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To Be, or Not to Be: Stereotypes, Identity Choice and Group Inequality

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1 Introduction

Social information is valuable, and many people seek it in daily life. One of the ways that we generate and store social information is to classify the persons we encounter on the basis of their common possession of visible marks or other observable characteristics, i.e., form broad categories between which contrasts can be drawn and about which generalizations can be made. Through classification (race, nationality, ethnicity, religion, political position, accent, occupation, smoker/non-smoker, etc.), we can better understand what is to be expected from those with whom we must interact but about whom all too little can be discerned. The information-hungry observers, in making pragmatic judgments, have such an incentive to use group-average information to assess a subject's functionally relevant traits when they are not directly observable. The 'collective reputations' are this sort of rational formation by external observers of beliefs about the unobserved traits of varied population aggregates. This phenomenon, sometimes referred to as 'stereotyping', has long been of interest to economists (e.g., Arrow, 1971; Coate and Loury, 1993; Tirole, 1996; Fang, 2001; Chaudhuri and Sethi, 2008; Kim and Loury, 2018), sociologists (e.g., Goffman, 1959; Anderson, 1990; Sampson and Raudenbush, 2004), and social psychologists (e.g., Fiske, 1998; Greenwald and Banaji, 1995; Steele and Aronson, 1995). In this paper, we extend the economics literature about collective reputations and stereotypes by allowing observed agents to exert control over their perceived identities.

When a stranger comes into our presence, first appearances are likely to enable us to anticipate his category and attributes, though the true attributes he could, in fact, possess are different from the anticipated ones (Goffman, 1963). This implies a fundamental distinction between *social identity*, which addresses how an individual is perceived and categorized by others, and *personal identity*, which is the distinct personality of an individual regarded as a persisting entity (Tajfel, 1974). An individual's success in everyday life can be influenced substantially by the social identity attached to him. Then, incurring some cost, individuals may take actions that affect the way in which they are categorized and perceived by observers. The choice of perceived social identity is a rational behavior of economic agents in societal settings.

Developing an identity choice model, we use a stereotyping-cum-signaling framework pioneered by Arrow (1973) and Coate and Loury (1993): when a job candidate's productivity is not perfectly observable, employers in the screening process have an incentive to use the collective reputations of the identity groups to which the job applicants belong. This can generate multiple self-confirming prior beliefs on the part of employers about different social identity groups. Individuals belonging to a group with a better collective reputation have a greater incentive to acquire the attributes valued in the marketplace than do those who belong to a group with a poor reputation. However, given its greater acquisition rate of valued attributes, the group can maintain this better collective reputation. On the other hand, individuals belonging to a group with a poor collective reputation have a smaller incentive to acquire the valued attributes, and with the lower acquisition rate, the employers' negative stereotype against this group is also self-confirmed. Therefore, in this framework, the multiple self-confirming beliefs explain the inequality of collective reputations between exogenous and equally endowed identity groups as being due to the positive feedback between a group's reputation and its members' investment incentives.

We extend this set of arguments by relaxing the immutability assumption: instead of exogenously given identities, people are able to control how they are categorized or perceived by others. If they are different in terms of an economically relevant dimension such as ability and if they anticipate that one type of identity will be better treated than another in the marketplace, the incentive for people to join the favored identity group may vary according to the ability. The identity choice behaviors will systematically induce a positive selection along the ability parameter in the group that is anticipated to be better treated. The result is that human capital investment cost (i.e., innate ability) distributions between groups endogenously diverge, which reinforces incentive-feedbacks. This creates an additional type of self-fulfilling prophecy that can generate inequality between identity groups, which is a clearly different mechanism than the positive complementarities between collective reputation and skill investment incentives. When these two mechanisms, *positive selection* and *positive complementarities*, are jointly operative, we have greater inequality between two identity groups than would have been the case in the absence of the endogeneity of identity choice.

There are many situations in which identity choice and group stereotypes operate in tandem. Among the human behaviors potentially illuminated by our theory are: (1) the selective 'out-migration' from a stigmatized group associated with 'passing' and (2) the production of the indices of differentiation by better-off members of the negatively stereotyped group, which is termed as 'partial passing' in this paper. In these identity manipulation activities, incurring some cost, they may change their names and nationality, learn an unfamiliar language, adopt new customs and habits, modify their accent and appearance, establish the newly developed social networks or even disconnect the ties to his past. The concrete examples relevant for these phenomena are introduced extensively in the next section.

In our theoretical framework, we define two distinct equilibria: PSE and ESE. A standard statistical discrimination framework (e.g., Coate and Loury, 1993) entails no selection into or out of the groups. We call the self-confirming belief equilibrium with exogenous social identities a Phenotypic Stereotyping Equilibrium (PSE), using the term 'phenotype' to indicate exogenously determined immutable appearance. When membership is endogenous, however, the better-regarded group will, in equilibrium, come to consist disproportionately of high ability/low human capital investment cost types. We call such a group-disparate equilibrium with endogenous identities an Endogenous Stereotyping Equilibrium

(ESE).

Comparing PSE and ESE, we find that, while inequality in PSE is due to the positive feedback between the reputation and investment incentive, inequality in ESE is due to the positive selection into the favored group as well as the reputation-incentive feedback. This ensures that the maximum group inequality that derives from the environment in which people have options to migrate between categorized memberships is greater than the group inequality that can emerge from the phenotypic stereotyping. We prove the existence and the stability of such unequal ESE, given the presence of multiple PSE. In addition, we show that when identity manipulation is sufficiently easier to undertake, an equality across social groups may not be achievable even with strong egalitarian interventions: an equilibrium with identical reputations is not stable as any small perturbation would motivate individuals to switch groups, thereby leading to a divergence.

Applying this theory to the passing and 'partial passing' phenomena, we find that the reputations of non-passers (or non-partial passers) who are left behind are adversely affected by the selective out-migrations (or the usage of the indices of differentiations), while the most flexible passers gain from the categorization change. Therefore, the identity manipulation activities may undermine solidarity or even engender some conflicts within a stereotyped population. The 'acting white' phenomenon in the US racial context is one of the examples: the worse-off members of the population accuse the partial passers of some kind of immoral betrayal and try to hold such people back by stigmatizing their action to adopt the indices of differentiation. This reasoning provides an alternative explanation of the phenomenon to those offered by other scholars (e.g., Austen-Smith and Fryer, 2005; Eguia, 2017).

Through the decomposition of the societal efficiency gain into reputational externalities and passing (partial passing) premium, however, we show that these identity manipulation activities can increase the total welfare of the society under some limited conditions. Furthermore, we demonstrate that when a stereotyped group is severely discriminated against, the activities can improve the societal efficiency even without hurting the welfare of the left-behind.

The similar inequality-amplifying effects of heterogeneous incentives for mobility are also found in other areas of the inequality literature, such as that on school vouchers, which reduce the switching costs for bright kids in moving from poor public schools to affluent private ones (Epple and Romano, 1998); socioeconomic stratification in a city, which arises due to middle-class flight to the suburbs (Benabou, 1993); and brain drain, which is caused by immigrant self-selection (Borjas, 1987). In these cases, the better-off types (i.e., those of high income or high ability) choose their school, neighborhood, or country without taking into account the external effects of their choice on others. Our approach contributes to these threads of research, as we add a case that the better-off types choose the perceived *social identity*, enlarging the divergence in the reputation across groups.

Various concepts of "identity" have been developed in the growing literature on the economics of identity. Akerlof and Kranton (2000) propose a theoretical framework in which social identity is associated with different prescriptions.[1] Since violating these prescriptions may evoke anxiety and discomfort in oneself and in others, identity is likely to affect one's decision-making and economic outcomes. Benjamin, Choi and Strickland (2010) also argue that social identity affects fundamental economic preferences, revealing the effect of social category norms on time and risk preferences. In these and other relevant developments, such authors consider the psychological effects stemming from the prescriptions or norms tied to social categories. Our approach to the concept of "identity" differs significantly from theirs, in the sense that we focus on the economic incentives for adopting one identity or another. That is, the "identity" in our

[1]The prescriptions in social psychology indicate the behavior appropriate for people in different social categories in different situations.

approach represents a social category's collective reputation that affects labor market opportunity: employers in the screening process have an incentive to use a job applicant's "identity" and job applicants have an incentive to engage in identity manipulation.

On the other hand, some previous works deal with the choice of *personal identity*, which is all about self-perception or self-representation.[2] Fang and Loury (2005) argue that people who interact frequently may end up embracing similar categories of self-representation, implying that a "bad" (dysfunctional or self-destructive) collective identity can be sustained in equilibrium for one group of people. More recently, Benabou and Tirole (2011) develop a theory of moral behavior, based on a general model of identity management, in which moral identity is modeled as beliefs about one's deep "value." Unlike these approaches on the choice of *personal identity*, we deal with the choice of perceived *social identity*. In this sense, our approach is most relevant to Fang's (2001) examination of the economic meaning of social culture, in which he finds that a skilled worker can be more willing than an unskilled worker to undertake a specific cultural activity,[3] and Shayo's (2009) development of a model of social identity with its application to the political economy of income redistribution, in which he proposes an equilibrium concept where an agent's voting action and his or her social identity (e.g., classes and nations) are endogenously determined.

The remainder of this paper is structured as follows. Section 2 introduces various real life examples that are relevant for passing and 'partial passing' phenomena. Section 3 describes the basic structure of the signaling model, in which

[2]Arguing that the concept of individual's personal identity has been omitted in the Akerlof and Kranton's framework, Aguiar et al. (2010) analyze the role of personal identity in altruism modifying their utility functions. Ben-Ner et al. (2009) also investigate the existence and relative strength of favoritism for in-group versus out-group along multiple identity categories.

[3]While Fang(2001) assumes a setting with unique PSE and displays the emergence of an elite group with differentiated cultural traits out of the population, we assume a setting with multiple PSE and displays the greater divergence in the reputation across unequal groups due to the endogenous group switching. Unlike his work, we also provide complete characterization of the equilibria outcomes and their welfare properties.

agents decide on the perceived identity as well as the skill acquisition. Section 4 defines both PSE and ESE. Section 5 studies the properties of the identity choice behaviors, and Section 6 examines the existence and the stability of ESE. Section 7 follows with a discussion of the welfare properties of the equilibria. Section 8 presents the study's conclusion.

2 Examples of Identity Choice Behaviors

Young members in a stereotyped group may consider "passing" into the better-regarded group when the return for "passing" (e.g., better treatment in the labor market) outweighs its cost (e.g., loss of ties to one's own kind; learning of an unfamiliar language; adoption of new customs and habits; change of appearance.) These stereotyped social groups are identified in various ways around the world: along racial lines in societies such as the United States, South Africa, Australia and many Latin American countries, along religious lines in Pakistan and Israel, along ethnic lines in Singapore and the Balkan states, with caste-like social division in the Indian sub-continent and the treatment of Gypsies and immigrants in Europe. The selective out-migration occurs as more talented members in the disadvantaged groups cross the color/religious/ethnic/caste lines disproportionately.

A manifest example is the ethnic Koreans in Japan (referred to as "Zainichi"), many of whom are descended from forced laborers in mines and factories who were brought to Japan from the Korean peninsula during the period of Japanese imperialism.[4] To escape the negative stereotypes and prejudices against the Zainichi, many of the naturalized Koreans conceal their ethnicity, giving up their names and pretending that they have no knowledge about Korean culture and language (Fukuoka et al., 1998).

[4]Every year, approximately 10,000 Koreans, of approximately 600,000 Korean descendants holding Korean nationality, choose to be naturalized as 'official' Japanese mostly when seeking formal employment or marriage.

Other than the Zainichi, who share a similar appearance with the Japanese, passing is harder for blacks and other minorities in the United States due to their physical makeup. However, the light-skinned minorities with mixed ancestry have been crossing the boundaries of color and racial identity.[5] In old Hollywood, for instance, talented movie stars were expected to downplay their ethnic origins when they were not solely of European extraction.[6]

Unlike the United States, which had defined concepts of race due to the 'one drop rule,' racial classifications in Latin American and Caribbean countries are based primarily on skin tone and on other physical characteristics such as facial features, hair texture, etc. In these countries, some of which might be classified as white supremacist societies, a dark skinned person is more likely to be discriminated against, and a light skinned person is considered more privileged (Telles, 2004). In their everyday life, the black-looking mixed race people tend to refuse to identify as Black, but the white-looking mixed race people gladly identify as White. The journalists report that the fascination with becoming "white" has increased over the last decades with the prevailing "whitening" practices (e.g., the use of skin bleaching cosmetics and treatments to straighten hair) among the mixed-race youngsters.

In other situations, discriminated groups may modify their accents, word choices, manner of dress and even custom in an attempt to appear to be members of a privileged group. A good example is My Fair Lady, a musical based upon George Bernard Shaw's *Pygmalion*, which concerns a Cockney flower girl (Eliza Doolittle) who takes speech lessons from a phonetician so that she may pass as

[5]According to the NLS79 National Longitudinal Survey conducted by the Department of Labor in the US, 1.87 percent of those who had originally answered "Black" in 1979 (when they were 14 to 22 years old) switched to answering the interviewer's race question with either "white," "I don't know," or "other" by 1998 (Sweet, 2004).

