\sigma \cdot \left(\frac{\rho\gamma x_j}{n} + \frac{\theta y}{kn}\right)^\tau. \tag{18}$$

Hence, the reproductive success of the male writes

$$M^D(\boldsymbol{x}, y) = \sum_{j=1}^{k} F(x_j, y) = \sum_{j=1}^{k} a(n) \cdot n \cdot s(x_j, y), \tag{19}$$

where $\boldsymbol{x} = (x_1, ..., x_k)$. The letter D (for decentralization) is used to differentiate the survival and male success functions from the ones in the analysis above; note that I here omit the arguments n and k, since those are given when time allocations are chosen.

The following proposition shows that this game has a unique Nash equilibrium, which coincides with the male's preferred labor division described in the previous section:

Proposition 6 *Suppose that in each household the male imposes some fertility rate n on his wives, and that the male as well as each wife then chooses his or her own time allocation. For any ecology $\boldsymbol{\xi}$ and any household composition (k, n), there exists a unique Nash equilibrium time allocation, $\left(\boldsymbol{x}^D, y^D\right)$, which is female-symmetric and such that $x^D = x^*(k)$ and $y^D = y^*(k)$, where $(x^*(k), y^*(k))$ is the male's preferred labor division (see Proposition 3).*

In this model the interests of the male and the females are aligned when it comes to the division of labor: for a given family composition they all want to maximize the probability that the children survive.[27] This explains why there exists a Nash equilibrium that implements the male's preferred division of labor. Of course, alignment of interest does not necessarily imply that there is a unique equilibrium; however, the proposition shows that uniqueness does obtain in this setting.

In sum:

Proposition 7 *Consider a population in which males play the R-round fighting game, and where, in each household formed following the fighting game, the male imposes his preferred fertility rate on his wives, and the male as well as each wife then chooses his or her own time allocation. Suppose further that in each household the male and his wives play the unique Nash equilibrium time allocation. Then the set of sustainable polygyny rates is the same as in a population where the male imposes his preferred division of labor.*

3.2 Allomothering

In the analysis above, it was assumed that each mother provides only for her own children; I will refer to this as private provisioning. Would the results be qualitatively different if, instead, women had to share the food and the care that they produce equally among all the children in the household? Here I characterize male and female behaviors and the ensuing implications for the set of sustainable polygyny rates under the latter assumption, a state of affairs I will refer to as allomothering (Hrdy, 2009).

Using a superindex A for "allomothering," the reproductive success of each female is

$$F^A(\boldsymbol{x}, y) = n \cdot a(n) \cdot s^A(\boldsymbol{x}, y), \tag{20}$$

where

$$s^A(\boldsymbol{x}, y) = \left(\frac{1-y}{(kn)^{\alpha}}\right)^{\lambda} \cdot \left(\frac{\sum_{j=1}^{k}(1-\phi x_j)}{(kn)^{\beta}}\right)^{\sigma} \cdot \left(\frac{\rho\gamma\sum_{j=1}^{k} x_j}{kn} + \frac{\theta y}{kn}\right)^{\tau}. \tag{21}$$

Compared to the expression in (10), the amount of care that a child receives now depends on the sum of the females' contributions towards the production of care. It follows that the

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[27] In particular, there is no opportunity cost of time. The results would likely be different in a model that allows for unfaithfulness, an issue that is left for future research.

reproductive success of a male is

$$M^{A}(\boldsymbol{x},y)=k\cdot n\cdot a(n)\cdot s^{A}(\boldsymbol{x},y)\,. \tag{22}$$

The following proposition characterizes the set of Nash equilibrium time allocations:[28]

Proposition 8 *Suppose that in each household the male imposes some fertility rate n on his wives, that the male as well as each wife then chooses his or her own time allocation, and that allomothering applies. For any ecology $\boldsymbol{\xi}$ and any household composition (k,n), there exists a unique Nash equilibrium time allocation, $\left(\boldsymbol{x}^{A},y^{A}\right)$, which is female-symmetric and identical to the one under private provisioning: $x^{A}=x^{*}(k)$ and $y^{A}=y^{*}(k)$.*

It may come as a surprise that the equilibrium time allocation is the same as under private provisioning, since intuition suggests that under allomothering the females would have an incentive to free ride on each other. However, the result is readily explained by the fact that, by assumption, females must devote all of their time to either food or care production: in other words, there is no opportunity cost of time, and therefore no incentive to free ride.

Although the equilibrium time allocations are the same as under private provisioning, reproductive success is generally higher under allomothering, since now each child benefits from the care produced by all the females. Formally, let $\tilde{M}^{A}(k)$ denote male reproductive success as a function of the polygyny rate k under allomothering. Then:

$$\begin{aligned}\tilde{M}^{A}(k) &= k\cdot a(n)\cdot n\cdot s^{A}\left(x^{*}(k),y^{*}(k)\right) \\ &= k^{\sigma(1-\beta)}\cdot\widetilde{M}(k)\,,\end{aligned} \tag{23}$$

where $\widetilde{M}(k)$ is the male reproductive success under private provisioning (see (2)). An immediate implication of this equation is that the preferred fertility rate is the same as under private provisioning. Another immediate remark is that as soon as care is a somewhat public good ($\beta<1$), reproductive success is higher than under private provisioning. Will this have implications for the set of sustainable polygyny rates? In the next proposition, which states the result,

$$\bar{\alpha}\equiv\hat{\alpha}+\frac{\sigma(1-\beta)}{\lambda},$$

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[28] Formally, the game that is being analyzed is a simultaneous-move game with $k+1$ players, where each player has strategy set $[0,T]$, where the strategy of female $j=1,...,k$ is denoted x_{j}, the strategy of the male is denoted y, and the payoff to female j is given in (20) while the payoff to the male in (22).

and $\bar{k}$ is implicitly defined by the equation

$$[1 - \lambda\alpha + \sigma(1-\beta)] \cdot \rho\gamma x^*\left(\bar{k}\right) = [\tau - 1 + \lambda\alpha - \sigma(1-\beta)] \cdot \frac{\theta y^*\left(\bar{k}\right)}{\bar{k}}. \tag{24a}$$

Proposition 9 *Consider a population in which males play the R-round fighting game, and where, in each household formed following the fighting game, the male imposes his preferred fertility rate on his wives, that the male as well as each wife then chooses his or her own time allocation, and that allomothering applies. Then the set of sustainable polygyny rates is as follows:*
(i) if $y^*(1) = 0$ *or if* $\alpha \leq \bar{\alpha}$*, the only sustainable polygyny rate is* $k = 2^R$*;*
(ii) if $y^*(1) > 0$ *and* $\alpha > \bar{\alpha}$*, any polygyny rate* $k \leq \bar{k}$ *in the set* $\left\{1, 2, 4, ..., 2^R\right\}$ *is sustainable, and so is* $k = 2^R$*;*
(iii) if $\lambda\alpha\beta = 1$*, any polygyny rate in the set* $\left\{1, 2, 4, ..., 2^R\right\}$ *is sustainable.*

In sum, the qualitative features of the set of sustainable polygyny rates obtained under private female provisioning, are maintained under allomothering.

3.3 Cost of fighting

In the analysis above, fighting entailed no cost beyond that associated with losing a fight. Consider again the baseline model studied in Section 2, but suppose now that fighting entails a cost even for the winners. Specifically, assume that each round of successful fighting means that any gain is discounted by a factor $\delta \in (0, 1]$, the baseline model corresponding to the special case $\delta = 1$. Clearly, Proposition 2 then generalizes to:

Proposition 10 *In the R-round fighting game,* $r = R$ *is locally evolutionarily stable, and any* $r \in \{0, 1, ..., R-1\}$ *such that* $\widetilde{M}(2^r) > \delta \cdot \widetilde{M}(2^{r+1})$ *is also locally evolutionarily stable.*

Any polygyny rate such that a male would not have an incentive to fight more than other males is sustainable. Hence, whenever male reproductive success $\widetilde{M}$ is decreasing in the polygyny rate, the presence of a discount factor $\delta \in (0, 1]$ makes no difference: whether $\delta = 1$ or $\delta < 1$, a male does not have an incentive to fight more than other males. By contrast, such a discount factor can make a difference when $\widetilde{M}$ is increasing in the polygyny rate. The following additional result on the shape of $\widetilde{M}$ will be instrumental to state the general result.

Proposition 11 *Whenever male reproductive success is increasing in the number of wives, the rate at which it increases is bounded above:* $\frac{d\widetilde{M}(k)}{dk} \leq [n^* \cdot a(n^*) \cdot s(x^*(k), y^*(k), n^*, k)]$*, with a strict inequality if and only if* $\alpha > 0$ *or* $y^*(k) > 0$*. Furthermore, the value of the upper bound declines as* k *increases unless* $\alpha = y^*(k) = 0$*, in which case* $\frac{d\widetilde{M}(k)}{dk} = n^* \cdot a(n^*) \cdot s(x^*(k), y^*(k), n^*, k)$*, which is then a constant.*

Stated differently, the marginal benefit of adding more wives cannot exceed the first-order effect thereof. Moreover, this first-order effect becomes smaller as k increases as soon as such an increase means that each child gets less protection and/or food from the father. I am now in a position to show:

Proposition 12 *Consider a population in which males play the* R*-round fighting game, and where, in each household formed following the fighting game, the male imposes his preferred fertility rate as well as his preferred division of labor on his wives. Suppose further that fighting entails a cost, modeled by way of the discount factor* $\delta \in (0,1]$*. Then there exists* $\hat{\delta}$ *such that:*
(A) For any $\delta \in (0, \hat{\delta}]$*, any polygyny rate* k *in the set* $\{1, 2, 2^2, ..., 2^R\}$ *is sustainable;*
(B) For any $\delta \in (\hat{\delta}, 1]$*, three cases arise:*
(B.i) if $y^*(1) = 0$ *or* $\alpha \leq \hat{\alpha}$*, there exists* $r^P \in \{0, 1, 2, ..., R\}$ *such that any polygyny rate* k *in the set* $\{1, 2, 2^2, ..., 2^R\}$ *which is larger or equal to* 2^{r^P} *is sustainable;*
(B.ii) if $y^*(1) > 0$ *and* $\alpha > \hat{\alpha}$*, there exists* $r^M \in \{0, 1, 2, ..., R\}$ *and* $r^N \in \{r^M, r^M + 1, ..., R\}$ *such that any polygyny rate* k *in the set* $\{1, 2, 2^2, ..., 2^R\}$ *which is smaller or equal to* 2^{r^M} *or larger or equal to* 2^{r^N} *is sustainable;*
(B.iii) if $\alpha\lambda = 1$*, any polygyny rate in the set* $\{1, 2, ..., 2^R\}$ *is sustainable.*

An intrinsic fighting cost tilts the balance in favor of lower polygyny rates: for instance, even when a male would always benefit from having more wives, polygyny rates below the highest possible one (2^R) are sustainable. In fact, if the cost of fighting is severe enough (δ is close to 0—case (A) of the proposition), males have an incentive to stop fighting at all polygyny levels, and all polygyny levels are then sustainable in any ecology. Still, for δ large enough, the qualitative results obtained in the baseline model are confirmed, in the sense that the bang-bang effect is still present: either only a subset of the highest polygyny rates are sustainable (case *(B.i)*), or a subset of the highest rates as well as the lowest rates are sustainable (case *(B.ii)*).

4 Discussion

4.1 Female preferences

The approach adopted in the baseline model was a sexist one: the males are the sole decision-makers in the fighting as well as in the childbearing and child-rearing stages, while the females obey the orders of their husbands. It was shown in Section 3.1 that this assumption entails no loss of generality when it comes to decisions pertaining to the childbearing and child-rearing stages, because in this model the female interests are aligned with those of their husbands at that point. However, is this also true when it comes to household composition? Here I characterize the female's preferences over polygyny rates, by studying the shape of female reproductive success

$$\widetilde{F}(k) = \frac{\widetilde{M}(k)}{k}, \tag{25}$$

where $\widetilde{M}(k)$ is defined in (2).

The following proposition shows that female preferences differ starkly from male ones:

Proposition 13 *Suppose that in each household composed of one male and k wives the preferred fertility rate* n^* *and the preferred time allocation* $(x^*(k), y^*(k))$ *are used, and that private provisioning applies. Then, in each ecology* $\boldsymbol{\xi}$ *the reproductive success of a female is strictly decreasing in the polygyny rate* k*, unless* $\alpha = y^*(1) = 0$*, in which case it is constant in* k*.*

The intuition behind this result is clear: an increase in the number of wives implies that each female's brood gets a smaller fraction of male outputs, and this necessarily reduces female reproductive success. The only exception to this rule arises in the (arguably inexistent) case where the totality of the male output is a pure public good: this occurs if the male produces no food ($y^*(1) = 0$) and protection is a pure public good ($\alpha = 0$).

In sum, under private provisioning, in any ecology female reproductive success declines as the rate of polygyny rises, and the preferred family composition of females is monogamy. By contrast, conditional on mating, males seldom prefer monogamy. In light of Propositions 4, 5, and 13 I conclude that there exists only one case in which both males and females are fully satisfied with a sustainable polygyny rate, in the sense that they would not prefer any other polygyny rate: this occurs when $\alpha\lambda = 1$, in which case both males and females prefer monogamy to any positive rate of polygyny. For ecologies in which $\alpha\gamma \neq 1$, one of the following three cases applies. (A) Females are fully satisfied but males are not: this happens when monogamy is sustainable and males do not fight ($r - 0$), although conditional on mating males would in fact prefer the largest sustainable polygyny rate ($k = 2^R$). (B) Males are fully satisfied and females are maximally dissatisfied: this happens when $r = R$ in the fighting game. (C) Both males and females are dissatisfied: this happens when males play some $0 < r < R$ in the fighting game; males may in fact be maximally dissatisfied in this case.

The conclusions are different under allomothering, as is shown in the next proposition (the proof is omitted, for it is readily seen that female reproductive success under allomothering equals $k^{\sigma(1-\beta)-1}$ times those of a male under private provisioning, so the proof of Proposition 4 can be directly adapted to the situation at hand). Let

$$\tilde{\alpha} \equiv \hat{\alpha} - \frac{1 - \sigma\left(1 - \beta\right)}{\lambda}. \tag{26}$$

Proposition 14 *Suppose that in each household composed of one male and k wives the preferred fertility rate n^* and the preferred time allocation $\left(x^*\left(k\right), y^*\left(k\right)\right)$ are used, and that allmothering applies. Then, females:*
(i) prefer monogamy to any polygyny rate if $\alpha \geq \sigma\left(1-\beta\right)/\lambda$, the preference being strict if and only if $y^*\left(1\right) > 0$;
(ii) strictly prefer monogamy to low polygyny rates, and high polygyny rates to monogamy, if $y^*\left(1\right) > 0$ and $\alpha \in \left(\tilde{\alpha}, \sigma\left(1-\beta\right)/\lambda\right)$;
(iii) (weakly) prefer any polygyny rate $k > 1$ to monogamy if $y^*\left(1\right) = 0$ or if $\alpha \in \left[0, \tilde{\alpha}\right]$.

While polygyny still entails a reduction in the contribution of the male to each female's brood, females now also derive some benefit from there being other wives as long as care is a somewhat public good ($\beta < 1$). Hence, female preferences under allmothering are qualitatively similar to male preferences under private provisioning. Importantly, however, here also the privateness of care (β) matters. If care is very private (i.e., if β is high), a female benefits only a little from other wives, and she then prefers monogamy over all polygyny rates even if protection is almost a fully public good (i.e., even for low valus of α), while under private provisioning, protection had to be fully private ($\alpha = 1/\lambda$) for males to prefer monogamy over all polygyny rates. But if both α and β are small, females no longer have a strict preference for monogamy. Thus, for intermediary degrees of "aggregate privateness" ($\lambda\alpha + \sigma\beta$), female preferences are non-monotonic in the polygyny rate, and for low levels of aggregate privateness, females prefer polygyny to monogamy.

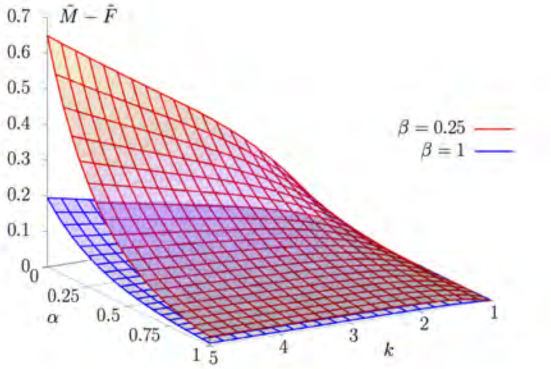

Figure 1: The difference between male and female reproductive success under allomothering, as a function of the polygyny rate (k) and the degree of publicness of male care (α), for $\lambda = \sigma = 1$, $\tau = 0.5$, $\phi = 0.75$, $\rho = 1$, $\gamma = 0.1$, $\theta = 0.6$, and $n = 5$, and for two different degrees of publicness of female care: in blue, a fully private good ($\beta = 1$), and in red, a somewhat public good ($\beta = 0.25$)

4.2 Battle of the sexes

While the fact that females and males typically have different preferences over polygyny rates is not surprising,[29] an arguably interesting by-product of the model is its ability to provide insights about how the magnitude of the disagreement between females and males depends on the ecology. This disagreement being literally about life and death, it may be expected to affect the quality of relations between men and women. As an illustration, Figure 1 shows the difference between the reproductive success of a lucky male and that of a female, as a function of the degree of publicness of male care and the polygyny rate. The graph suggests that for any α, the difference increases (from zero for $k = 1$) as k increases. Furthermore, for any k, the difference is smaller the higher is α: in sum, in this example relations between males and females may be expected to be more pacific in ecologies where male care is a relatively private good.

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[29] This is consistent with the observation that the degree of polygyny in a society may be expected to depend on whether females are free to choose or are coerced into polygynous marriages (Gibson and Mace, 2007, Winking et al., 2013).

4.3 Efficiency

In a distant enough evolutionary past, there were no institutions. However, once institutions emerged, they opened the door to reflection about what is good for society as a whole. This might have been particularly relevant in populations with intense competition between groups, since more efficient groups may have given their members an edge in the evolutionary race. Modeling group competition is beyond the scope of this paper.[30] Here I simply derive an efficiency result, which may provide some insights regarding the direction in which group competition may have pushed rules on polygyny rates.