[6]Some of them successfully estranged themselves from their roots and achieved fame and fortune in the movies, including Carol Channing, who did not reveal her African American ancestry (a quarter Black) until when she released her memoir, and Merle Oberon, who concealed that she was born to an Indian mother. Besides, it was not uncommon for stars of even European extraction to downplay their roots by adopting American sounding names.

a lady in the high society of Edwardian London. This type of passing in the context of caste is called *Sanskritization*, which is a process by which a low or middle Hindu caste seeks upward mobility by emulating the rituals and practices of the upper or dominant castes (Srinivas, 1952).[7]

Passing into the better-regarded group is not always possible for every stigmatized group. It would be very hard when the pertinent physical traits passed on across generations are easily discerned and are not readily disguised. To inhibit being stereotyped, the most talented of the visibly distinct stigmatized population, who gain most by separating themselves from the mass, may develop the indices of differentiation that can send signals that they are different from the average of the stigmatized mass. Taking the example of the blacks in the United States, whereby people with any known African ancestry were automatically classified as Black, the strategies of social identity manipulation that can be adopted by better-off members are: affectations of speech, dressing formally rather than wearing casual clothes, spending more on conspicuous consumption and migration to affluent residential areas (Goffman, 1959). In short, these self-presentation methods for 'partial passing' aim to communicate "I'm not one of THEM; I'm one of YOU!" (Loury, 2002).

There is systematic empirical evidence regarding the styles of self-presentation for social identity manipulation. For instance, Charles et al. (2009) report that blacks and Hispanics spend 30 percent more than similar whites on visible goods such as clothing, cars and jewelry. They conclude that blacks and Hispanics earning a higher income, who live in an area where the community income is relatively lower, have greater incentives to differentiate themselves and signal their high status by acquiring visible goods. Grogger (2011) finds that, among blacks, speech patterns are highly correlated with the wages of young workers: black speakers whose voices were distinctly identified as black earn approximately 12

[7] A caste may rise to a higher position in the hierarchy, in a generation or two, by adopting the Sanskritic theological ideas and the Brahminic way of life such as vegetarianism and teetotalism.

percent less than whites with similar observable skills, while indistinctly identified blacks earn essentially the same as comparable whites. Such speech-related wage premia may provide incentives for talented blacks to adopt standard American English rather than African American English. Then, in the labor market, speech patterns can signal the worker's underlying abilities.[8]

The theoretical model developed below explains the rationale behind these identity manipulation activities and explores implications of such fact that the distribution of abilities within distinct identity groups becomes endogenous.

3 Framework of the Model

Imagine a large number of identical employers and a large population of workers, in which each employer is randomly matched to many workers. The workers not only make an investment decision on skill acquisition but also choose how to be perceived by others before they are matched with an employer.

A worker's skill acquisition decision is denoted by $e \in \{0, 1\}$. The cost of obtaining a skill varies among the workers: $c \in [0, \infty]$. Workers with less cost are more capable individuals, and they can acquire skills more easily. Let $G(c)$ be the fraction of workers with a skill acquisition cost no greater than c. The cumulative distribution function (CDF) of the cost, $G(c)$, satisfies $G(0) > 0$, implying the existence of a fraction of highly capable workers whose skill acquisition cost is sufficiently low. We impose that the related density function of the cost, $g(c)$, is a single-peaked function of c, increasing (decreasing) for any c less (greater) than $\hat{c}$ (e.g., normal distribution).[9] An agent with cost c invests in skills if and only

[8]Charles et al. (2009) made a careful examination of the Consumer Expenditure Survey (CES) by the U.S. Department of Labor. Grogger (2011) used audio data from interviews administered to the National Longitudinal Survey of Youth (NLSY) respondents.

[9]Most ability-related test scores reveal single-peaked distributions of intelligence. For instance, SAT scores are approximately normally distributed over the tested population. The widely-used intelligence quotient (IQ) scores are also distributed normally about 100, with a standard deviation of 15. If a person's intelligence is affected by a large number of independent causes, each of which has a small effect, intelligence can be argued to be distributed normally across the population (Hunt, 2011).

if the anticipated return from doing so exceeds this cost for the skill acquisition.

The workers are also allowed to choose, prior to being matched with an employer, how they are categorized and perceived in the labor market. There are two types of affects that they can assume, either A or B: $i \in \{A, B\}$. They can choose how to present themselves either way incurring some cost $k \in R$. The variable k, so called identity "switching" cost, can be positive or negative. If it is positive, he is naturally inclined to be perceived as B and should incur cost k to be perceived as A. If it is negative, he is naturally inclined to be perceived as A and should incur cost $-k$ to be perceived as B. In other words, the relative cost of being perceived as A rather than B is k; equivalently, the relative cost of being perceived B rather than A is $-k$. Therefore, the variable k and its sign successfully reflects a cost to adopting a different identity than one's own natural one.

The cumulative distribution function (CDF) of the cost is denoted by $H(k)$. For the sake of simplicity, we assume the symmetry of the distribution: $H(k) = 1 - H(-k)$. (i.e., its related density function, $h(k)$, satisfies $h(k) = h(-k)$.) The whole population does not incline to one way or the other, implying that half of the population is naturally inclined toward A and the other half is naturally inclined toward B. An agent with cost k chooses to be perceived as A if the incentive for electing the A-type rather than the B-type exceeds the relative cost for being perceived as A; Otherwise, he chooses to be perceived as B.

We state that there is no connection between the two exogenous cost variables, c and k. The economic ability of an individual and the natural affect orientation of an individual are distributed independently in the population, implying that a person's identity orientation cannot be used to predict his or her economic ability.

For the wage setting mechanism, we adopt a statistical discrimination framework originally proposed in Coate and Loury(1993), which links the reputation of a group and the skill acquisition incentives for the group members. Employers cannot observe the skill level e of a person, but they can observe the group to which the person belongs and a noisy signal $t \in [0,1]$ that is generated out of the hiring process. The signal might be the result of the test, an interview by employers, internship, or on-the-job training. The distribution of the signal depends on whether the person acquires the skill. Let $F_1(t)$ $[F_0(t)]$ be the probability that the signal does not exceed t, given that a worker is skilled [unskilled], and let $f_1(t)$ $[f_0(t)]$ be the related density function. Define $\psi(t) \equiv f_1(t)/f_0(t)$, to be the likelihood ratio at t. We assume that $\psi(t)$ is a monotonically increasing function in t, which is defined as the Monotonic Likelihood Ratio Property (MLRP). This property implies $F_1(t) < F_0(t)$ for any $t \in (0,1)$.[10] Thus, higher values of the signal are more likely if the worker is skilled, and for a given prior, the posterior likelihood that a worker is skilled is larger if his signal takes a higher value.

Employers start with a prior belief about the actual rate of skill acquisition of a group π. Let us define the function $f(\pi,t) \equiv \pi f_1(t) + (1-\pi) f_0(t)$, which indicates the distribution of the signal t of agents belonging to a group with the skill level π. The employers' posterior belief of the likelihood that an agent who presents the test score t is in fact skilled is achieved using the Bayes' rule: $\rho(\pi,t) (\equiv Pr[e=1|\pi,t]) = \frac{\pi f_1(t)}{f(\pi,t)}$. We assume a simple economy in which the value of a skilled worker to employers is w and the value of an unskilled worker to employers is zero. The competitive wage denoted by W will be the workers' expected productivity: $W \equiv w \cdot \rho(\pi,t)$, as assumed in Moro and Norman (2004). Then, the anticipated wage for an individual who belongs to a group with the believed skill acquisition rate of π and whose test score is realized as t is

$$W(\pi,t) = w \cdot \frac{\pi f_1(t)}{\pi f_1(t) + (1-\pi) f_0(t)}. \tag{1}$$

Given this framework, we can readily express the expected payoff from acquiring

[10]Denote $\bar{t}$ which satisfies $\frac{f_1(\bar{t})}{f_0(\bar{t})} = 1$. For any $t \in (0,\bar{t})$, the following holds $F_1(t) - F_0(t) = \int_0^t f_1(x)(1 - \frac{f_0(x)}{f_1(x)})\,\mathrm{d}x < 0$. For any $t \in [\bar{t},1)$, the following holds $F_1(t) - F_0(t) = -\int_{\bar{t}}^1 f_1(x)(1 - \frac{f_0(x)}{f_1(x)})\,\mathrm{d}x < 0$.

a skill ($e = 1$) and that without acquiring a skill ($e = 0$) as follows:

$$V_e(\pi) = \int_0^1 f_e(t)W(\pi, t)\,\mathrm{d}t,\ \forall e \in \{0, 1\}, \tag{2}$$

in which both derivatives $V_0'(\pi)$ and $V_1'(\pi)$ are always positive, indicating that they are increasing functions of the believed skill acquisition rate π, as depicted in Panel A of Figure 1.[11] We can also derive that $\lim_{\pi\to 0} V_0'(\pi) = w$ and $\lim_{\pi\to 1} V_1'(\pi) = w$.

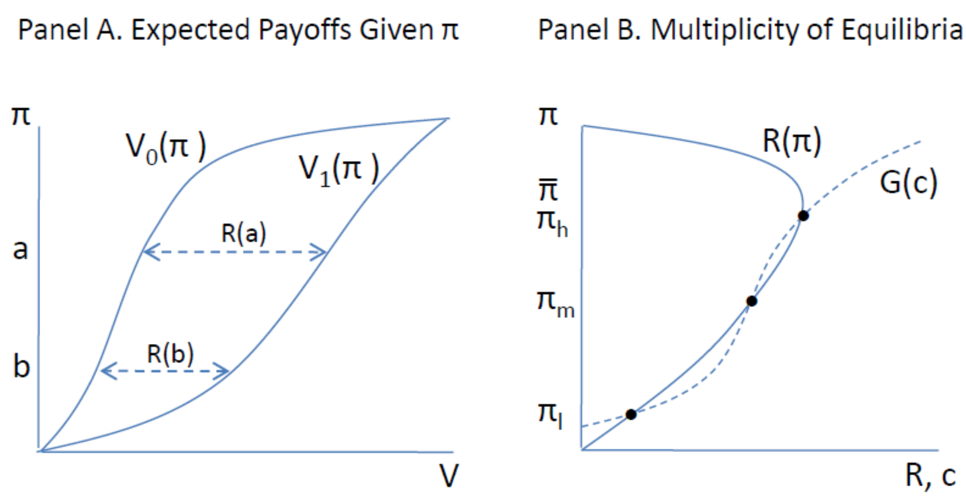

Figure 1. Phenotypic Stereotyping Equilibria

Workers' expected economic return from being skilled, which is denoted by $R(\pi)$, is equivalent to the difference between the expected payoff from acquiring a skill and that without acquiring a skill: $R(\pi) \equiv V_1(\pi) - V_0(\pi)$. Given $\pi = 0$, both the expected payoff from acquiring a skill and that without acquiring a skill are zero, implying that the expected economic return from being skilled is zero: $V_1(0) = V_0(0) = 0$ and $R(0) = 0$. Using a similar logic, given $\pi = 1$, we have $V_1(1) = V_0(1) = w$ and $R(1) = 0$.

Using the above equations, the expected economic return from skill invest-

[11]Note that the first derivatives are derived as $V_0'(\pi) = \int_0^1 wf_1(t)f_0(t)^2 f(\pi, t)^{-2}\,\mathrm{d}t$ and $V_1'(\pi) = \int_0^1 wf_1(t)^2 f_0(t)f(\pi, t)^{-2}\,\mathrm{d}t$.

ment for an individual who belongs to a group with the believed skill investment rate of π is expressed as

$$R(\pi) = w\pi \int_0^1 \frac{(f_1(t) - f_0(t))f_1(t)}{f(\pi, t)}\,\mathrm{d}t. \tag{3}$$

The first and second derivatives of the return function can be directly seen as:

$$R'(\pi) = w \int_0^1 \frac{(f_1(t) - f_0(t))f_1(t)f_0(t)}{f(\pi, t)^2}\,\mathrm{d}t, \tag{4}$$

$$R''(\pi) = -2w \int_0^1 \frac{(f_1(t) - f_0(t))^2 f_1(t)f_0(t)}{f(\pi, t)^3}\,\mathrm{d}t \ \ (< 0). \tag{5}$$

Using MLRP property, we can derive that $\lim_{\pi\to 0} R'(\pi) > 0$ and $\lim_{\pi\to 1} R'(\pi) < 0$.[12] Because the second derivative of the return function is negative for any π, $R(\pi)$ is concave. The return is maximized at $\bar{\pi}$, which satisfies $R'(\bar{\pi}) = 0$. Panel B of Figure 1 illustrates how this return function $R(\pi)$, an agent's skill acquisition incentive, depends upon his group's collective reputation π.

Finally, a worker with skill acquisition cost c, who belongs to a group believed to be investing at rate π, has the anticipated net reward of $V_1(\pi) - c$ if he decides to be a skilled person and that of $V_0(\pi)$ if he decides not to be skilled. Thus, the anticipated net reward in the labor market for such a worker, $U(\pi, c)$, is summarized as

$$U(\pi, c) = \max\{V_1(\pi) - c,\ V_0(\pi)\}, \tag{6}$$

in which the function $U(\pi, c)$ is increasing in π for both $V_1(\pi)$ and $V_0(\pi)$ are increasing in π. The function is non-increasing in c given any fixed level of π.

[12] $\lim_{\pi\to 0} R'(\pi) = w\int_0^1 [f_1(t) - f_0(t)] \cdot \frac{f_1(t)}{f_0(t)}\,\mathrm{d}t = w\int_0^{\bar{t}} [f_1(t) - f_0(t)] \cdot \frac{f_1(t)}{f_0(t)}\,\mathrm{d}t + w\int_{\bar{t}}^1 [f_1(t) - f_0(t)] \cdot \frac{f_1(t)}{f_0(t)}\,\mathrm{d}t > w\int_0^{\bar{t}} [f_1(t) - f_0(t)]\,\mathrm{d}t + w\int_{\bar{t}}^1 [f_1(t) - f_0(t)]\,\mathrm{d}t = 0$, in which $\bar{t}$ satisfies $\frac{f_1(\bar{t})}{f_0(\bar{t})} = 0$. In the same way, we can indicate that $\lim_{\pi\to 1} R'(\pi) < 0$.

4 Phenotypic vs Endogenous Stereotyping Equilibria

In this section, we define both the Phenotypic and Endogenous Stereotyping Equilibria. The contrasting difference between PSE and ESE is that in the former, group identity is exogenous, and, in the latter, it is endogenous.

4.1 Phenotypic Stereotyping Equilibria

Imagine that society consists of exogenous, visibly distinct and equally endowed groups, the membership of which is immutable. Then, employers can discriminate among individuals based upon this observable 'phenotype'.

If employers anticipate that the probability that a randomly drawn individual from a population group i has invested in a skill is π_i, the return from investing in skill for the individual belonging to this group is $R(\pi_i)$. Then, the fraction of the group who will invest is $G(R(\pi_i))$, given the skill acquisition cost distribution $G(c)$. Thus, when a prior belief π_i satisfies $G(R(\pi_i)) = \pi_i$, such a belief about any group is self-confirming. Let us denote an equilibrium belief by $\hat{\pi} \in [0, 1] : \hat{\pi} = G(R(\hat{\pi}))$. The set of all such equilibrium beliefs is denoted by Ψ_{CL} (Coate an Loury, 1993). We call such outcomes "Phenotypic Stereotyping Equilibria (PSE)." An example of such equilibria is described in Panel B of Figure 1, in which $R(\pi)$ is concave and $G(c)$ is S-shaped.

Multiple equilibria create the possibility of phenotypic stereotyping wherein exogenously and visibly distinct groups fare unequally in the equilibrium. Unequal reputations of the groups can be sustained in equilibrium despite the groups being equally well endowed (i.e., having the same $G(c)$). In this case, inequality of collective reputation between the exogenous groups in equilibrium is due to the feedback between the group reputation and individual skill investment activities. The individuals in a group with a better collective reputation have a greater incentive to invest in skills, and with their greater skill investment rate, the group maintains a better collective reputation (and vice versa).

4.2 Endogenous Stereotyping Equilibria

Now consider a society in which workers can choose a perceived group membership, A or B, though at some cost k (either positive or negative) of affecting identity "A" rather than "B" before entering the labor market. Let a and b be employers' beliefs about human capital investment rates in affective groups A and B. $U(a,c)$ ($U(b,c)$) is the anticipated net reward in the labor market for an agent who is perceived as a member of group A (group B) and whose skill acquisition cost is given as c. Let us define a function $\Delta U(a,b;c)$ as the net reward difference between an A-type worker and a B-type worker given their skill acquisition cost level c: $\Delta U(a,b;c) \equiv U(a,c) - U(b,c)$. This indicates the incentive for electing type-A rather than type-B for an agent whose skill acquisition cost is c. Symmetrically, $\Delta U(b,a;c) \equiv U(b,c) - U(a,c)$, indicating the incentive for electing type-B rather than type-A. When $a > (<)\ b$, $\Delta U(a,b;c)$ is positive (negative) because $U(\pi,c)$ is increasing in π. Note also that $\Delta U(a,b;c) = -\Delta U(b,a;c)$ and $\Delta U(a,b;c) = 0$ when $a = b$.

An agent with the endowed cost set (c,k) elects to be an A-type worker if and only if $\Delta U(a,b;c) \geq k$. Otherwise, he elects to be a B-type worker. Because c and k are independently distributed, the fraction of workers who elect to be A-type is $H(\Delta U(a,b;c))$ among the population segment with skill acquisition cost level c. Thus, among the whole population, the fraction of agents who elect to be A-type is given by using the two cumulative distribution functions $H(k)$ and $G(c)$,

$$\Sigma^A \equiv \int_0^\infty H(\Delta U(a,b;c))\,\mathrm{d}G(c). \tag{7}$$

Among the agents who will elect to be A-type, the higher capability population whose skill acquisition cost is not greater than the incentives for skill investment (i.e., $c \leq R(a)$) will elect to be skilled. Then, the fraction of workers who elect to be A-type and become skilled is given by

$$\sigma^A \equiv \int_0^{R(a)} H(\Delta U(a, b; c))\, \mathrm{d}G(c). \tag{8}$$

Among the population whose skill acquisition cost level is c, the fraction of agents who elect to be B-type is $1-H(\Delta U(a, b; c))$, which is equivalent to $H(\Delta U(b, a; c))$ by the symmetry assumption of $H(k) = 1 - H(-k)$. Thus, among the total population, the fraction of agents who elect to be B-type is given by

$$\Sigma^B \equiv \int_0^{\infty} H(\Delta U(b, a; c))\, \mathrm{d}G(c). \tag{9}$$

Consequently, the fraction of workers who elect to be B-type and become skilled is given by

$$\sigma^B \equiv \int_0^{R(b)} H(\Delta U(b, a; c))\, \mathrm{d}G(c). \tag{10}$$

Therefore, given the employers' belief about human capital investment rates (a, b), the actual investment rates for the endogenously constructed groups A and B are denoted by $\phi(a; b)$ $(= \sigma^A/\Sigma^A)$ and $\phi(b; a)$ $(= \sigma^B/\Sigma^B)$ for each, where the function $\phi(x; y)$ is defined as follows:

$$\phi(x; y) \equiv \frac{\int_0^{R(x)} H(\Delta U(x, y; c))\, \mathrm{d}G(c)}{\int_0^{\infty} H(\Delta U(x, y; c))\, \mathrm{d}G(c)}, \tag{11}$$

in which $\phi(x; x) = G(R(x))$.

An equilibrium in this society with endogenous group membership is defined as a pair of investment rates for the endogenously constructed groups $(a^*, b^*) \in [0, 1]^2$ such that $a^* = \phi(a^*; b^*)$ and $b^* = \phi(b^*; a^*)$. We call such outcomes "Endogenous Stereotyping Equilibria (ESE)," and the set of all such equilibria is denoted by Ω_{KL}.

4.3 Correspondence and the Set of Equilibria

In order to analyze the equilibria effectively, we introduce a correspondence $\Gamma(y)$: $\Gamma(y) = \{x \mid x = \phi(x;y)\}$. By definition, the correspondence indicates interceptions between the $\phi(x;y)$ curve and 45 degree line, at which a group's actual skill investment rate $\phi(x;y)$ becomes equal to the employers' prior belief about the group's skill level x, given the employers' prior belief about the other group's skill level y. (For example, given b_1, the $\phi(a;b_1)$ curve intercepts 45 degree line three times in Figure 3. The three crossing points marked with tiny triangles represent the correspondence $\Gamma(b_1)$.)

First, note that any $\hat{\pi} \in \Psi_{CL}$ satisfies $\hat{\pi} \in \Gamma(\hat{\pi})$ and any $\hat{\pi} \in \Gamma(\hat{\pi})$ satisfies $\hat{\pi} \in \Psi_{CL}$. Thus, the set of phenotypic stereotyping equilibria (PSE) is represented as follows using the correspondence: $\Psi_{CL} = \{x \mid x \in \Gamma(x)\}$. On the other hand, the set of endogenous stereotyping equilibria (ESE) is expressed as $\Omega_{KL} = \{(x,y) \mid x \in \Gamma(y) \text{ and } y \in \Gamma(x)\}$, because an ESE is defined as a pair (x,y) that satisfies both $x = \phi(x;y)$ and $y = \phi(y;x)$. This also implies that every PSE corresponds to trivial ESE where differences in affect are uninformative: $(\hat{x},\hat{x}) \in \Omega_{KL}$ if $\hat{x} \in \Psi_{CL}$.

Before we start to search for PSE/ESE in the given framework, readers may review Appendix A first to grasp the key mechanism and the main intuitions of the model, in which those equilibria are determined in a setup with the simplest possible cost structures: agents are composed of only three types of human capital cost (c_l, c_m, c_h) and only four types of identity manipulation cost $(K_l, K_h, -K_l, -K_h)$. The set of ESE in such setup is depicted in Panel D of Appendix Figure 1: given two PSE, Π_l and Π_h, there exist two trivial ESE, (Π_l, Π_l) and (Π_h, Π_h), and two non-trivial ESE, (Π'_l, Π'_h) and (Π'_h, Π'_l), which satisfy $\Pi'_l < \Pi_l < \Pi_h < \Pi'_h$, implying that the inequality between endogenously constructed groups can be greater than the inequality that can emerge between exogenous groups.

5 Properties of Identity Choice Behaviors

In this section, we examine the key properties of the identity choice behaviors in the given original framework. Acknowledge that the expected net reward difference between an A-type worker and a B-type worker in the labor market, $\Delta U(a,b;c)$, can be expressed by, using the equation (6),

$$\Delta U(a,b;c) = \max\{R(a),c\} - \max\{R(b),c\} + V_0(a) - V_0(b). \tag{12}$$

This expression helps us to achieve the following lemma concerning the varying values of $\Delta U(a,b;c)$:

Lemma 1. *For any* $c \le \min\{R(a),R(b)\}$*,* $\Delta U(a,b;c) = V_1(a) - V_1(b)$*. For any* $c \ge \max\{R(a),R(b)\}$*,* $\Delta U(a,b;c) = V_0(a) - V_0(b)$*. For any* c *such that* $\min\{R(a),R(b)\} < c < \max\{R(a),R(b)\}$*, we have*

$$\Delta U(a,b;c) = \begin{cases} V_1(a) - V_0(b) - c & \text{if } R(a) \ge R(b) \\ V_0(a) - V_1(b) + c & \text{if } R(a) < R(b) \end{cases}. \tag{13}$$

The above lemma is summarized in Figure 2, in which the full-fledged four panels describe $\Delta U(a,b;c)$ curves with respect to skill acquisition cost level c for the following four distinct cases: $a > b$ and $R(a) > R(b)$ (Panel A), $a > b$ but $R(a) < R(b)$ (Panel B), $a < b$ but $R(a) > R(b)$ (Panel C) and $a < b$ and $R(a) < R(b)$ (Panel D).

From the above lemma, we achieve two valuable propositions concerning the identity choice behaviors of economic agents. First, it is directly seen that $\Delta U(a,b;c) > (<)\, 0$ for any given cost level c if and only if $a > (<)\, b$, as displayed in Panels A and B (Panels C and D). This implies that all the agents whose naturally oriented identity is favored in the labor market do not "switch", only some of those whose naturally oriented identity is less favored choose to "switch".