The male preferences derived above apply to lucky males, i.e., preferences conditional on being successful at mating. But evolutionary theory predicts that at birth the expected reproductive success of males must be equal to that of females, and that the sex ratio equals one (Fisher, 1930). Hence, the polygyny rate that maximizes the reproductive success of males at birth—and thus behind the veil of ignorance as to who will be lucky and who will be unlucky in the fighting game—coincides with the polygyny rate that maximizes female reproductive success. Female preferences are thus of particular interest when it comes to understanding how resources may be utilized efficiently at the level of the group. In a nutshell, my analysis suggests that while group competition may be expected to favor monogamy in many ecologies, allomothering sometimes makes polygyny attractive for the group, even absent the fact that polygyny makes some male labor available for fighting wars. Formally, Propositions 13 and 14 imply:

Corollary 1 *Under private provisioning, monogamy* ($k = 1$) *is efficient in any ecology* $\boldsymbol{\xi}$*, and it is the unique efficient polygyny rate for any ecology such that* $\alpha \cdot y^{*}(1) > 0$*. Under allomothering, monogamy is efficient in any ecology* $\boldsymbol{\xi}$ *such that* $\alpha \geq \sigma(1-\beta)/\lambda$ *(and it is the unique efficient polygyny rate if, moreover,* $y^{*}(1) > 0$*), while maximal polygyny* ($k = 2^{R}$) *is the unique efficient polygyny rate if* $\alpha < \sigma(1-\beta)/\lambda$*.*

There are two striking aspects here. First, polygyny can be efficient only under allomothering. Second, there is bang-bang result under allomothering: it is either minimal or maximal polygyny that is efficient. The intuition is clear: either the negative effect of diluting male outputs over more than one female's children always outweighs the positive effect

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[30] Henrich, Boyd, and Richerson (2012) show that monogamy is favored by between-group competition if it leads to a reallocation of resources from wasteful activties to paternal investment.

of sharing total female outputs among all the children in the household, or the former effect is small enough to be outweighed for high enough polygyny rates (when the male devotes all his time to protection), which happens if protection is public enough in nature.[31]

5 Conclusion

This paper proposes a novel approach to the study of the evolution of polygyny. The key argument is that in our evolutionary past male preferences over polygyny rates would have depended on the ecology in which the population at hand was evolving, that these preferences in turn determined males' incentives to fight with each other to acquire more wives, incentives that ultimately may have shaped effective polygyny rates. The originality of the model stems from its ability to link the preferences to both the ecology and the division of labor between males and females. In particular, the model predicts that either high or low, but not intermediate, polygyny rates will tend to arise absent institutional involvement. Furthermore, the analysis reveals that the outcome is intimately linked to the extent to which males engage in food production, monogamy being sustainable only in ecologies in which males engage in food production. This is because food is a private good, which implies that the most severe drop in average child survival probability that results from adding wives arises when the male spends time on food production.

The results challenge the view that the rise of monogamy is a mystery in a world driven by evolutionary forces. Indeed, they instead suggest that there are two types of environments: one in which natural selection favors both monogamy and high polygyny rates over intermediate polygyny rates, and one in which natural selection favors a high polygyny rate. In the former type of environment, in our evolutionary past males simply did not achieve a higher reproductive success by fighting at the margin to get more than one wife; the associated lack of incentive for males to compete against each other may have led to egalitarian societies and the rise of monogamy a long time ago, prior to any intervention by rulers (Lagerlöf, 2010, De La Croix and Mariani, 2015). Interestingly, in this type of environment, there is typically also a preference for low fertility rates. Hence, in such environments, even if male heterogeneity eventually arose thanks to the intergenerational transmission of wealth, the

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[31] Interestingly, the propensity for monogamy to be more efficient than some levels of polygyny goes hand in hand with the propensity for low fertility rates to be efficient, since both are driven by a low average publicness of care.

population growth may not have been high enough to sustain polygyny through a mismatch between male and female mating age, as it may have done in Africa (Tertilt, 2006). In sum, the results suggest that human societies may have split into essentially two groups a long time ago, the (mainly) monogamous ones and the highly polygynous ones, depending on the ecology in which their populations had evolved. This split may in turn have laid the basis for differential success of religions, depending on their attitude towards polygynous marriage. Indeed, I would even like to argue that this is consistent with the pattern observed today in the two most represented religions today, Christianity and Islam: one imposes strict monogamy while the other allows not for two or three wives, but four.

The analysis relies on a model from which many arguably important aspects were left out. These should be included in future research. Specifically, it would be interesting to embed households more explicitly in a market economy and allow for wealth accumulation over generations. Allowing for uncertainty in food production, risk sharing within the extended family, and teamwork between males might also be fruitful. Finally, the theoretical model may perhaps also be extended in view of studying the evolution of preferences of mothers and fathers for their children and for each other.

6 Appendix

6.1 Proof of Proposition 2

Begin by considering some resident strategy $r \in \{0, 1, ..., R-1\}$, and suppose that the mutant strategy $r' = r + 1$ is present in a share $\varepsilon > 0$ of the population. In this population all males play *Fight* in the first r rounds; hence, in each round until round r, the male population halves, but the *share* of residents who are still in the race is $1 - \varepsilon$. Consider now round $r + 1$. In this round, residents play *Peace* while mutants play *Fight*. Thus, the expected reproductive success of a resident who is alive at the beginning of round $r + 1$ is

$$(1-\varepsilon) \cdot \frac{1}{2^r} \cdot \widetilde{M}(2^r) + \varepsilon \cdot 0.$$

With probability $1 - \varepsilon$ he is matched with another resident, in which case both play *Peace*, both survive and each gets to keep the 2^r women that he had at the beginning of round $r + 1$; otherwise he is matched with a mutant, in which case he gets 0 since he plays *Peace* while the mutant plays *Fight*. The expected payoff to a mutant in round $r + 1$ is

$$(1-\varepsilon) \cdot \frac{1}{2^r} \cdot \widetilde{M}\left(2^{r+1}\right) + \varepsilon \cdot \frac{1}{2^{r+1}} \cdot \widetilde{M}\left(2^{r+1}\right).$$

If matched with a resident, he wins for sure, and gets 2^{r+1} women; if matched with another mutant, he gets 2^{r+1} women with probability 1/2. Since $\frac{1}{2^{r+1}} \cdot \widetilde{M}\left(2^{r+1}\right) > 0$, a necessary and sufficient condition for strategy r to be evolutionarily stable against $r' = r + 1$ is that

$$\widetilde{M}(2^r) > \widetilde{M}\left(2^{r+1}\right). \tag{27}$$

Turning now to the mutant strategy which consists in starting to play *Peace* one round before residents do, consider some $r \in \{1, ..., R\}$, and suppose that the mutant strategy $r' = r - 1$ is present in a share ε. In this population all males play *Fight* for $r - 1$ rounds, and at the beginning of round r there is still a share $1 - \varepsilon$ of residents and a share ε of mutants. In round r the expected payoff to a resident is

$$(1-\varepsilon) \cdot \frac{1}{2^r} \cdot \widetilde{M}(2^r) + \varepsilon \cdot \frac{1}{2^{r-1}} \cdot \widetilde{M}(2^r).$$

With probability ε he is matched with a mutant, in which case he survives with certainty to round $r + 1$. Otherwise he is matched with another resident, in which case both play *Fight*. In round $r + 1$ there will only be residents around, so any resident who survives to round $r + 1$ gets 2^r women. The expected payoff to a mutant in round r is

$$(1-\varepsilon) \cdot 0 + \varepsilon \cdot \frac{1}{2^{r-1}} \cdot \widetilde{M}\left(2^{r-1}\right).$$

Since $\frac{1}{2^r} \cdot \widetilde{M}(2^r) > 0$, it is immediate that any $r \in \{1, ..., R\}$ is evolutionarily stable against mutant strategy $r' = r - 1$.

6.2 Proof of Proposition 3

To begin, note that the male's maximization problem can be treated as two separate maximization problems, one for n and one for (x, y), since the objective function can be written

$$M(x, y, n, k) = k^{1-\alpha\lambda} \cdot g(n) \cdot \tilde{s}(x, y), \tag{28}$$

where

$$g(n) = n^{1-\alpha\lambda-\beta\sigma-\tau} \cdot a(n) \tag{29}$$

and

$$\tilde{s}(x, y) = (T - y)^{\lambda} \cdot (T - \phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}. \tag{30}$$

Specifically, choosing $(x, y, n) \in [0, T]^2 \times [1, +\infty)$ to maximize $M(x, y, n, k)$ boils down to choosing $n \in [1, +\infty)$ to maximize $g(n)$ and $(x, y) \in [0, T]^2$ to maximize $\tilde{s}(x, y)$.