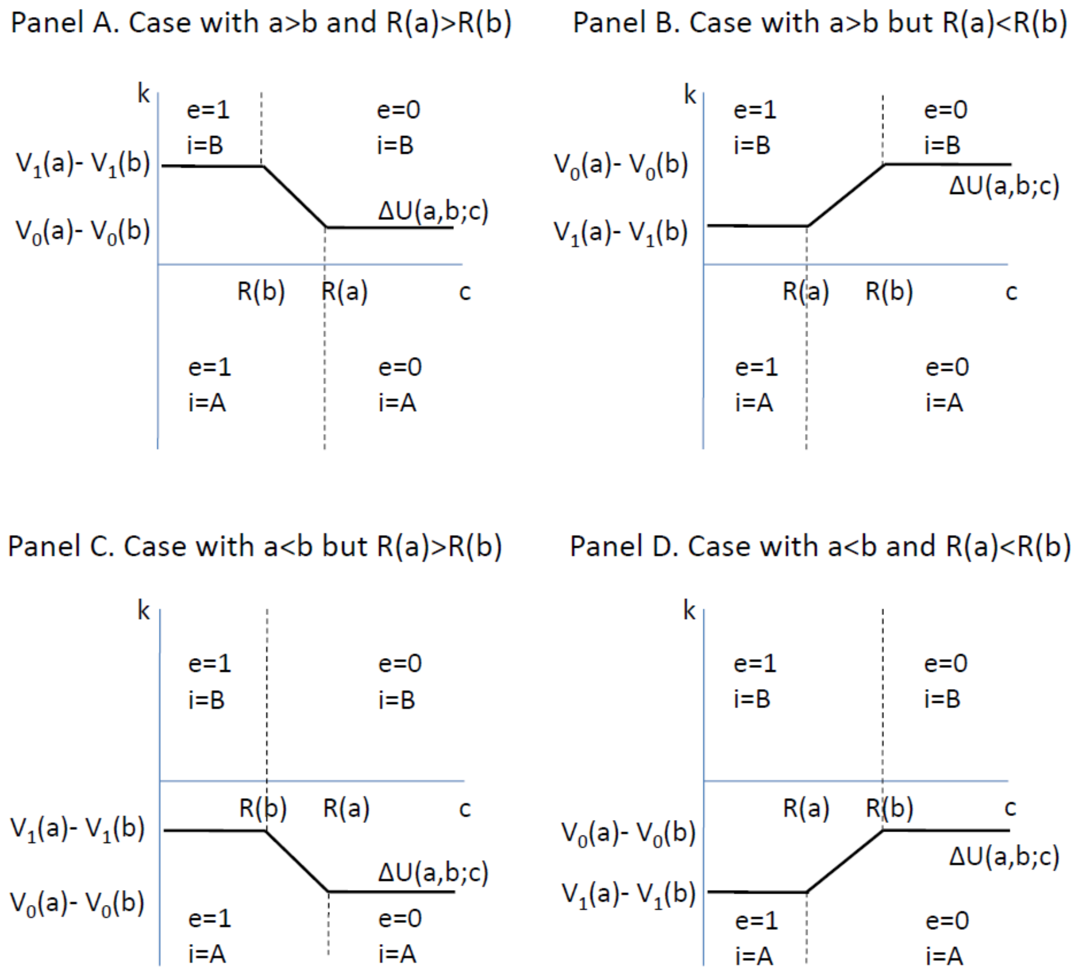

Figure 2. Human Capital Investment and Identity Choice Behavior

That is, in the current setting with a symmetric cost distribution of $h(k)$, the fraction of workers who adopt the 'affect' corresponding to the favored employers' belief is greater than that of workers who adopt the 'affect' with the less favored employers' belief as summarized in the following proposition.[13]

Proposition 1. *When employers have different beliefs about two affective groups($a \neq b$), all the agents whose naturally oriented identity is favored in the labor market do not "switch" to the less favored group, only some of those whose natu-*

[13]In other words, more than half of workers adopt the 'affect' that corresponds to the more favorable employers' belief: $\Sigma^A > (<) \, 0.5$ and $\Sigma^B < (>) \, 0.5$ if $a > (<) \, b$.

rally oriented identity is less favored choose to "switch" to the favored group: $\Sigma^A > (<) \Sigma^B$ *if* $a > (<) b$.

Lemma 1 also indicates that whenever $R(a) > R(b)$, $\Delta U(a, b; c)$ is non-increasing with respect to c regardless of $a > b$, as depicted in Panels A ($a > b$) and C ($a < b$). This implies that whenever $R(a) > R(b)$, the disproportionately more talented workers choose affect A that corresponds to the greater return to human capital investment, regardless of whether the affect is more favored or not in the labor market.[14] Thus, the actual skill investment rate for the endogenously constructed group A (B) is greater (smaller) than that for the exogenously given group with the same collective reputation level: $\phi(a; b) > G(R(a))$ and $\phi(b; a) < G(R(b))$.

In a symmetric way, whenever $R(a) < R(b)$, $\Delta U(a, b; c)$ is non-decreasing with respect to c regardless of $a > b$, as depicted in Panels B ($a > b$) and D ($a < b$). This implies that whenever $R(a) < R(b)$, the disproportionately more talented workers choose affect B that corresponds to the greater return to human capital investment: $\phi(a; b) < G(R(a))$ and $\phi(b; a) > G(R(b))$. These properties are summarized by the following proposition.

Proposition 2. *The disproportionately more talented workers, whose human capital investment costs (c) are relatively lower, choose the 'affect' that corresponds to the greater return to human capital investment: given* $R(i) > R(j)$, $\phi(i; j) > G(R(i)) > G(R(j)) > \phi(j; i)$ *for each combination* $(i, j) \in \{(a, b), (b, a)\}$.[15]

The overall shape of $\phi(a; b)$ with respect to a given a fixed level of b is

[14] Note that when $R(a) > R(b)$ but $a < b$, some of those whose naturally oriented group is the less privileged group A "switch" to the favored group B, but the disproportionately more talented workers do not "switch" and choose to stay with their less privileged group identity (A), as depicted in Panel C in Figure 2.

[15] This proposition implies that, given $i > j$ but $R(i) < R(j)$, it is even possible that the disproportionately less talented workers choose the 'affect' that corresponds to the favored employer belief i, resulting in $\phi(i; j) < \phi(j; i)$, for each combination $(i, j) \in \{(a, b), (b, a)\}$, as depicted in Panels B and C of Figure 2. This is because it is embedded in the given statistical discrimination framework that those who are talented gain less than those who are less talented with adopting the favored 'affect' i in such case: $V_1(i) - V_1(j) < V_0(i) - V_0(j)$.

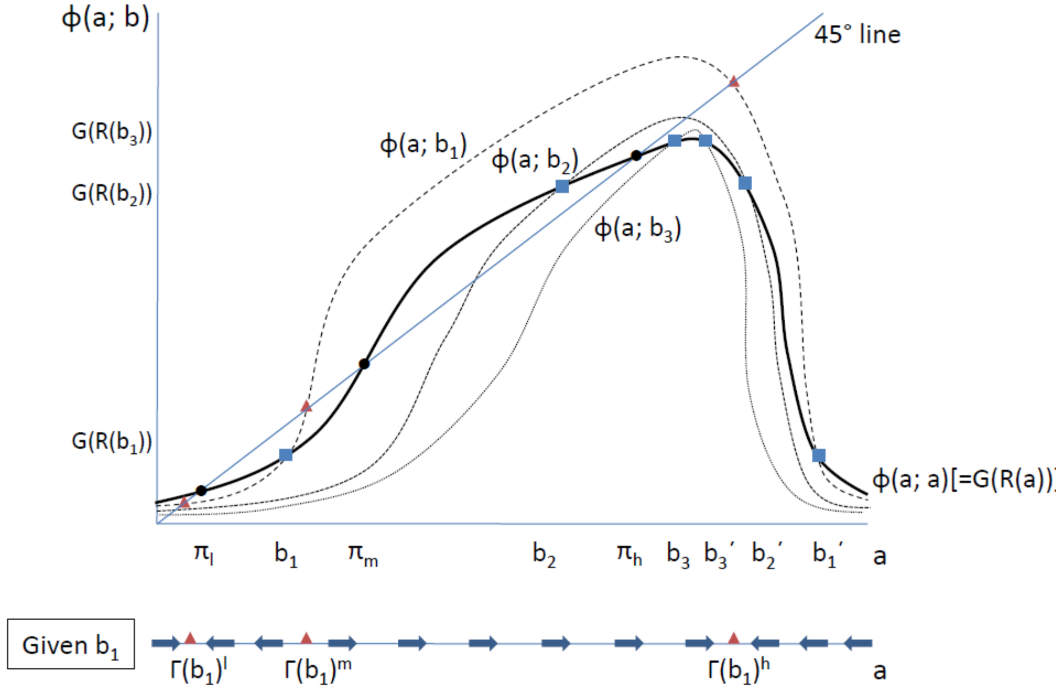

Figure 3. Human Capital Investment Rate $\phi(a; b)$

displayed in Figure 3 for the three exemplary levels of b below $\bar{\pi}$, $b_1 < b_2 < b_3 < \bar{\pi}$, together with its benchmark curve $\phi(a; a)(= G(R(a)))$. The less attractive the choice of affect B promising a smaller return to human capital investment $R(b)$, the more talented workers are willing to take affect A, leading to a greater skill investment rate for the endogenously constructed group A. For example, b_1, b_2, and b_3 satisfying $b_1 < b_2 < b_3 < \bar{\pi}$ "usually" satisfy $\phi(a; b_1) > \phi(a; b_2) > \phi(a; b_3)$ for a specific $a \in [0, 1]$, which implies that the $\phi(a; b_1)$ curve is "usually" placed above the $\phi(a; b_2)$ curve, which is also "usually" placed above the $\phi(a; b_3)$ curve, as exemplified in Figure 3. (The same is true for b_1, b_2, and b_3 that satisfy $\bar{\pi} < b_3 < b_2 < b_1$.)

Also, note that for any b except for $\bar{\pi}$, we always find $b'(\neq b)$ such that $R(b) = R(b')$. According to Lemma 1, $\Delta U(b', b; c)$ is constant with respect to c ($\because V_1(b) - V_0(b) = V_1(b') - V_0(b')$), implying that the following should hold for the combination (b', b): $\phi(b; b) = \phi(b'; b) = \phi(b; b') = G(R(b)) = G(R(b'))$. Therefore, we know that a dotted $\phi(a; b)$ curve must intercept the solid $\phi(a; a)(= G(R(a)))$ curve at both $a = b$ and $a = b'$, as described in the figure. From the above proposition, we find that the dotted $\phi(a; b)$ curve must be placed above (below) the solid $G(R(a))$ curve inside (outside) the a range between b and b': given $b < \bar{\pi} < b'$, $\phi(a; b) > G(R(a))$, $\forall a \in (b, b')$; $\phi(a; b) < G(R(a))$, $\forall a \in (0, b)$; $\phi(a; b) < G(R(a))$, $\forall a \in (b', 1)$.

Finally, the following lemma helps us understand the curvature of the $\phi(a; b)$ curve when it crosses over the $\phi(a; a)$ curve:

Lemma 2. *The slope of the $\phi(a; b)$ curve at the point where it crosses over the benchmark $\phi(a; a)$ curve is*

$$\frac{\partial \phi(a; b)}{\partial a}\bigg|_{a=b} \approx g(R(b))\, R'(b) + 2H'(0)\, R'(b)\, G(R(b)) \cdot [1 - G(R(b))]. \tag{14}$$

Proof. Refer to the proof of lemmas in the online appendix. ■

The above lemma implies that the slope of $\phi(a; b)$ at the crossing point is positive (negative) whenever $R'(b)$ is positive (negative), i.e., whenever b is less (greater) than $\bar{\pi}$. Furthermore, the slope of $\phi(a; b)$ at the crossing point is greater (smaller) than the slope of $\phi(a; a)$, which equals $g(R(b))R'(b)$, whenever $R'(b)$ is positive (negative). These facts indicate that the slope of $\phi(a; b)$ is always "steeper" than $\phi(a; a)$ at such crossing point.

6 Characteristics of Endogenous Stereotyping Equilibria

Now, we are ready to examine both the existence and the stability of Endogenous Stereotyping Equilibria. This analysis is inevitably complex, though still tractable, due to the mutual interaction between the two groups' collective reputations. First of all, we show that allowing for endogenous group "switching" can increase the divergence in the reputation and actual skill acquisition rates across groups above the maximum divergence possible in a setting where there are multiple equilibria in the exogenous-groups case.

Since the hump-shaped $R(\pi)$ is concave and $G(c)$ is S-shaped, usually there exist three or less PSE. (Refer to the equilibria described in Panel B of Figure 1.) The case with one (or two) PSE is not very interesting. In the following discussion, we assume that there exist three PSE that satisfy $G(R(\pi)) = \pi$: π_l, π_m and π_h, with the ordering of $\pi_l < \pi_m < \pi_h$.[16] Then, two of them (π_l and π_m) must be below $\bar{\pi}$ because $G(c)$ is a non-decreasing function and it is assumed that $G(0) > 0$. Another one (π_h) can be greater or smaller than $\bar{\pi}$. For the concise presentation of our key arguments, we focus on a representative case where the three equilibria are placed below $\bar{\pi}$: $\pi_l < \pi_m < \pi_h < \bar{\pi}$. However, readers will find that the main results do not change for the other possible case with $\pi_l < \pi_m < \bar{\pi} < \pi_h$, which is not presented in this manuscript but can be provided upon request.

6.1 Existence of Endogenous Stereotyping Equilibria

We introduce the following notation rule to examine correspondence $\Gamma(y)$ effectively in the $(y, \Gamma(y))$ plane. When there are three unique values in a correspondence $\Gamma(y)$, let us denote the greatest, the middle and the smallest one of them by $\Gamma(y)^h$, $\Gamma(y)^m$ and $\Gamma(y)^l$ for each. When the correspondence $\Gamma(y)$ contains just one value which is greater (smaller) than π_m, it is denoted by $\Gamma(y)^h$ ($\Gamma(y)^l$). (Refer to the solid curve $\Gamma(b)^i$ in Figure 4 to see this unique notation rule.)

Using the relative positions of the $\phi(x; y)$ curves for different levels of y, the properties of which are concretely discussed in the previous section, we can derive

[16] There always exists at least one PSE because $0 < G(R(0)) = G(0) < 1$ and $0 < G(R(1)) = G(0) < 1$. At least three PSE are guaranteed by the condition that there exist π_1 and π_2, in which $\pi_1 < \pi_2$, such that $G(R(\pi_1)) < \pi_1$ and $G(R(\pi_2)) = \pi_2$.

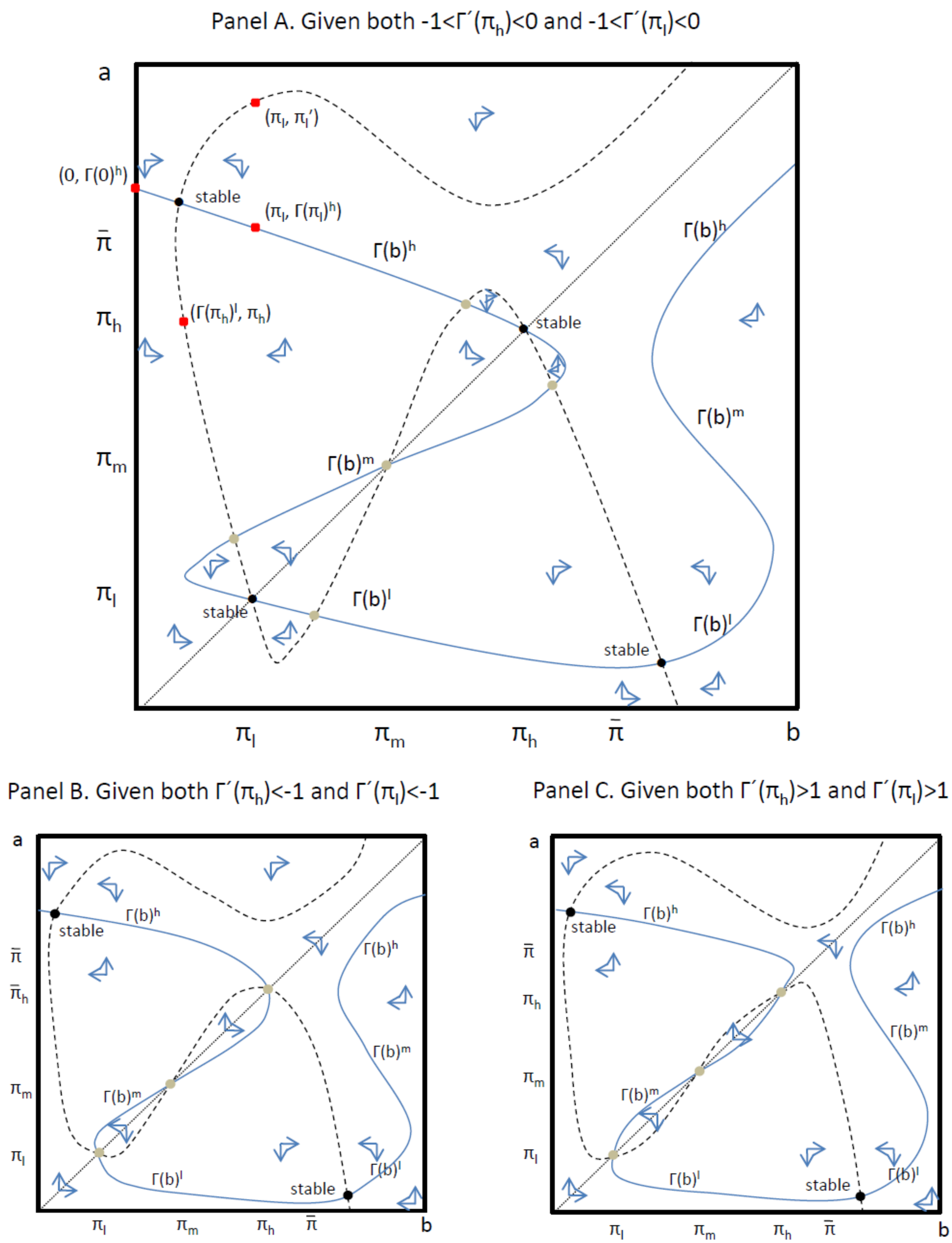

Figure 4. ESE given Multiple PSE (π_l, π_m, π_h)

the generic patterns of $\Gamma(y)^i$ for $i \in \{h, m, l\}$: for any y below $\bar{\pi}$, $\Gamma(y)^h$ and $\Gamma(y)^l$ are "usually" downward slopping in y and $\Gamma(y)^m$ is "usually" upward slopping in y. Owing to the hump-shaped $R(\pi)$ ($\because$ equation (5)), the patterns are repeated in a reverse way: for any y above $\bar{\pi}$, $\Gamma(y)^h$ and $\Gamma(y)^l$ are "usually" upward sloping in y and $\Gamma(y)^m$ is "usually" downward sloping in y. Thus, $\Gamma(\bar{\pi})^l$ is minimized around $\bar{\pi}$, as exemplified in Figure 4. More specifically, using Proposition 2, we achieve the following lemma that is useful for the further analysis:

Lemma 3. *Given $R(\pi_h) = R(\pi_h')$ and $R(\pi_l) = R(\pi_l')$, in which $\pi_h \neq \pi_h'$ and $\pi_l \neq \pi_l'$, the correspondence $\Gamma(y)$ satisfies $\pi_h < \Gamma(y)^h < 1$, $\forall y \notin [\pi_h, \pi_h']$, and $0 < \Gamma(y)^l < \pi_l$, $\forall y \in (\pi_l, \pi_l')$.*

Proof. Refer to the proof of lemmas in the online appendix.■

Based on the above findings, the two correspondences $\Gamma(b)$, which is a set $\{a | a = \phi(a; b)\}$, and $\Gamma(a)$, which is a set $\{b | b = \phi(b; a)\}$, are depicted in solid and dashed curves for each and overlapped in each panel of Figure 4. Let us call them *correspondence curves*, which satisfy the conditions of $\pi_h < \Gamma(0)^h < 1$ and $\pi_h < \Gamma(1)^h < 1$ according to the above lemma. Using the local linearization process, we can calculate the slope of *correspondence curve* at each trivial ESE $(\hat{x}, \hat{x})$, which satisfies $\hat{x} \in \Gamma(\hat{x})$, as follows:

Lemma 4. *The slope of the "correspondence curve" at a trivial ESE $(\hat{x}, \hat{x})$, which is denoted by $\Gamma'(\hat{x})$, is approximated by*

$$\Gamma'(\hat{x}) \approx \frac{2H'(0)\, R'(\hat{x})\, \hat{x}\, (1-\hat{x})}{g(R(\hat{x}))\, R'(\hat{x}) - 1 + 2H'(0)\, R'(\hat{x})\, \hat{x}\, (1-\hat{x})}. \tag{15}$$

Proof. Refer to the proof of lemmas in the online appendix.■

Using the above lemma, we conclude that the slope of the *correspondence curve*, $\Gamma'(\hat{x})$, varies according to the density of the identity choice cost distribution around zero, $H'(0)$:

Lemma 5. *While the slope of the "correspondence curve" at a trivial ESE* (π_m, π_m) *always satisfies* $0 < \Gamma'(\pi_m) < 1$, *the slope of the "correspondence curve" at a trivial ESE, either* (π_h, π_h) *or* (π_l, π_l), *depends on the density of the identity choice cost distribution around zero,* $H'(0)$*:*

$$\begin{cases} -1 < \Gamma'(\hat{x}) < 0 & \textit{for} \quad H'(0) < \frac{1-g(R(\hat{x}))\, R'(\hat{x})}{4R'(\hat{x})\, \hat{x}\, (1-\hat{x})} \\ \Gamma'(\hat{x}) < -1 & \textit{for} \quad \frac{1-g(R(\hat{x}))\, R'(\hat{x})}{4R'(\hat{x})\, \hat{x}\, (1-\hat{x})} < H'(0) < \frac{1-g(R(\hat{x}))\, R'(\hat{x})}{2R'(\hat{x})\, \hat{x}\, (1-\hat{x})}, \quad \forall \hat{x} \in \{\pi_h, \pi_l\}. \\ \Gamma'(\hat{x}) > 1 & \textit{for} \quad H'(0) > \frac{1-g(R(\hat{x}))\, R'(\hat{x})}{2R'(\hat{x})\, \hat{x}\, (1-\hat{x})} \end{cases}$$

Proof. Refer to the proof of lemmas in the online appendix.■

This lemma implies that when the sensitivity of identity choice activities represented by $H'(0)$ is sufficiently high that it is greater than some threshold $\frac{1-g(R(\hat{x}))\, R'(\hat{x})}{4R'(\hat{x})\, \hat{x}\, (1-\hat{x})}$, the absolute value of the slope of *correspondence curve* $|\Gamma'(\hat{x})|$ at a trivial ESE $(\hat{x}, \hat{x})$, $\forall \hat{x} \in \{\pi_h, \pi_l\}$, is greater than one.

The above lemmas help us to develop some meaningful theoretical conclusions. First, the following can be proved directly using the overlapped shapes of $\Gamma(a)$ and $\Gamma(b)$ *correspondence curves* in the (b, a) coordination plane:

Proposition 3. *Given multiple PSE (*π_l*,* π_m *and* π_h*), there always exist at least two non-trivial ESE.*

Proof. Using Lemma 3, given multiple PSE (π_l, π_m and π_h) and the condition of $\pi_h < \bar{\pi}$, the *correspondence curve* $\Gamma(b)$ "passes through" the symmetric point (π_h, π_h) and a-intercept $(b, a) = (0, \Gamma(0)^h)$, in which $\pi_h < \Gamma(0)^h < 1$. The *correspondence curve* of $\Gamma(a)$ also "passes through" the symmetric point (π_l, π_l) and b-intercept $(a, b) = (1, \Gamma(1)^h)$, in which $\pi_h < \Gamma(1)^h < 1$. (Refer to the panels of Figure 4.) Thus, there must be at least one ESE (b^*, a^*) that satisfies $a^* > b^*$. In a symmetric way, there exists at least one ESE that satisfies $b^* > a^*$. ■

In general, whether there are more than two non-trivial ESE depends on the curvatures of $\Gamma(a)$ and $\Gamma(b)$ *correspondence curves* around trivial ESE $(\hat{x}, \hat{x})$.

The slope of the *correspondence curve* at a trivial ESE, $\Gamma'(\hat{x})$, plays a key role in the number of non-trivial ESE. WLOG, the condition $|\Gamma'(\hat{x})| < 1$ for $\hat{x} \in \{\pi_h, \pi_l\}$ generates (at least) two additional non-trivial ESE around the trivial ESE $(\hat{x}, \hat{x})$, while the condition $|\Gamma'(\hat{x})| > 1$ for $\hat{x} \in \{\pi_h, \pi_l\}$ does not generate such additional non-trivial ESE around it. For instance, refer to Panel A of Figure 4 for a possible case with both $|\Gamma'(x_h)| < 1$ and $|\Gamma'(x_l)| < 1$ being satisfied, in which the total six non-trivial ESE are generated, and Panels B and C of the figure for a possible case with both $|\Gamma'(x_h)| > 1$ and $|\Gamma'(x_l)| > 1$ being satisfied, in which only two non-trivial ESE are generated.

Therefore, we can imagine (at least) two non-trivial ESE that exist regardless of the curvatures of the *correspondences curves* of $\Gamma(a)$ and $\Gamma(b)$. Let us call them "Persistent ESE" and denote them (π_L^*, π_H^*) and (π_H^*, π_L^*), in which both $\pi_H^* > \pi_h$ and $\pi_L^* < \pi_l$ are satisfied as proved in the following theorem.

Theorem 1. *Given multiple PSE (π_l, π_m and π_h), there always exist (at least) two "Persistent ESE", (π_L^*, π_H^*) and (π_H^*, π_L^*), which satisfy $\pi_L^* < \pi_l < \pi_h < \pi_H^*$.*

Proof. From Lemma 3, we know $\pi_h < \Gamma(b)^h < 1$, $\forall b \in [0, \pi_h)$. From Proposition 2, we have $\phi(a; \pi_l) > G(R(a))$, $\forall a \in (\pi_l, \pi_l')$, in which $R(\pi_l) = R(\pi_l')$. This implies that $\Gamma(\pi_l)^h < \pi_l'$. Consequently, we obtain that the *correspondence curve* $\Gamma(b)^h$ passes through the following two points $(0, \Gamma(0)^h)$ and $(\pi_l, \Gamma(\pi_l)^h)$ in the (b, a) coordination plane, in which $\pi_h < \Gamma(0)^h < 1$ and $\pi_h < \Gamma(\pi_l)^h < \pi_l'$, as demonstrated in Panel A of Figure 4. From Lemma 3, we know $0 < \Gamma(a)^l < \pi_l$, $\forall a \in [\pi_h, \pi_l']$. Since $\phi(\pi_l; \pi_l') = \pi_l$, we know $\pi_l \in \Gamma(\pi_l')$. Consequently, we obtain that the *correspondence curve* $\Gamma(a)^h$ passes through the following two points $(\Gamma(\pi_h)^l, \pi_h)$ and (π_l, π_l') in the (b, a) coordination plane, as demonstrated in Panel A of Figure 4. Therefore, there must be (at least) one intercept of the continuous *correspondence curves* $\Gamma(b)$ and $\Gamma(a)$, (π_L^*, π_H^*), which satisfies both $\pi_L^* < \pi_l$ and $\pi_H^* > \pi_h$. Out of the symmetry, there must be (at least) one more ESE (π_H^*, π_L^*). ■

The theorem implies that the inequality between endogenously constructed social groups in these "Persistent ESE" is greater than the inequality that can emerge between exogenously given groups: e.g., $|\pi_H^* - \pi_L^*| > |\pi_h - \pi_l|$. This main result is fairly intuitive. In a setting where there are multiple equilibria (PSE) in the exogenous-groups case, allowing for endogenous group switching can increase the divergence in the reputation across groups because the group with the better reputation not only provides higher return to investment, but also attracts relatively more talented workers from the disadvantaged group. Thus, the composition of the groups changes in a way that further reinforces the disparity. That is, the inequality in the "Persistent ESE" is not only due to the positive complementarities between a group's reputation and its members' investment activities but also due to the positive selection along the ability parameter.