Starting with the choice of (x, y), note first that $y^* \neq T$, since for any $x \in [0, T]$, $\tilde{s}(x, T) = 0$ while $\tilde{s}(x, y) > 0$ for any $y \in (0, T)$. Moreover, $(x^*, y^*) \neq (0, 0)$, since $\tilde{s}(0, 0) = 0$ while $\tilde{s}(x, y) > 0$ for any $(x, y) \in [0, T] \times (0, T)$. Next, since $\tilde{s}$ is strictly concave in (x, y), it is sufficient to study the two first-order partial derivatives to determine (x^*, y^*). Starting with y:

$$\begin{aligned} \frac{\partial \tilde{s}(x, y)}{\partial y} = {} & \frac{\tau\theta}{k} \cdot (T - y)^{\lambda} \cdot (T - \phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1} \\ & -\lambda (T - y)^{\lambda-1} (T - \phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}. \end{aligned} \tag{31}$$

Dividing this by the strictly positive term $(T - y)^{\lambda-1} \cdot (T - \phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x_j, y)}{\partial y}$ is the same as the sign of

$$T\theta\tau - k\lambda\rho\gamma x - \theta(\lambda + \tau) y. \tag{32}$$

This expression is strictly decreasing in y. It is non-negative for all $y \in [0, T]$ iff it is non-negative for $y = T$, i.e., if $T\theta\tau - k\lambda\rho\gamma x - \theta(\lambda + \tau) \geq 0$, which is false. The expression in

(32) is non-positive for all $y \in [0, T]$ iff it is non-positive for $y = 0$, i.e., iff $T\theta\tau - k\lambda\rho\gamma x \leq 0$, or

$$x \geq T\theta\tau / (k\lambda\rho\gamma) \equiv x_1. \tag{33}$$

Thus, if $x \in (0, x_1)$, there exists a unique y such that the expression in (32) equals zero:

$$y = \frac{T\tau}{\lambda + \tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda + \tau)} x. \tag{34}$$

In sum,

$$\begin{cases} y^* = 0 \text{ if } x \geq x_1 \\ y^* = \frac{T\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} x \text{ if } x \in [0, x_1]. \end{cases} \tag{35}$$

Turning now to x:

$$\begin{aligned} \frac{\partial \tilde{s}(x, y)}{\partial x} &= \tau\rho\gamma \cdot (T - y)^\lambda \cdot (T - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau - 1} \\ &\quad - (T - y)^\lambda \cdot \phi\sigma \cdot (T - \phi x)^{\sigma - 1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^\tau. \end{aligned} \tag{36}$$

Dividing this by the strictly positive term $(T - y)^\lambda \cdot (T - \phi x)^{\sigma-1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}(x,y)}{\partial x}$ is the same as the sign of

$$Tk\tau\rho\gamma - (\sigma + \tau) k\rho\gamma\phi x - \sigma\phi\theta y. \tag{37}$$

This expression is strictly decreasing in x. It is non-negative for all $x \in [0, T]$ iff it is non-negative for $x = T$, i.e., if $Tk\tau\rho\gamma - (\sigma + \tau) k\rho\gamma\phi X - \sigma\phi\theta y \geq 0$, i.e., iff

$$y \leq \left[\frac{k\tau\rho\gamma}{\sigma\phi\theta} - \frac{(\sigma + \tau) k\rho\gamma}{\sigma\theta}\right] T \equiv y_0. \tag{38}$$

Note that $y_0 \geq 0$ iff $\frac{\tau}{\tau+\sigma} \geq \phi$. The expression in (37) is non-positive for all $x \in [0, T]$ iff it is non-positive for $x = 0$, i.e., iff $Tk\tau\rho\gamma - \sigma\phi\theta y \leq 0$, or

$$y \geq Tk\tau\rho\gamma / (\sigma\phi\theta) \equiv y_1. \tag{39}$$

Thus, if $y \in (\max\{0, y_0\}, y_1)$, there exists a unique x such that the expression in (37) equals zero:

$$x = \frac{T\tau}{(\sigma + \tau)\phi} - \frac{\sigma\theta}{(\sigma + \tau) k\rho\gamma} y. \tag{40}$$

In sum,

$$\begin{cases} x^* = 0 \text{ if } y \geq y_1 \\ x^* = \frac{T\tau}{(\sigma+\tau)\phi} - \frac{\sigma\theta}{(\sigma+\tau)k\rho\gamma} y \text{ if } y \in [\max\{0, y_0\}, y_1] \\ x^* = T \text{ if } y \leq \max\{0, y_0\}. \end{cases} \tag{41}$$

Combining (35) and (41), two cases may be distinguished, depending on whether $y_0 \leq 0$ or $y_0 > 0$. First, if $\frac{\tau}{\sigma+\tau} \leq \phi$ (i.e., if $y_0 \leq 0$),

$$\left(x^*\left(k\right), y^*\left(k\right)\right) = \begin{cases} \left(\frac{\tau T}{\phi(\sigma+\tau)}, 0\right) \text{ if } k \geq \frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda} \\ \left(\frac{(\lambda+\tau)kT-\sigma\phi\mu T}{k\phi(\lambda+\sigma+\tau)}, \frac{\mu\phi(\sigma+\tau)T-\lambda kT}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}, \frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda}\right] \\ \left(0, \frac{\tau T}{\lambda+\tau}\right) \text{ if } k \leq \frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}; \end{cases} \tag{42}$$

Second, if $\frac{\tau}{\sigma+\tau} > \phi$ (i.e., if $y_0 > 0$)

$$\left(x^*\left(k\right), y^*\left(k\right)\right) = \begin{cases} \left(T, 0\right) \text{ if } k \geq \frac{\theta\tau}{\rho\gamma\lambda} \\ \left(T, \frac{\tau T}{\lambda+\tau} - \frac{k\lambda T}{\mu(\lambda+\tau)}\right) \text{ if } k \in \left[\frac{\theta\sigma\phi}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}, \frac{\theta\tau}{\rho\gamma\lambda}\right] \\ \left(\frac{(\lambda+\tau)kT-\sigma\phi\mu T}{k\phi(\lambda+\sigma+\tau)}, \frac{\mu\phi(\sigma+\tau)T-\lambda kT}{\mu\phi(\lambda+\sigma+\tau)}\right) \text{ if } k \in \left[\frac{\theta\sigma\phi}{\rho\gamma(\lambda+\tau)}, \frac{\theta\sigma\phi}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}\right] \\ \left(0, \frac{\tau T}{\lambda+\tau}\right) \text{ if } k \leq \frac{\theta\sigma\phi}{\rho\gamma(\lambda+\tau)}. \end{cases} \tag{43}$$

Turning now to the choice of n, one obtains

$$g'\left(n\right) = \left(1-\lambda\alpha-\sigma\beta-\tau\right)\left(1-bn\right) \cdot n^{-\lambda\alpha-\sigma\beta-\tau} - bn^{1-\lambda\alpha-\sigma\beta-\tau},$$

which implies that two cases arise. First, if $1-\lambda\alpha-\sigma\beta-\tau \leq 0$, $g'\left(n\right) < 0$ for all $n \in [1, +\infty)$, which implies that $n^* = 1$. Second, if $1-\lambda\alpha-\sigma\beta-\tau > 0$,

$$g''\left(n\right) = \left(1-\lambda\alpha-\sigma\beta-\tau\right)\left[-\left(\lambda\alpha+\sigma\beta+\tau\right)\left(1-bn\right)n^{-\lambda\alpha-\sigma\beta-\tau-1} - 2bn^{-\lambda\alpha-\sigma\beta-\tau}\right]$$

is strictly negative, so that either $n^* = 1$ (if $g'\left(1\right) \leq 0$), or there exists $n > 1$ that satisfies the necessary first-order condition for an interior solution, $g'\left(n\right) = 0$. Since $g'\left(n\right) = 0$ iff $n = \frac{1-\lambda\alpha-\sigma\beta-\tau}{(2-\lambda\alpha-\sigma\beta-\tau)b}$, the solution writes:

$$n^* = \max\left\{1, \frac{1-\lambda\alpha-\sigma\beta-\tau}{\left(2-\lambda\alpha-\sigma\beta-\tau\right)b}\right\}.$$

6.3 Proof of Proposition 4

Using the notation introduced in the proof of Proposition 3, and letting

$$s^*\left(x^*\left(k\right), y^*\left(k\right), k\right) = \left[T - y^*\left(k\right)\right]^{\lambda} \cdot \left[T - \phi x^*\left(k\right)\right]^{\sigma} \cdot \left[\rho\gamma x^*\left(k\right) + \frac{\theta y^*\left(k\right)}{k}\right]^{\tau},$$

one obtains the following expression for male reproductive success as a function of k (see equation (2)):

$$\begin{aligned} \widetilde{M}\left(k\right) &= M\left(x^*\left(k\right), y^*\left(k\right), n^*, k\right) \qquad (44) \\ &= g\left(n^*\right) \cdot k^{1-\lambda\alpha} \cdot s^*\left(x^*\left(k\right), y^*\left(k\right), k\right). \end{aligned}$$

Upon dividing the expression in (44) by the strictly positive constant $g(n^*)$, one obtains that $\frac{d\widetilde{M}(k)}{dk}$ has the same sign as

$$\begin{aligned} & (1-\lambda\alpha)\cdot k^{-\lambda\alpha}\cdot s^*\left(x^*(k),y^*(k),k\right) \\ & +k^{1-\lambda\alpha}\cdot\frac{ds^*\left(x^*(k),y^*(k),k\right)}{dk}, \end{aligned} \tag{45}$$

where

$$\begin{aligned} \frac{ds^*\left(x^*(k),y^*(k),k\right)}{dk} &= \left.\frac{\partial\tilde{s}(x,y)}{\partial x}\right|_{(x,y)=(x^*(k),y^*(k))}\cdot\frac{\partial x^*(k)}{\partial k} \\ &\quad+\left.\frac{\partial\tilde{s}(x,y)}{\partial y}\right|_{(x,y)=(x^*(k),y^*(k))}\cdot\frac{\partial y^*(k)}{\partial k} \\ &\quad+\frac{\partial s^*\left(x^*(k),y^*(k),k\right)}{\partial k} \end{aligned} \tag{46}$$