6.2 Stability of Endogenous Stereotyping Equilibria

The key theoretical outcomes are achieved above. Readers anxious to get to the welfare analysis can skip this section without loss of continuity. In order to examine the stability of ESE, we consider the following intergenerational population structure. A worker is subject to the "Poisson death process" with parameter α: in a unit period, each individual faces α chances of dying and the α proportion of the population are newly born.[17] The newborn agents incur the cost c of skill achievement, and the cost k to choose the affect A (rather than the affect B). Each newborn agent with his cost set (c, k) decides whether to invest for skills and which 'affect' to choose among A and B in the early days of his life. Right after the days of education and affect adoption, newborns join the labor market and receive wages set by employers. However, newborns expect that their wages are affected by employers' prior belief (π_j) about the overall

[17]Refer to the "poisson death process" adopted by pervious works such as Tirole (1996), Derviz (2004) and Kim and Loury (2014).

skill rate of the population belonging to each identity group $j \in \{A, B\}$. Then, the actual skill investment rate of the entering newborns who adopt the affect j, $\phi(\pi_j; \pi_{-j})$, follows the rule described in equation (11).

In order to update their belief π_j, employers compare the realized actual skill acquisition rate of the newborns, $\phi(\pi_j; \pi_{-j})$, and their prior belief π_j.[18] When the realized skill acquisition rate of the newborns adopting the affect j, $\phi(\pi_j; \pi_{-j})$, is greater (smaller) than their prior belief about the skill level of identity group j, π_j, their posterior belief about the group's overall skill level becomes greater (smaller) than their prior one, as summarized in the following dynamics:

$$\dot{\pi}_j > (<)\, 0 \Leftrightarrow \phi(\pi_j; \pi_{-j}) > (<)\, \pi_j. \tag{16}$$

For readers' convenience, at the bottom of Figure 3, we present the law of motions of a given an arbitrary b_1: $\dot{a} > 0$ for any $a \in (0, \Gamma(b_1)^l)$ and any $a \in (\Gamma(b_1)^m, \Gamma(b_1)^h)$ ($\because \phi(a; b_1) > a$), and $\dot{a} < 0$ for any $a \in (\Gamma(b_1)^l, \Gamma(b_1)^m)$ and any $a \in (\Gamma(b_1)^h, 1)$ ($\because \phi(a; b_1) < a$). Therefore, the direction arrows of $\dot{a}$ are upward below $\Gamma(b)^l$ and between $\Gamma(b)^m$ and $\Gamma(b)^h$ in the (b, a) coordination plane and downward between $\Gamma(b)^l$ and $\Gamma(b)^m$ and above $\Gamma(b)^h$, as displayed in Figure 4. In a symmetric way, the direction arrows of $\dot{b}$ are rightward at the left-hand side of $\Gamma(a)^l$ and between $\Gamma(a)^m$ and $\Gamma(a)^h$ in the (b, a) coordination plane and leftward between $\Gamma(a)^l$ and $\Gamma(a)^m$ and at the right-hand side of $\Gamma(a)^h$. From the described direction arrows in Figure 4, we can infer the following result without any difficulty.

Theorem 2. *Given multiple PSE (π_l, π_m and π_h), (at least) two "Persistent ESE", (π_L^*, π_H^*) and (π_H^*, π_L^*), are stable.*

For instance, when the total number of non-trivial ESE is exactly six, as exemplified in Panel A of Figure 4, two "Persistent ESE" are stable and other

[18] We assume that employers have correct information about the actual skill acquisition rate of the newborns belonging to each identity group.

four non-trivial ESE are unstable. Combining Theorems 1 and 2, we come to the conclusion that there always exist two stable ESE in which the between-group inequality is greater than the maximum inequality possible in the exogenous groups case, regardless of the curvatures of the *correspondence curves* of $\Gamma(a)$ and $\Gamma(b)$.

Furthermore, using the direction arrows, we can easily confirm the stability condition of trivial ESE that the middle trivial ESE (π_m, π_m) is always unstable. Other trivial ESE, (π_h, π_h) and (π_l, π_l), are stable if $|\Gamma'(\hat{x})| \leq 1$ and unstable if $|\Gamma'(\hat{x})| > 1$. Using Lemma 5, we know that $|\Gamma'(\hat{x})| > 1$ if and only if $H'(0) > \frac{1-g(R(\hat{x}))\,R'(\hat{x})}{4R'(\hat{x})\,\hat{x}\,(1-\hat{x})}$, for $\hat{x} \in \{\pi_h, \pi_l\}$. Therefore, when the society consists of a sufficiently large fraction of newborns whose identity choice cost is very low (i.e., $H'(0)$ is sufficiently large), the balanced skill rates between two identity groups cannot be sustainable as any small perturbation would motivate a significant fraction of talented members to choose the "affect" associated with the better collective reputation, thereby leading to a divergence in the human capital cost distributions across groups that reinforces the disparity. Thus, we arrive at the following worthwhile result:

Proposition 4. *All the balanced skill rates are unstable if and only if* $H'(0) > \frac{1-g(R(\hat{x}))\,R'(\hat{x})}{4R'(\hat{x})\,\hat{x}\,(1-\hat{x})}, \forall \hat{x} \in \{\pi_h, \pi_l\}$.

This means that when the affordability of identity choice activities is sufficiently high (e.g., an adoption of accent of a favored regional group), the skill composition of the society inevitably converges to an unequal ESE in the long run. From a policy perspective, this result provides a meaningful conclusion that, even with strong egalitarian government interventions, if the more talented individuals adopt the more highly regarded group's identity to a disproportionately very large extent, then the between-group difference will never be vanished and the interventions will be like throwing water on thirsty soil. In facing these challenges, a government pursing an equal society may consider the policies that may affect the sensitivity of identity choices (as captured by $H'(0)$), such as group specific (religious, ethnic, cultural or regional) self-esteem programs or public promotion of social events celebrating specific identity categorizations.[19]

7 Welfare Properties and Implications

So far, we have provided an explicit micro-foundation for the endogenous group formation, which is embedded in a statistical discrimination framework with endogenous beliefs about skill acquisition. This allows for some welfare analysis, highlighting the winners and losers from the assimilation process. Among various situations in which identity choice and stereotypes operate in tandem, we focus on the following two identity manipulation activities introduced in Section 2: passing and 'partial passing' behaviors.

7.1 Selective Out-migration (Passing)

Consider two social groups, a privileged group (A) and a stigmatized group (B). The selective out-migration from the stigmatized group to the privileged group occurs when the return for "passing" such as better treatment in the labor market outweighs its cost such as losing ties to ones' own kind, learning unfamiliar customs and adopting a new culture. According to Theorems 1 and 2, there always exist (at least) one stable "Persistent ESE," in which the groups' collective reputations are self-confirmed at π_H^* and π_L^* for each.

The welfare effects of the passing behavior can be examined by comparing the welfare at this stable equilibrium to the welfare at the benchmark economy in which the perceived identity is not malleable and each group's collective reputation is self-confirmed at one of the stable PSE: π_h for the privileged group and

[19] For instance, a change of accent (dialect) is one of the most affordable methods for 'regional identity' manipulation. If talented members of a stereotyped regional group tend to modify their accents to avoid the anticipated disadvantages in the labor market, the once-developed stereotypes against the group will never disappear, even when their government makes strong commitments for establishing national unity and reconciliation between the groups.

π_l for the stigmatized group.[20] Refer to Figure 5 for this benchmark economy without the passing activities.[21]

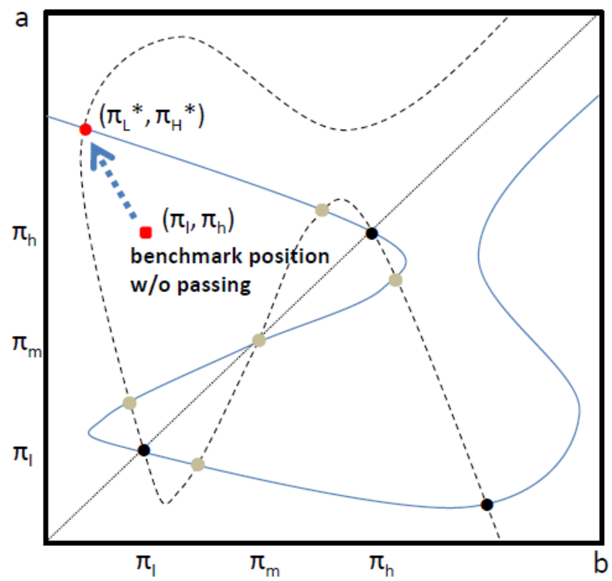

Figure 5. Passing Behavior (Group B to Group A)

Now, let us clarify who benefits and who suffers from the prevalence of passing activities. The total population in the "Persistent ESE" can be classified into three population aggregates according to their identity manipulation incentives: "passers" who give up their natural orientation type-B to be perceived as type A ($0 < k < \Delta U(\pi_H^*, \pi_L^*; c)$), "non-passers" who maintain their natural orientation type-B although being stigmatized in the marketplace ($k \geq \Delta U(\pi_H^*, \pi_L^*; c)$) and "the advantaged" who keep their privileged type-A membership ($k \leq 0$).

Because the anticipated net reward $U(\pi, c)$ is monotonically increasing in π

[20] Note that π_m is not stable in the sense that the group's overall skill acquisition rate $G(R(\pi))$ diverges away from π_m with any little perturbation: $G(R(\pi_m + \epsilon)) > \pi_m + \epsilon$ and $G(R(\pi_m - \epsilon)) > \pi_m - \epsilon$, for any small $\epsilon > 0$.

[21] In the theoretical model, the identity manipulation cost k is symmetrically distributed around zero. In the real world, however, we see that many stereotyped groups are in fact minorities. Acknowledging this reality does not make a qualitative difference in terms of model interpretations. The only difference is that the decline in the reputation of the minority group is affected more by existing passing activities, while the increased reputation of the dominant group is less affected by them.

and the condition $\pi_L^* < \pi_l < \pi_h < \pi_H^*$ holds according to Theorem 1, we can infer that "non-passers" suffer from the prevalent out-migration activities as much as $U(\pi_l, c) - U(\pi_L^*, c)$, while "the advantaged" benefit from such activities as much as $U(\pi_H^*, c) - U(\pi_h, c)$. It is noteworthy that not all of passers benefit from the prevalence of out-migrations. A passer's anticipated net reward changes as much as $U(\pi_H^*, c) - U(\pi_l, c) - k$ between the two distinct economies. Only those whose identity manipulation cost is sufficiently small that it is less than some threshold $\tilde{k}(c)$ benefit, while those whose identity manipulation cost is above the threshold suffer, in which $\tilde{k}(c) \equiv U(\pi_H^*, c) - U(\pi_l, c)$.