(for the definition of $\tilde{s}(x,y)$, see (30)). From the proof of Proposition 3, one obtains

$$\left.\frac{\partial\tilde{s}(x,y)}{\partial x}\right|_{(x,y)=(x^*(k),y^*(k))}\cdot\frac{\partial x^*(k)}{\partial k}=\left.\frac{\partial\tilde{s}(x,y)}{\partial y}\right|_{(x,y)=(x^*(k),y^*(k))}\cdot\frac{\partial y^*(k)}{\partial k}=0,$$

so that (46) reduces to

$$\begin{aligned} \frac{ds^*\left(x^*(k),y^*(k),k\right)}{dk} &= \frac{\partial s^*\left(x^*(k),y^*(k),k\right)}{\partial k} \\ &= -\frac{\tau\theta y^*(k)}{k^2}\cdot\left[T-y^*(k)\right]^{\lambda}\cdot\left[T-\phi x^*(k)\right]^{\sigma}\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]^{\tau-1} \\ &= -\frac{\tau\theta y^*(k)}{k^2}\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]^{-1}\cdot s^*\left(x^*(k),y^*(k),k\right). \end{aligned} \tag{47}$$

Plugging this expression into that in (45), dividing by the strictly positive term $s^*\left(x^*(k),y^*(k),k\right)\cdot k^{-\lambda\alpha}\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]^{-1}$, and rearranging the terms, one obtains that $\frac{d\widetilde{M}(k)}{dk}$ has the same sign as

$$A(k)\equiv(1-\lambda\alpha)\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]-\tau\cdot\frac{\theta y^*(k)}{k}. \tag{48}$$

Note the following:

Remark 1. For any $(\alpha,\lambda)\in[0,1]\times(0,1]$, $1-\lambda\alpha\geq 0$. Furthermore, $1-\lambda\alpha=0$ if and only if $\alpha=\lambda=1$, in which case $A(k)$ has the same sign as $-y^*(k)$. The remaining remarks all pertain to the case $\alpha\cdot\lambda\neq 1$.

Remark 2. The term $\rho\gamma x^*(k) + \frac{\theta y^*(k)}{k}$, which is the total amount of food produced for each brood of n^* children, writes

$$\rho\gamma x^*(k) + \frac{\theta y^*(k)}{k} = \begin{cases} \frac{\rho\gamma\tau T}{\phi(\sigma+\tau)} \text{ if } k \geq \frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda} \\ \frac{\tau(\rho\gamma k+\theta\phi)T}{k\phi(\lambda+\sigma+\tau)} \text{ if } k \in \left[\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}, \frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda}\right] \\ \frac{\theta\tau T}{k(\lambda+\tau)} \text{ if } k \leq \frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)} \end{cases} \tag{49}$$

if $\frac{\tau}{\sigma+\tau} \leq \phi$, and

$$\rho\gamma x^*(k) + \frac{\theta y^*(k)}{k} = \begin{cases} \rho\gamma T \text{ if } k \geq \frac{\theta\tau}{\rho\gamma\lambda} \\ \frac{\tau(\rho\gamma k+\theta)T}{k(\lambda+\tau)} \text{ if } k \in \left[\frac{\theta\phi\sigma}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}, \frac{\theta\tau}{\rho\gamma\lambda}\right] \\ \frac{\tau(\rho\gamma k+\theta\phi)T}{k\phi(\lambda+\sigma+\tau)} \text{if } k \in \left[\frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)}, \frac{\theta\phi\sigma}{\rho\gamma[\lambda+\tau-\phi(\lambda+\sigma+\tau)]}\right] \\ \frac{\theta\tau T}{k(\lambda+\tau)} \text{ if } k \leq \frac{\theta\phi\sigma}{\rho\gamma(\lambda+\tau)} \end{cases} \tag{50}$$

if $\frac{\tau}{\sigma+\tau} > \phi$. In either case, this term is always strictly positive. Furthermore, it is strictly decreasing in k whenever $y^*(k) > 0$ and constant in k whenever $y^*(k) = 0$. For further use below, let $\tilde{k} \geq 0$ denote the threshold value such that $y^*(k) > 0$ iff $k < \tilde{k}$, and $y^*(k) = 0$ iff $k \geq \tilde{k}$ (note that $\tilde{k}$ may be smaller than 1):

$$\tilde{k} \equiv \begin{cases} \frac{\theta\tau}{\rho\gamma\lambda} \text{ if} \frac{\tau}{\sigma+\tau} > \phi \\ \frac{\theta\phi(\sigma+\tau)}{\rho\gamma\lambda} \text{if } \frac{\tau}{\sigma+\tau} \leq \phi. \end{cases} \tag{51}$$

Remark 3. $\frac{y^*(k)}{k}$ is strictly decreasing in k for any $k < \tilde{k}$ and constant in k for any $k \geq \tilde{k}$.

Remark 4. Suppose that $\tilde{k} > 1$. Then $A(k)$ changes sign at most once for $k \in [1, \tilde{k})$. Indeed, suppose that there exists some $\hat{k} \in [1, \tilde{k})$ such that $A\left(\hat{k}\right) = 0$, i.e.,

$$(1-\lambda\alpha) \cdot \rho\gamma x^*\left(\hat{k}\right) = (\tau - 1 + \lambda\alpha) \cdot \frac{\theta y^*\left(\hat{k}\right)}{\hat{k}}. \tag{52}$$

(Note that $\hat{k}$ must indeed be strictly smaller that $\tilde{k}$ since $A(k) > 0$.for any $k \geq \tilde{k}$.) Note that since $\rho\gamma x^*(k) + \frac{\theta y^*(k)}{k} > 0$ for all k, and since $(1-\lambda\alpha)\cdot\rho\gamma > 0$, both the left-hand side and the right-hand side of this expression must be strictly positive. Then, since $x^*(k)$ is increasing in k and $\frac{\theta y^*(\hat{k})}{\hat{k}}$ is decreasing in k, it must be that $(1-\lambda\alpha)\cdot\rho\gamma x^*\left(\hat{k}\right) > (\tau - 1 + \lambda\alpha) \cdot \frac{\theta y^*(\hat{k})}{\hat{k}}$ for any $k > \hat{k}$.

Remarks 1 - 4 together imply:

A. If $\alpha\lambda = 1$ and $y^*(1) = 0$, then $\frac{d\widetilde{M}(k)}{dk} = 0$ for all $k \geq 1$.

B. If $\alpha\lambda = 1$ and $y^*(1) > 0$, then $\frac{d\widetilde{M}(k)}{dk} < 0$ for all $k \in [1, \tilde{k})$ and $\frac{d\widetilde{M}(k)}{dk} = 0$ for all $k \geq \tilde{k}$.

C. If $\alpha\lambda < 1$ and $y^*(1) = 0$, then $\frac{d\widetilde{M}(k)}{dk} > 0$ for all $k \geq 1$.

D. If $\alpha\lambda < 1$ and $y^*(1) > 0$, then $\frac{d\widetilde{M}(k)}{dk} > 0$ for all $k \geq \tilde{k} > 1$. Moreover, a sufficient condition for $\widetilde{M}$ to be non-monotonic in k is that $\left.\frac{d\widetilde{M}(k)}{dk}\right|_{k=1} < 0$, which is true if and only if

$$(1 - \lambda\alpha)\left[\rho\gamma x^*(1) + \theta y^*(1)\right] < \tau\theta y^*(1). \tag{53}$$

Because $x^*(1)$ and $y^*(1)$ do not depend on α, and since $\rho\gamma x^*(1) + \theta y^*(1) > 0$, the left-hand side can be viewed as an affine and strictly decreasing function of α, which takes the value 0 for $\alpha = 1/\lambda$ and the value $\rho\gamma x^*(1) + \theta y^*(1) > 0$ for $\alpha = 0$. Hence, (53) is equivalent to

$$\alpha > \frac{\rho\gamma x^*(1) + (1-\tau)\theta y^*(1)}{\lambda\left[\rho\gamma x^*(1) + \theta y^*(1)\right]} \equiv \hat{\alpha}. \tag{54}$$

Because $A(k)$ changes sign at most once (see Remark 4), the condition $\alpha > \hat{\alpha}$ is also necessary for $\widetilde{M}$ to be non-monotonic in k. Furthermore, if $\alpha > \hat{\alpha}$, there exists $\hat{k} \in \left(1, \tilde{k}\right)$ such that $\frac{d\widetilde{M}(k)}{dk} < 0$ for all $k \in [1, \hat{k})$, $\frac{d\widetilde{M}(k)}{dk} < 0$ if $k = \hat{k}$, and $\frac{d\widetilde{M}(k)}{dk} > 0$ for all $k > \hat{k}$. Finally, if $\alpha \leq \hat{\alpha}$, $\frac{d\widetilde{M}(k)}{dk} \geq 0$ for all $k \geq 1$, with a strict inequality for all $k \geq 1$ if and only if $\alpha < \hat{\alpha}$.

6.4 Proof of Proposition 5

Recall from Proposition 2 that any $r \in \{0, 1, ..., R-1\}$ such that $\widetilde{M}(2^r) > \widetilde{M}(2^{r+1})$ is locally evolutionarily stable. Thus, any $k \in \left\{2^0, 2^1, 2^2, ..., 2^R\right\}$ such that $\frac{d\widetilde{M}(k)}{dk} < 0$ is sustainable, and so is $k = 2^R$ (see Proposition 2).

6.5 Proof of Proposition 6

A strategy profile $\left(x_1^D, x_2^D, ..., x_k^D, y^D\right) \equiv \left(\boldsymbol{x}^D, y^D\right)$ is a Nash equilibrium if and only if

$$\left\{\begin{array}{c} x_j^D \in \arg\max_{x_j \in [0,1]} a(n) \cdot n \cdot s^D\left(x_j, y^D\right) \qquad \forall j = 1, ..., k \\ y^D \in \arg\max_{y \in [0,1]} a(n) \cdot n \cdot \sum_{j=1}^k s^D\left(x_j^D, y\right), \end{array}\right. \tag{55}$$

where (from (18))

$$s^D(x_j, y) = \left(\frac{T-y}{(kn)^\alpha}\right)^\lambda \cdot \left(\frac{T-\phi x_j}{n^\beta}\right)^\sigma \cdot \left(\frac{\rho\gamma x_j}{n} + \frac{\theta y}{kn}\right)^\tau. \tag{56}$$

To begin, note that $y = T$ cannot be part of an equilibrium strategy profile. Suppose, to the contrary, that $\left(\boldsymbol{x}^D, y^D\right) = \left(\boldsymbol{x}^D, T\right)$ for some $\boldsymbol{x}^D \in [0, T]^k$. Then $s^D\left(x_j^D, y^D\right) = 0$ for any $x_j^D \in [0, T]$, while for any $y \in (0, T)$, $s^D\left(x_j^D, y\right) > 0$. Similarly, $\left(\boldsymbol{x}^*, y^D\right) = (\mathbf{0}, 0)$, where $\mathbf{0} = (0, 0, ..., 0)$, cannot be an equilibrium strategy profile. Indeed, $s^D(\mathbf{0}, 0) = 0$, while, for any $y \in (0, T)$, $s^D(\mathbf{0}, y) > 0$.

Next, it is straightforward to verify that $s^D(x_j, y)$ is strictly concave in x_j. This implies that for each $y \in [0, T)$ there is a unique x_j that maximizes $s^D(x_j, y)$, implying that any equilibrium must be female-symmetric. Hence, it is sufficient to use a two-dimensional vector, $\left(x^D, y^D\right)$, to describe any equilibrium strategy profile. Noting that the term $a(n) \cdot n$ is irrelevant, and that for $x_1^D = ... = x_k^D = x^D$, $\sum_{j=1}^k s^D\left(x_j^D, y\right) = k \cdot s^D\left(x^D, y\right)$, the system of $k+1$ equations (6) reduces to the following system of two equations:

$$\begin{cases} x^D \in \arg\max_{x\in[0,T]} s^D\left(x, y^D\right) \\ y^D \in \arg\max_{y\in[0,T]} s^D\left(x^D, y\right), \end{cases} \tag{57}$$

where

$$s^D(x, y) = \left(\frac{T - y}{(kn)^\alpha}\right)^\lambda \cdot \left(\frac{T - \phi x}{n^\beta}\right)^\sigma \cdot \left(\frac{\rho\gamma x}{n} + \frac{\theta y}{kn}\right)^\tau. \tag{58}$$

Note that, in fact, then, the original $k+1$-player game may be viewed as a game between one male and one female; since the payoff functions are continuous and the strategy spaces are compact and convex, equilibrium existence is guaranteed.

It has already been noted that for each male time allocation $y \in [0, T)$, there exists a unique x that is a best response for the female to y. Likewise, for each female time allocation $x \in [0, T]$, s is strictly concave in y, implying that for each $x \in [0, T]$, there exists a unique y that is a best response for the male to x. Moreover,

$$s^D(x, y) = \frac{1}{(kn)^{\lambda\alpha} n^{\sigma\beta} n^\tau} \cdot (T - y)^\lambda \cdot (T - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^\tau.$$

Letting $\tilde{s}^D(x, y) = (T - y)^\lambda \cdot (T - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^\tau$, it is clearly sufficient to study the first-order partial derivatives of $\tilde{s}^D(x, y)$ to determine the best response functions.

Thus, for the male:

$$\begin{aligned} \frac{\partial \tilde{s}^D(x, y)}{\partial y} &= \frac{\tau\theta}{k} \cdot (T - y)^\lambda \cdot (T - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau - 1} \\ &\quad -\lambda (T - y)^{\lambda - 1} (T - \phi x)^\sigma \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^\tau. \end{aligned} \tag{59}$$

Dividing this by the strictly positive term $(T-y)^{\lambda-1} \cdot (T-\phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}^D(x_j,y)}{\partial y}$ is the same as the sign of

$$T\theta\tau - k\lambda\rho\gamma x - \theta\left(\lambda+\tau\right)y. \tag{60}$$

This expression is strictly decreasing in y. It is non-negative for all $y \in [0, T]$ iff it is non-negative for $y = T$, i.e., if $T\theta\tau - k\lambda\rho\gamma x - \theta\left(\lambda+\tau\right) \geq 0$, which is false. The expression in (60) is non-positive for all $y \in [0, T]$ iff it is non-positive for $y = 0$, i.e., iff $T\theta\tau - k\lambda\rho\gamma x \leq 0$, or

$$x \geq x_1,$$

where x_1 was defined in (33). Thus, if $x \in (0, x_1)$, there exists a unique y such that the expression in (60) equals zero:

$$y = \frac{T\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta\left(\lambda+\tau\right)}x. \tag{61}$$

Hence, the male's best response to the female strategy x (where x is chosen by each of the k females) is:

$$\begin{cases} y^m = 0 \text{ if } x \geq x_1 \\ y^m = \frac{T\tau}{\lambda+\tau} - \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)}x \text{ if } x \in [0, x_1]. \end{cases} \tag{62}$$

Turning now to the female:

$$\begin{aligned} \frac{\partial \tilde{s}^D(x,y)}{\partial x} &= \tau\rho\gamma \cdot (T-y)^{\lambda} \cdot (T-\phi x)^{\sigma} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1} \\ &\quad - (T-y)^{\lambda} \cdot \phi\sigma \cdot (T-\phi x)^{\sigma-1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau}. \end{aligned} \tag{63}$$

Dividing this by the strictly positive term $(T-y)^{\lambda} \cdot (T-\phi x)^{\sigma-1} \cdot \left(\rho\gamma x + \frac{\theta y}{k}\right)^{\tau-1}$ and simplifying, one obtains that the sign of $\frac{\partial \tilde{s}^D(x,y)}{\partial x}$ is the same as the sign of

$$Tk\tau\rho\gamma - \left(\sigma+\tau\right)k\rho\gamma\phi x - \sigma\phi\theta y. \tag{64}$$

This expression is strictly decreasing in x. It is non-negative for all $x \in [0, T]$ iff it is non-negative for $x = T$, i.e., if $Tk\tau\rho\gamma - \left(\sigma+\tau\right)k\rho\gamma\phi T - \sigma\phi\theta y \geq 0$, i.e., iff

$$y \leq y_0,$$

where y_0 was defined in (38). The expression in (64) is non-positive for all $x \in [0, T]$ iff it is non-positive for $x = 0$, i.e., iff $Tk\tau\rho\gamma - \sigma\phi\theta y \leq 0$, or

$$y \geq y_1,$$

where y_1 was defined in (39). Thus, if $y \in (\max\{0, y_0\}, y_1)$, there exists a unique x such that the expression in (64) equals zero:

$$x = \frac{T\tau}{(\sigma+\tau)\phi} - \frac{\sigma\theta}{(\sigma+\tau)k\rho\gamma}y. \tag{65}$$

Hence, a female's best response to the male strategy y is:

$$\begin{cases} x^f = 0 \text{ if } y \geq y_1 \\ x^f = \frac{T\tau}{(\sigma+\tau)\phi} - \frac{\sigma\theta}{(\sigma+\tau)k\rho\gamma}y \text{ if } y \in [\max\{0, y_0\}, y_1] \\ x^f = 1 \text{ if } y \leq \max\{0, y_0\}. \end{cases} \tag{66}$$

Noting that y^m is linear and strictly decreasing in x (whenever $x \in [0, x_1]$), and that x^f is linear and strictly decreasing in y (whenever $y \in [y_0, y_1]$), a necessary and sufficient condition for equilibrium to be unique is that the (absolute value of the) slope of the male's best response curve be strictly smaller than the (absolute value of the) slope of the female's best response curve, which is true: $\left|\frac{dy^m}{dx}\right| = \frac{k\lambda\rho\gamma}{\theta(\lambda+\tau)} < \frac{(\sigma+\tau)k\rho\gamma}{\sigma\theta} = \left|1/\left(\frac{dx^f}{dy}\right)\right|$.

Hence, by combining (62) and (66), one gets the unique Nash equilibrium strategy profile (x^D, y^D). By comparing (62) to (35) and (66) to (41), it is immediate that $x^D = x^*(k)$ and $y^D = y^*(k)$.

6.6 Proof of Proposition 8

For the purpose of this proof some additional notation is introduced. From equation (21) it is clear that, for any given female it is only the sum of the efforts of the other females that matter, and not how these efforts are distributed among these other females. As a result, when it is necessary to single out how s^A depends on the effort of some female under consideration, say female j, one can write $s^A(x_j, \boldsymbol{x}_{-j}, y)$, where x_j is the effort of the female at hand, and $\boldsymbol{x}_{-j}$ is the vector of the other females' efforts.