Note that the threshold $\tilde{k}(c)$ satisfies $0 < \tilde{k}(c) < \Delta U(\pi_H^*, \pi_L^*; c)$ for any specific level of c. Then, we achieve the following welfare property that denies the possibility of *Pareto improvement.*

Proposition 5. *The individuals (with skill investment cost c) whose identity manipulation cost k is above the threshold $\tilde{k}(c)$ suffer due to the prevalence of passing activities, while those whose identity manipulation cost k is below the threshold benefit from it.*

Second, let us examine the conditions under which the selective out-migration may improve the social efficiency. We can compute the societal efficiency gain (ΔW_{total}) by the double integrations of the welfare changes of the three population aggregates (non-passers, passers and the advantaged):[22]

$$\begin{aligned}\Delta W_{total} &= \int_0^{\infty} \Big[\int_{\Delta U}^{\infty} [U(\pi_L^*, c) - U(\pi_l, c)] \, \mathrm{d}H(k) + \int_0^{\Delta U} [U(\pi_H^*, c) - U(\pi_l, c) - k] \, \mathrm{d}H(k) \\ &\quad + \int_{-\infty}^{0} [U(\pi_H^*, c) - U(\pi_h, c)] \, \mathrm{d}H(k) \Big] \, \mathrm{d}G(c), \text{ where } \Delta U \equiv \Delta U(\pi_H^*, \pi_L^*; c)\end{aligned}$$

A utilitarian government may take actions that encourage (or discourage) selective out-migration behaviors depending on the sign of the societal efficiency

[22]The employers' expected payoffs are always zero because they are assumed to pay exact wages to workers according to their expected productivity.

gain (ΔW_{total}). Through the decomposition, we obtain[23]

$$\begin{aligned}\Delta W_{total} &= \underbrace{0.5\int_0^{\infty}[U(\pi_H^*,c)-U(\pi_h,c)]\,\mathrm{d}G(c)}_{\text{“positive reputational externality”}} - \underbrace{0.5\int_0^{\infty}[U(\pi_l,c)-U(\pi_L^*,c)]\,\mathrm{d}G(c)}_{\text{“negative reputational externality”}} \\ &+ \underbrace{\int_0^{\infty}\int_0^{\Delta U}[\Delta U-k]\,\mathrm{d}H(k)\,\mathrm{d}G(c)}_{\text{“passing premium”}},\ \text{using the symmetry of } H(k). \qquad (17)\end{aligned}$$

The change from the PSE benchmark economy (π_l, π_h) to the “passing” equilibrium (π_L^*, π_H^*) generates the positive reputational externality for the population aggregate whose natural orientation is type A and the negative reputational externality for the population aggregate whose natural orientation is type B. The sizes of both externalities are summarized in the first and second terms in the above equation. The third term in the equation plays a significant role in the determination of the positive societal efficiency gain, which reflects the passing premium for the passers who choose to elect type A although their natural orientation is type B. The positive efficiency gain is achieved only when the passing premium is sufficiently great that it is bigger than the net loss in terms of the reputational externalities—the size of the negative reputational externailty minus the size of the positive reputational externality.[24]

Generally, under some limited conditions, the “passing” activities can cure to some extent the societal inefficiency caused by labor market imperfection and the

[23] Use the following decomposition: $\int_0^{\Delta U}[U(\pi_H^*,c)-U(\pi_l,c)-k]\,\mathrm{d}H(k) = \int_0^{\Delta U}[U(\pi_L^*,c)-U(\pi_l,c)]\,\mathrm{d}H(k) + \int_0^{\Delta U}[U(\pi_H^*,c)-U(\pi_L^*,c)-k]\,\mathrm{d}H(k) = \int_0^{\Delta U}[U(\pi_L^*,c)-U(\pi_l,c)]\,\mathrm{d}H(k) + \int_0^{\Delta U}[\Delta U-k]\,\mathrm{d}H(k)$, where $\Delta U \equiv \Delta U(\pi_H^*, \pi_L^*; c)$.

[24] In the given model, the wage rate per unit of efficient labor is fixed as w for high-skilled labor and 0 for low-skilled labor. However, if we allow skill complementarities between high and low skill labor in production, the wage rate per unit of high-skilled (low-skilled) labor would depend negatively (positively) on the total level of human capital in the economy. Since selective out-migration tends to raise the total level of human capital, this would reduce the benefits of the “passing premium” and the size of the positive reputational externality as well as the size of the negative reputational externality. The societal efficiency gain of passing would then reduce.

consequent low skill acquisition rate of the stigmatized group. In this respect, we derive some useful findings from the above decomposition as follows.

The passing premium is directly transformed into the following form, $\int_0^\infty \int_0^{\Delta U} [H(k) - 0.5]\,\mathrm{d}k\,\mathrm{d}G(c)$, where $\Delta U \equiv \Delta U(\pi_H^*, \pi_L^*; c)$,[25] which implies that the size of the passing premium is largely governed by how much the distribution of the identity manipulation cost is concentrated around zero. That is, the more dense around zero the distribution of identity manipulation cost k is, the greater the societal efficiency gain from the selective out-migration (ΔW_{total}) will be. Accordingly, the positive efficiency gain is more likely to be achieved when identity manipulation is easier to undertake.

It is also notable that the negative reputational externalities can even vanish when the disadvantaged group is so severely stigmatized that the believed skill acquisition rate of group B is close to zero in the PSE benchmark economy (i.e., $\pi_l \approx 0$)[26]: $0.5 \int_0^\infty [U(\pi_l, c) - U(\pi_L^*, c)]\,\mathrm{d}G(c) \approx 0$ as $\pi_L^* < \pi_l \,(\approx 0)$. That is, in this extreme case, there is almost no reputation to lose for the disadvantaged group members, at least not as a result of the endogenous out-migration. Therefore, a positive societal efficiency gain is achieved with "non-passers" who are little worse off, and all other agents who are better off:

Proposition 6. *The net welfare effect of the selective out-migration is positive when the disadvantaged group is severely discriminated in a society.*

There are many real-life examples in which passing improves social efficiency. For instance, the living conditions of the Zainichi were the worst in Japan, and they were severely stigmatized even after Japanese imperialism ended. However, their identity manipulation was relatively easier to achieve, given how their appearance was similar to that of Japanese individuals. Their selective out-migration presumably improved social efficiency: the passing premium was

[25] $\int_0^{\Delta U} [\Delta U - k]\,\mathrm{d}H(k) = \int_0^{\Delta U} [\Delta U - k] h(k)\,\mathrm{d}k = [(\Delta U - k)H(k)]_0^{\Delta U} + \int_0^{\Delta U} H(k)\,\mathrm{d}k = -\Delta U \cdot H(0) + \int_0^{\Delta U} H(k)\,\mathrm{d}k = \int_0^{\Delta U} [H(k) - 0.5]\,\mathrm{d}k$.

[26] This is possible when $G(0)$ is positive but sufficiently small: i.e., $G(0) = +\epsilon$ and $\epsilon \to 0$.

high, but the negative impact on those left behind was minimal. Given the severe discrimination against the Zainichi at those days, the net welfare effect of the passing activities was doubtlessly positive.

7.2 The Indices of Differentiation (Partial Passing)

In this section, we analyze the case where a sub-hierarchy of identities can be adopted by a stigmatized population. Consider a stigmatized population for which the pertinent physical traits are not readily disguised or the distinct culture and customs cannot be given up without paying a very high cost (e.g., dark-skinned blacks in the Americas or orthodox Islamic immigrants in Europe). Most of the better-off members of this stigmatized population will not be able to pass for a better regarded social group. Instead, they may seek other ways of artful self-presentations to send signals that they are different from the average of the stigmatized mass.[27] In this way, a "visible" subgroup can be constructed around any cluster of markers which are evidently informative though functionally irrelevant traits (such as affectations of speech, dressing up and consumption habits). Acknowledge that anything that is costly to acquire, say even dressing in funny clothes, can be one of the markers, but the most effective ones to send signal that they are different will be cultural or behavioral traits of the better regarded group.[28]

Imagine a specific set of indices that is used for the differentiation. Suppose that employers, who are doing their best under trying circumstances, partition the stigmatized population into two subgroups along these indices: subgroup Z' composed of the agents adopting the set of indices and subgroup Z composed of the agents who do not adopt the indices. Assume that the stigmatized population consists of a subpopulation whose natural orientation is not to adopt the indices

[27] By using a more refined set of indices to guide their discrimination, observers may also encourage the production of those very indices of differentiation by the more talented members.

[28] One of the reasons is that successfully adopting those traits will signal a person's willingness to put in effort to "confirm", which is valuable to employers.

($k > 0$) and the other subpopulation whose natural orientation is to adopt the indices ($k < 0$): an agent with positive k should incur the cost k to be equipped with those indices, while an agent with negative k should incur the cost $-k$ to discard their naturally adopted indices.

The theory developed earlier is directly applied to this altered setting, replacing groups A and B with subgroups Z' and Z. The most talented members of the population, who gain most by separating themselves from the mass, will disproportionately elect to join the subgroup Z', adopting the indices, inducing the positive selection into this subgroup and making the human capital cost distributions of the two subgroups diverge endogenously. Denoting the believed skill acquisition rates of the two subgroups by z and z', the stable unequal ESE of (z, z') can be (π_L^*, π_H^*), given the existence of multiple PSE (π_l, π_m and π_h), in which $\pi_L^* < \pi_l < \pi_h < \pi_H^*$ holds.

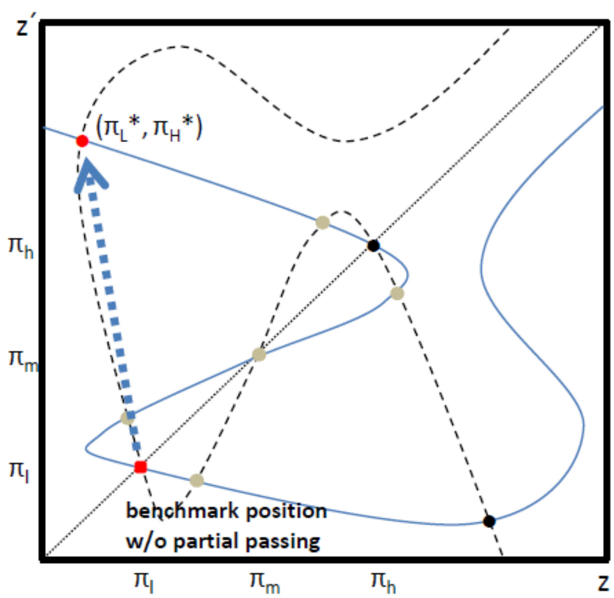

Figure 6. Partial Passing Behavior (Subgroup Z to Subgroup Z')

The welfare effects of this partial passifng behavior can be examined comparing the welfare at the stable ESE (π_L^*, π_H^*) to the welfare at the PSE benchmark economy in which agents in the stigmatized population do not make a strategic decision on whether to adopt the indices. In this benchmark economy, there should be no clear difference in terms of the skill acquisition rates between the two subgroup Z and Z': $(z, z') = (\pi_l, \pi_l)$. (Refer to Figure 6 for this benchmark economy.) Then, we obtain the following welfare changes of three population aggregates:

Proposition 7. *Comparing an unequal stable economy with the prevalent partial passing activities* (π_L^*, π_H^*) *with its benchmark economy without such activities* (π_l, π_l)*, "Non-partial passers* $(k > \Delta U(\pi_H^*, \pi_L^*; c))$*" suffer from the prevalence of the activities as much as* $U(\pi_l, c) - U(\pi_L^*, c)$*, while the population who adopt the indices naturally* $(k < 0)$ *is benefited from it as much as* $U(\pi_H^*, c) - U(\pi_l, c)$*. The welfare change of a "partial passer* $(0 < k < \Delta U(\pi_H^*, \pi_L^*; c))$*" is* $U(\pi_H^*, c) - U(\pi_l, c) - k$*, which is positive (negative) for those whose cost to adopt the indices is less (greater) than the threshold* $\tilde{k}(c) (\equiv U(\pi_H^*, c) - U(\pi_l, c))$*, which satisfies* $0 < \tilde{k}(c) < \Delta U(\pi_H^*, \pi_L^*; c), \forall c$*.*

The societal efficiency gain from the prevalence of partial passing activities is computed by the double integrations of the welfare changes summarized in the above proposition, or, alternatively, by the replacement of $U(\pi_h, c)$ with $U(\pi_l, c)$ in ΔW_{total} (equation (17)). In general, the findings in the previous section work for this altered setting: the positive efficiency gain is achieved only when the premium obtained by the partial passers is greater than the net loss in terms of the reputational externalities. The positive efficiency gain is more likely to be achieved when the adoption of the indices of differentiation is easier to undertake.[29]

The welfare results could help shed light on the conflict within a stereotyped population. While some partial passers whose identity manipulation cost is lower benefit from the identity manipulation activities, the non-partial passers suffer

[29]The mathematical expression of the partial passing premium is identical to that of the passing premium in equation (17).

from them. Thus, social identity manipulation through partial passing can be a way to undermine solidarity in the visibly distinct stigmatized population. The adverse impact on the left-behind may generate the resentment against the partial passers. Moreover, it may trigger internal reactions that have the intended goal of increasing the identity manipulation cost. This may explain the emergence of collective institutions (e.g., gangs, religious or ethnic associations) within the disadvantaged group that will try to shift the distribution of the cost k. The emergence of such "oppositional" institutions is likely to be more active when the decline in the reputation of the disadvantaged group is greater, threatening to the welfare of those left behind.

The worse-off members of the group may even accuse the partial passers of some kind of immoral betrayal, which is often referred to as "acting white" accusation in the US racial context due to the partial passers' assuming the social expectations of white society.[30] This reasoning can be an alternative explanation of the "acting white" phenomenon to that offered by Austen-Smith and Fryer (2005). They propose a two-audience model in which the incumbents of the minority population reject their own members who acquire human capital for "acting white" because they think them low social ability types. Instead, we suggest that the group reject "partial passers" but not because these people are thought to be socially inept. The group rejects them because it feels betrayed by them and because their departure adversely affects the reputations of those who are left behind.[31] In this respect, our work complements Eguia's (2017) assimilation model that emphasizes peer effects at school, in which low-skilled disadvantaged students use sanctions to dissuade their more able co-ethics from becoming too

[30]For instance, the behaviors that lead to accusation of "acting white" include speaking standard English, wearing clothes from the Gap or Abercrombie & Fitch, wearing shorts in the winter, and enrolling in honors or advanced placement classes, according to Neal-Barnett's (2001) focus group interview. Among them, academic success is a functionally relevant index, which is valuable to employers. Thus, it will further exaggerate the positive selection effects.