A strategy profile $(x_1^A, x_2^A, ..., x_k^A, y^A)$ is a Nash equilibrium if and only if

$$\begin{cases} x_j^A \in \arg\max_{x_j \in [0,1]} n \cdot a(n) \cdot s\left(x_j, \boldsymbol{x}_{-j}^A, y^A\right) \qquad \forall j = 1, ..., k \\ y^A \in \arg\max_{y \in [0,1]} k \cdot n \cdot a(n) \cdot s\left(\boldsymbol{x}^A, y\right), \end{cases} \tag{67}$$

where (from (21))

$$\begin{aligned} s^A(x_j, \boldsymbol{x}_{-j}, y) &= \left(\frac{1-y}{(kn)^\alpha}\right)^\lambda \cdot \left(\frac{1-\phi x_j + \sum_{i\neq j}(1-\phi x_i)}{(kn)^\beta}\right)^\sigma \\ &\quad \cdot \left[\frac{\rho\gamma\left[x_j + \sum_{i\neq j} x_i\right]}{kn} + \frac{\theta y}{kn}\right]^\tau \end{aligned}$$

and

$$s^A(\boldsymbol{x},y)=\left(\frac{1-y}{(kn)^{\alpha}}\right)^{\lambda}\cdot\left(\frac{\sum_{j=1}^{k}(1-\phi x_j)}{(kn)^{\beta}}\right)^{\sigma}\cdot\left(\frac{\rho\gamma\sum_{j=1}^{k}x_j}{kn}+\frac{\theta y}{kn}\right)^{\tau}.$$

Note first that $y=1$ cannot be part of an equilibrium strategy profile. Suppose, to the contrary, that $\left(\boldsymbol{x}^A,y^A\right)=\left(\boldsymbol{x}^A,1\right)$ for some $\boldsymbol{x}^A\in[0,1]^k$. Then $s^A\left(\boldsymbol{x}^A,y^A\right)=0$, while, for any $y\in(0,1)$, $s^A\left(\boldsymbol{x}^A,y\right)>0$. Similarly, $\left(\boldsymbol{x}^A,y^A\right)=(\mathbf{0},0)$, where $\mathbf{0}=(0,0,...,0)$, cannot be an equilibrium strategy profile. Indeed, $s^A(\mathbf{0},0)=0$, while, for any $y\in(0,1)$, $s^A(\mathbf{0},y)>0$.

Next, it is straightforward to verify that for each $\left(\boldsymbol{x}_{-j}^A,y^A\right)\in[0,1]^{k-1}\times[0,1)$, $s^A\left(x_j,\boldsymbol{x}_{-j}^A,y^A\right)$ is strictly concave in x_j. This implies that any equilibrium must be female-symmetric. To see this, suppose to the contrary that there exists some equilibrium in which $k'\geq 1$ females choose x' while k'' females choose $x''>x'$ (and where $k'+k''=k$). Then, it must be that each female j is indifferent between x' and x''. But strict concavity of s^A in x_j implies that for any $x'''\in(x',x'')$, a female playing x''' rather than x' achieves a strictly higher payoff.

Finally, note that for each $\boldsymbol{x}^A\in[0,1]^k$, $s^A\left(\boldsymbol{x}^A,y\right)$ is strictly concave in y.

Because any equilibrium is female-symmetric, any equilibrium srategy profile may be fully described by a two-dimensional vector, denoted $\left(x^A,y^A\right)$. Furthermore, since $s^A\left(x_j,\boldsymbol{x}_{-j},y\right)$ is strictly concave in x_j for each $\left(\boldsymbol{x}_{-j},y\right)\in[0,1]^{k-1}\times[0,1)$, and $s^A(\boldsymbol{x},y)$ is strictly concave in y for each $\boldsymbol{x}\in[0,1]^k$ it is sufficient to analyze the first-order partial derivatives $\partial s^A\left(x_j,\boldsymbol{x}_{-j},y\right)/\partial x_j$ and $\partial s^A(\boldsymbol{x},y)/\partial y$ to determine the best response functions of the females and the male.

Writing $\mathbf{x}_{-j}^A$ to denote the $(k-1)$-dimensional vector whose components all equal x^A, and $\mathbf{x}^A$ to denote the k-dimensional vector whose components all equal x^A, and noting that the terms $n\cdot a(n)$, $k\cdot n\cdot a(n)$, and $\frac{1}{(kn)^{\lambda\alpha+\sigma\beta+\tau}}$ are irrelevant, female-symmetry implies that the system of $k+1$ equations (67) reduces to the following system of two equations:

$$\begin{cases} x^A\in\arg\max_{x_j\in[0,1]}(kn)^{\lambda\alpha+\sigma\beta+\tau}\cdot s^A\left(x_j,\mathbf{x}_{-j}^A,y^A\right)\\ y^A\in\arg\max_{y\in[0,1]}(kn)^{\lambda\alpha+\sigma\beta+\tau}\cdot s^A\left(\mathbf{x}^A,y\right),\end{cases}\tag{68}$$

where

$$\begin{aligned}(kn)^{\lambda\alpha+\sigma\beta+\tau}\cdot s^A\left(x_j,\mathbf{x}_{-j}^A,y^A\right) &= \left(1-y^A\right)^{\lambda}\cdot\left[1-\phi x_j+(k-1)\left(1-\phi x^A\right)\right]^{\sigma}\\ &\quad\cdot\left[\rho\gamma\left[x_j+(k-1)x^A\right]+\theta y^A\right]^{\tau}\end{aligned}\tag{69}$$

and

$$(kn)^{\lambda\alpha+\sigma\beta+\tau}\cdot s^A\left(\mathbf{x}^A,y\right)=(1-y)^{\lambda}\cdot\left[k\left(1-\phi x^A\right)\right]^{\sigma}\cdot\left(\rho\gamma kx^A+\theta y\right)^{\tau}.\tag{70}$$

Next, disregarding the strictly positive term $(kn)^{\lambda\alpha+\sigma\beta+\tau}$, the sign of $\partial s^A\left(x_j, \mathbf{x}_{-j}^A, y^A\right)/\partial x_j$ is studied in order to determine a female's best response to $\left(\mathbf{x}_{-j}^A, y^A\right)$. It is straightforward to verify that upon division by $\left(1-y^A\right)^\lambda > 0$ and simplification, the sign of $\left.\frac{\partial s^A\left(x_j, \mathbf{x}_{-j}^A, y^A\right)}{\partial x_j}\right|_{x_j=x^A}$ is the same as the sign of

$$\begin{aligned} & k^{\tau+\sigma-1}\cdot\tau\rho\gamma\cdot\left(1-\phi x^A\right)^\sigma\cdot\left(\rho\gamma x^A+\frac{\theta y^A}{k}\right)^{\tau-1} \\ & -k^{\tau+\sigma-1}\cdot\phi\sigma\cdot\left(1-\phi x^A\right)^{\sigma-1}\cdot\left(\rho\gamma x^A+\frac{\theta y^A}{k}\right)^{\tau}, \end{aligned} \tag{71}$$

which, since $k^{\tau+\sigma-1} > 0$, is proportional to and has the same sign as the expression in (63), which was used to determine the equilibrium time allocation for a female under private property. Hence, $x^A = x^*(k)$. It follows directly that $y^A = y^*(k)$ as well.

6.7 Proof of Proposition 9

The proof is based on the remark that the reproductive success achieved by a male living in a household with k females under allomothering in an ecology $\boldsymbol{\xi} = (\alpha, \beta, \gamma, \sigma, \theta, \lambda, \rho, \tau, \phi)$,

$$\begin{aligned} \tilde{M}^A(k) &= k^{\sigma(1-\beta)}\cdot\widetilde{M}(k) \qquad (72) \\ &= k^{1-\lambda\alpha+\sigma(1-\beta)}\cdot g(n)\cdot\left[1-y^*(k)\right]^\lambda\cdot\left[1-\phi x^*(k)\right]^\sigma\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]^\tau, \end{aligned}$$

is the same as that achieved by a male living in a household with k females under private provisioning in an ecology $\boldsymbol{\xi}' = (\alpha', \beta, \gamma, \sigma, \theta, \lambda, \rho, \tau, \phi)$ where $\alpha' = \alpha - \frac{\sigma(1-\beta)}{\lambda}$, as can be seen by replacing α' by $\alpha - \frac{\sigma(1-\beta)}{\lambda}$ in

$$\begin{aligned} \widetilde{M}(k) &= k\cdot g(n)\cdot\left(\frac{1-y^*(k)}{k^{\alpha'}}\right)^\lambda\cdot\left[1-\phi x^*(k)\right]^\sigma\cdot \\ & \left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]^\tau. \end{aligned} \tag{73}$$

The logic followed to determine the shape of $\widetilde{M}$ can therefore be applied here to determine the shape of $\tilde{M}^A$. Specifically, one obtains that $\frac{d\tilde{M}^A(k)}{dk}$ has the same sign as

$$B(k) \equiv \left[1-\lambda\alpha+\sigma(1-\beta)\right]\cdot\left[\rho\gamma x^*(k)+\frac{\theta y^*(k)}{k}\right]-\tau\cdot\frac{\theta y^*(k)}{k}. \tag{74}$$

Suppose that $\tilde{k} > 1$ (for the definition of $\tilde{k}$ see (51)). Then $B(k)$ changes sign at most once for $k \in [1, \tilde{k})$. Indeed, suppose that there exists some $\bar{k} \in [1, \tilde{k})$ such that $B\left(\bar{k}\right) = 0$, i.e.,

$$\left[1-\lambda\alpha+\sigma(1-\beta)\right]\cdot\rho\gamma x^*\left(\bar{k}\right) = \left[\tau-1+\lambda\alpha-\sigma(1-\beta)\right]\cdot\frac{\theta y^*\left(\bar{k}\right)}{\bar{k}}. \tag{75}$$

(Note that $\bar{k}$ must indeed be strictly smaller that $\tilde{k}$ since $B(k) > 0$.for any $k \geq \tilde{k}$.) Note that since $\rho\gamma x^*(k) + \frac{\theta y^*(k)}{k} > 0$ for all k, and since $[1 - \lambda\alpha + \sigma(1-\beta)] \cdot \rho\gamma > 0$, both the left-hand side and the right-hand side of this expression must be strictly positive. Then, since $x^*(k)$ is increasing in k and $\frac{\theta y^*(\hat{k})}{\hat{k}}$ is decreasing in k, it must be that $[1 - \lambda\alpha + \sigma(1-\beta)] \cdot \rho\gamma x^*(\bar{k}) > [\tau - 1 + \lambda\alpha - \sigma(1-\beta)] \cdot \frac{\theta y^*(\bar{k})}{\bar{k}}$ for any $k > \bar{k}$. Moreover, since $1 - \lambda\alpha + \sigma(1-\beta) > 1 - \lambda\alpha$ and $\tau - 1 + \lambda\alpha - \sigma(1-\beta) < \tau - 1 + \lambda\alpha$ for any $\sigma(1-\beta) > 0$, it follows that if $\sigma(1-\beta) > 0$ then $\bar{k} > \hat{k}$ (for the definition of $\hat{k}$, see (52)).

Arguments similar to those used in the proof of Proposition 5) lead to the following conclusions:

A. If $\alpha\lambda\beta = 1$ and $y^*(1) = 0$, then $\frac{d\tilde{M}^A(k)}{dk} = 0$ for all $k \geq 1$.

B. If $\alpha\lambda\beta = 1$ and $y^*(1) > 0$, then $\frac{d\tilde{M}^A(k)}{dk} < 0$ for all $k \in [1, \tilde{k})$ and $\frac{d\tilde{M}^A(k)}{dk} = 0$ for all $k \geq \tilde{k}$.

C. If $\alpha\lambda\beta < 1$ and $y^*(1) = 0$, then $\frac{d\tilde{M}^A(k)}{dk} > 0$ for all $k \geq 1$.

D. If $\alpha\lambda\beta < 1$ and $y^*(1) > 0$, then $\frac{d\tilde{M}^A(k)}{dk} > 0$ for all $k \geq \tilde{k} > 1$. Moreover, a sufficient condition for $\tilde{M}^A$ to be non-monotonic in k is that $\left.\frac{d\tilde{M}^A(k)}{dk}\right|_{k=1} < 0$, which is true if and only if

$$[1 - \lambda\alpha + \sigma(1-\beta)]\,[\rho\gamma x^*(1) + \theta y^*(1)] < \tau\theta y^*(1). \tag{76}$$

Because $x^*(1)$ and $y^*(1)$ do not depend on α, and since $\rho\gamma x^*(1) + \theta y^*(1) > 0$, the left-hand side can be viewed as an affine and strictly decreasing function of α, which takes the value 0 for $\alpha = [1 + \sigma(1-\beta)]/\lambda$ and the value $[1 + \sigma(1-\beta)]\,[\rho\gamma x^*(1) + \theta y^*(1)] > 0$ for $\alpha = 0$. Hence, (76) is equivalent to

$$\alpha > \frac{[1 + \sigma(1-\beta)]\,\rho\gamma x^*(1) + [1 + \sigma(1-\beta) - \tau]\,\theta y^*(1)}{\lambda\,[\rho\gamma x^*(1) + \theta y^*(1)]} \equiv \bar{\alpha}.$$

Note that

$$\bar{\alpha} = \frac{\rho\gamma x^*(1) + (1-\tau)\,\theta y^*(1)}{\lambda\,[\rho\gamma x^*(1) + \theta y^*(1)]} + \frac{\sigma(1-\beta)}{\lambda} = \hat{\alpha} + \frac{\sigma(1-\beta)}{\lambda},$$

where $\hat{\alpha}$ is defined in (54). Because $B(k)$ changes sign at most once (see above), the condition $\alpha > \bar{\alpha}$ is also necessary for $\tilde{M}^A$ to be non-monotonic in k. Furthermore, if $\alpha > \bar{\alpha}$, there exists $\bar{k} \in \left(\hat{k}, \tilde{k}\right)$ such that $\frac{d\tilde{M}^A(k)}{dk} < 0$ for all $k \in [1, \bar{k})$, $\frac{d\tilde{M}^A(k)}{dk} < 0$ if $k = \bar{k}$, and $\frac{d\tilde{M}^A(k)}{dk} > 0$ for all $k > \bar{k}$. Finally, if $\alpha \leq \bar{\alpha}$, $\frac{d\tilde{M}^A(k)}{dk} \geq 0$ for all $k \geq 1$, with a strict inequality for all $k \geq 1$ if and only if $\alpha < \bar{\alpha}$.

6.8 Proof of Proposition 11

Recall, from equation (3), that

$$\frac{d\widetilde{M}(k)}{dk} = n \cdot a(n) \cdot \left[s(x,y,n,k) + k \cdot \frac{\partial s(x,y,n,k)}{\partial k} \right]_{|(x,y,n,k)=(x^*(k),y^*(k),n^*(k),k)}.$$

Recalling the expression for $s(x,y,n,k)$, one obtains

$$\begin{aligned} \frac{\partial s(x,y,n,k)}{\partial k} &= -\lambda\alpha \cdot k^{-\lambda\alpha-1} \left(\frac{T-y}{n^\alpha} \right)^\lambda \cdot \left(\frac{(T-\phi x)}{n^\beta} \right)^\sigma \cdot \left(\frac{\rho\gamma x}{n} + \frac{\theta y}{kn} \right)^\tau \\ & \quad -\tau \cdot \frac{\theta y}{k^2 n} \cdot k^{-\lambda\alpha} \left(\frac{T-y}{n^\alpha} \right)^\lambda \cdot \left(\frac{(T-\phi x)}{n^\beta} \right)^\sigma \cdot \left(\frac{\rho\gamma x}{n} + \frac{\theta y}{kn} \right)^\tau. \end{aligned}$$

Since all the term in brackets are strictly positive, $\frac{\partial s(x,y,n,k)}{\partial k}$ has the same sign as $-\lambda\alpha \cdot k^{-\lambda\alpha-1} - \tau \cdot \frac{\theta y}{k^2 n} \cdot k^{-\lambda\alpha}$, an expression which is non-positive for all parameter values, and strictly negative if $\alpha > 0$ or $y^*(k) > 0$. By the same token, and since $n^*(k)$ is constant in k, this implies that the value of the upper bound, $[n \cdot a(n) \cdot s(x,y,n,k)]_{|(x,y,n,k)=(x^*(k),y^*(k),n^*(k),k)}$, is decreasing in k if $\alpha > 0$ or $y^*(k) > 0$, and is constant otherwise.

6.9 Proof of Proposition 12

Recall from Proposition 10 that any $r \in \{0,1,...,R-1\}$ such that $\widetilde{M}(2^r) > \delta \cdot \widetilde{M}(2^{r+1})$ is locally evolutionarily stable. Clearly, any $k \in \{2^0, 2^1, 2^2, ..., 2^R\}$ such that $\frac{d\widetilde{M}(k)}{dk} \leq 0$ is sustainable. Turning to values of k for which $\frac{d\widetilde{M}(k)}{dk} > 0$, Proposition 11 implies that there exists some $t \in \mathbb{N} \setminus \{0\}$ such that $\widetilde{M}(2^t) > \delta \cdot \widetilde{M}(2^{t+1})$ (where t may or may not be smaller than $R-1$, depending on the value of δ and the value of R).

6.10 Proof of Proposition 13

Since

$$\widetilde{F}(k) = \frac{\widetilde{M}(k)}{k} = g(n^*) \cdot k^{-\lambda\alpha} \cdot s^*(x^*(k), y^*(k), k), \tag{77}$$

and since $g(n^*) > 0$, one obtains that $\frac{d\widetilde{F}(k)}{dk}$ has the same sign as

$$-\lambda\alpha \cdot k^{-\lambda\alpha-1} \cdot s^*(x^*(k), y^*(k), k) + k^{-\lambda\alpha} \cdot \frac{ds^*(x^*(k), y^*(k), k)}{dk}. \tag{78}$$

The following arguments use some results shows in the proof of Proposition 5. First, the fact that $\frac{ds^*(x^*(k),y^*(k),k)}{dk} \leq 0$ (see equation (47)), together with the fact that $s^*(x^*(k), y^*(k), k) >$

0, implies that the expression in (78) is strictly negative for any $\alpha > 0$. Furthermore, the fact that $\frac{ds^*(x^*(k),y^*(k),k)}{dk} \leq 0$ if and only if $y^*(k) = 0$ implies that if $\alpha = 0$, $\frac{d\widetilde{F}(k)}{dk} = 0$ for all $k \geq 1$ if and only if $y^*(1) = 0$.

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