[31]In a similar spirit, scholars in Sociology (e.g., Wilson, 1987) argue that the movement of the black middle-class from black neighborhoods to suburbs (so-called "black flight") has had a detrimental impact on black poverty.

skilled and ending up assimilating into the advantaged group because they want them to stay close generating positive social network externalities.

However, as discussed earlier, the supposed "immoral" activities of which some are accused may improve the total welfare of society under some limited conditions. The improvement is clear when the adoption of those indices are not very costly and the disadvantaged population had been widely stigmatized in society, because then the premium obtained by partial passers will be great, but the size of the created negative reputational externalities will be relatively smaller.

8 Conclusion

Our theoretical model is based on a stereotyping-cum-signaling framework suggested by Arrow (1973) and Coate and Loury (1993), in which multiple self-confirming beliefs by employers about different social identity groups explain the between-group inequality in terms of the skill acquisition activities. Unlike the previous works and their subsequent developments, we handle the dynamics between the collective reputation and the identity choice problem. By relaxing the immutability assumption, the model explores the implications of the fact that the distribution of abilities within distinct identity groups becomes endogenous when individuals choose how they will be identified by external observers. The logic reveals that the low human capital cost types are disproportionately drawn to the group with a better collective reputation, causing a skill disparity between groups to endogenously diverge. An equal status is not sustainable when identity manipulation is sufficiently easier to undertake.

We have applied the theory to the passing and 'partial passing' phenomena, finding that these inequality-amplifying identity manipulation activities can improve the social efficiency either when the (partial) passing premium is maximized or when the loss in terms of the reputational externalities is minimized.

Identifying who benefits and who suffers from the phenomena, we provide the rationale behind conflicts within a stereotyped population such as the emergence of the "oppositional" institutions and the 'acting white' accusation. One might expect the winners and losers to take strategic actions accordingly—namely, the punishing activity by the losers, to deter selective out-migration, and the subsidies offered by the winners, to promote it.[32] The government may also consider policy measures that are more likely to mitigate (or amplify) the reinforcing effects between endogenous identity and investment incentives.[33] In case of the partial passing, the high-ability types of a disadvantaged group may strategically choose what traits identify the differentiated subgroup, in such a way that it is easy for them to acquire the traits, but hard for the low types. These reactions of the stakeholders and their economic implications are worthy of further and deeper examination, but are left for future study.

The developed micro-foundation of endogenous group formation has the potential to illuminate many other social phenomena involving the choice of the perceived identities (e.g., racial profiling in law enforcement, the coming out decision by LGBT people and effectively 'branding' a new consumer product). Above all, in the increasingly globalized and multicultural societies in which we live, the question of identity choice and how this interacts with human capital investment incentives and socio-economic discrimination processes is becoming an important topic with many policy implications. We look forward to seeing more developments in the economic research on this topic of endogenous identity.

[32]However, the advantaged group, besides being concerned about returns to skills, may care more about maintaining its social status. If this is the case, members of the advantaged group would create "subtle" socialization barriers to members of the other groups, making it more difficult for them to "pass"; this brings to mind sociologist Bourdieu's (1987) term "distinction."

[33]For instance, if the government introduces public polices that ease the assimilation or passing of under-privileged minorities, the result may well be that the low-ability types who are left behind end up more severely (statistically) discriminated, unlike the policy makers' original intentions.

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[For Online Publication: Appendixes A and B]

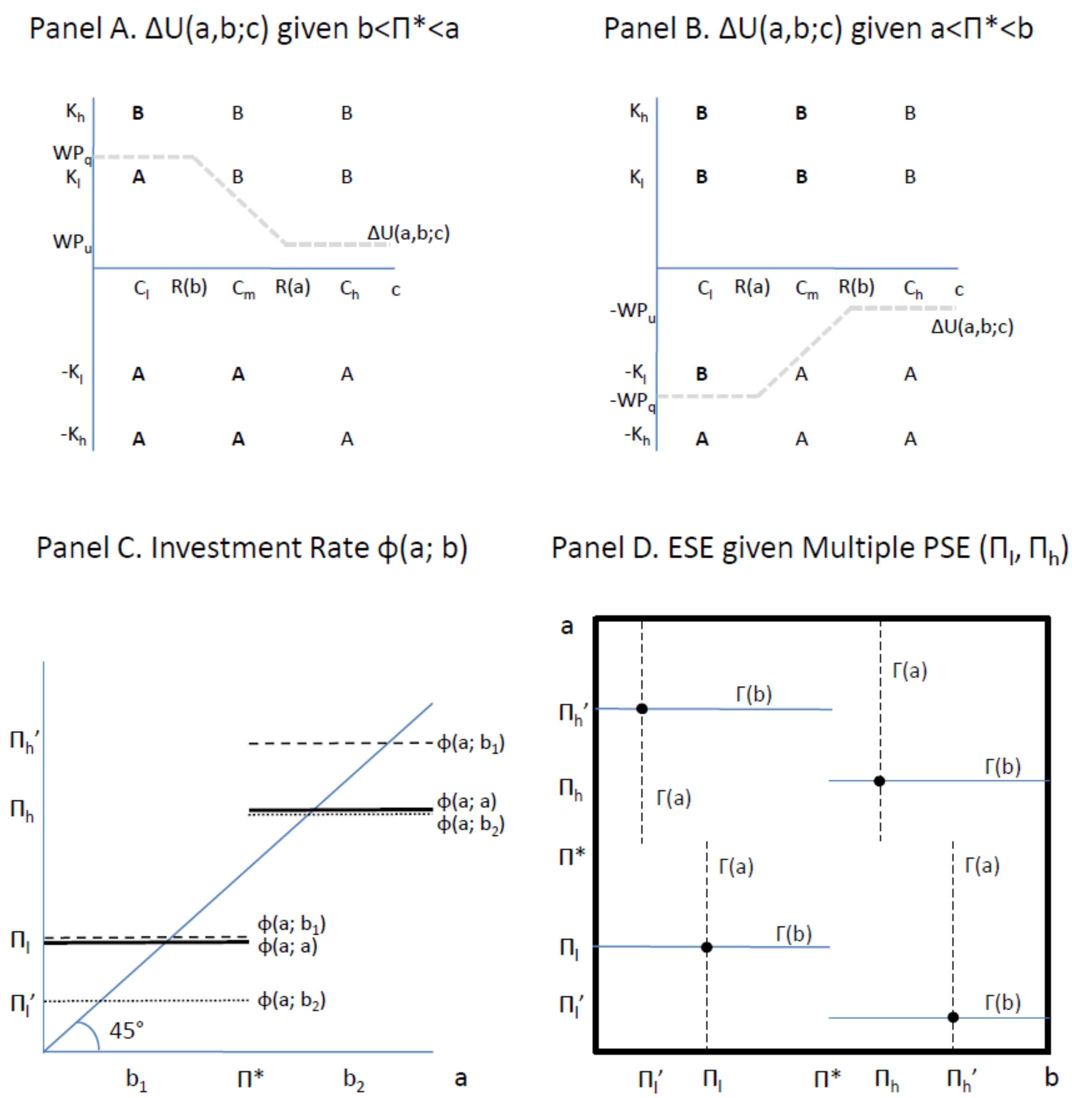
Appendix A: ESE in a Simple Model

In this appendix, we present the endogenous stereotyping equilibria in the simplest possible cost and signaling structure, in order to help readers better understand the elementary mechanism of endogenous group formation at work.

A1. The Simplest Cost and Signaling Structure

Let us adopt discrete cost and labor market signal distributions, instead of continuous ones. The population comprises three human capital investment cost (c) types: (1) Π_l fraction of agents whose investment cost (C_l) is close to zero and who will thus always invest in skills, (2) $\Pi_h - \Pi_l$ fraction of agents whose investment cost (C_m) is mediocre, and who will decide whether or not to invest based on the expected return to skill investment, and (3) $1 - \Pi_h$ fraction of agents whose investment cost (C_h) is very high and who will never invest in skills. Then, we have the step function of $G(c)$: $G(c) = \Pi_l, \forall c \in (0, C_m)$; $G(c) = \Pi_h, \forall c \in [C_m, C_h)$; $G(c) = 1, \forall c \in [C_h, \infty]$. In terms of the relative cost of being perceived as A rather than B (k), the population comprises four types: η fraction (η fraction) of agents who are naturally inclined toward B (A) and should incur a relatively lower cost K_l to be perceived as A (B), indicating that $k = K_l$ ($k = -K_l$), and $0.5 - \eta$ fraction ($0.5 - \eta$ fraction) of agents who are naturally inclined toward B (A) and should incur a very high cost K_h to be perceived as A (B), indicating that $k = K_h$ ($k = -K_h$). Thus, we have in total 12 different population aggregates, of which the cost combination (c, k) is represented by $(c, k) \in \{(C_i, K_j), (C_i, -K_j)\}, \forall i \in \{l, m, h\}, \forall j \in \{l, h\}$. (Refer to the distribution of those 12 aggregates, as seen in Panels A and B of Appendix Figure 1.)

The test of qualification (prior to assignment) yields one of the three signals, $t \in \{H, M, L\}$. The test outcome H (L) is achieved only by those who are



Appendix Figure 1. PSE and ESE in a Simple Set-up

qualified (unqualified). The test outcome M can be achieved by either those who are qualified or those who are unqualified. Let P_q (P_u) be the probability that if a worker does (does not) invest, his or her test outcome is M: $P_q \equiv Prob[M|skilled]$ and $P_u \equiv Prob[M|unskilled]$.

We further assume that workers receive a gross benefit of W if hired, and zero if unemployed. Employers gain a net return of X_q if they hire a skilled worker, and suffer a net loss of X_u if they hire an unskilled worker. Then, they will (will not) hire all who achieve signal H (L), and will hire a worker who achieves signal M if and only if the expected net return from doing so is nonnegative: $X_q \cdot Prob[skilled|M] - X_u \cdot Prob[unskilled|M] \geq 0$, in which the posterior probability that the worker with the unclear test outcome M is in fact skilled is $Prob[skilled|M] = \pi P_q/(\pi P_q + (1-\pi)P_u)$, using Bayes' rule, and given the believed skill investment rate of the group π. Hence, employers will hire a worker with signal M if and only if the employer is sufficiently optimistic about the rate of skill acquisition of a group from which the worker was drawn:

$$\textit{Hiring a worker with signal } M \iff \pi \geq \frac{P_u X_u}{P_q X_q + P_u X_u} (\equiv \Pi^*), \tag{18}$$

for which we assume that the threshold level Π^* satisfies $\Pi_l < \Pi^* < \Pi_h$.

Given this simplest framework, the expected payoff from acquiring a skill $V_1(\pi)$ is W if $\pi \geq \Pi^*$, and $W(1-P_q)$ if $\pi < \Pi^*$. That without acquiring a skill $V_0(\pi)$ is WP_u if $\pi \geq \Pi^*$, and 0 if $\pi < \Pi^*$. Thus, the expected economic return from being skilled $R(\pi)(\equiv V_1(\pi) - V_0(\pi))$ is

$$R(\pi) = \begin{cases} W(1-P_u), & \text{if } \pi \geq \Pi^* \\ W(1-P_q), & \text{if } \pi < \Pi^* \end{cases}. \tag{19}$$

In order to have multiple PSE, Π_l and Π_h, the human capital investment costs must satisfy the condition $C_l \leq W(1-P_q) < C_m \leq W(1-P_u) < C_h$, because $G(R(\Pi_l)) = G(W(1-P_q)) = \Pi_l$ only when $C_l \leq W(1-P_q) < C_m$, and $G(R(\Pi_h)) = G(W(1-P_u)) = \Pi_h$ only when $C_m \leq W(1-P_u) < C_h$.

A2. ESE Given Multiple PSE (Π_l and Π_h)

Now suppose that perceived identity is malleable and groups are endogenously constructed, given the existence of multiple PSE, Π_l and Π_h. The anticipated net reward for a worker who belongs to a group believed to be investing at rate

π, denoted by $U(\pi, c)$, is either $V_1(\pi) - c$ if he or she invests, or $V_0(\pi)$ if he or she does not. Hence, it is expressed as $\max\{V_1(\pi) - c, V_0(\pi)\}$:

$$U(\pi, c) = \begin{cases} \max\{W - c, WP_u\}, & \text{if } \pi \geq \Pi^* \\ \max\{W(1 - P_q) - c, 0\}, & \text{if } \pi < \Pi^* \end{cases}. \tag{20}$$

Given the employers' prior belief about human capital investment rates (a, b), we achieve a worker's incentive for electing type-A rather than type-B, denoted by $\Delta U(a, b; c)$, which is equivalent to $U(a, c) - U(b, c)$. Only the population aggregate whose cost set (c, k) satisfies $k \leq \Delta U(a, b_1; c)$ will elect type-A. All the other aggregates will elect type-B.

When both a and b are less (or greater) than Π^*, this incentive $\Delta U(a, b; c)$ is zero, indicating that those whose k is negative (positive) elect type-A (type-B). Given $b < \Pi^* < a$, however, this incentive is positive for every human capital cost type and non-increasing in c, as seen in Panel A of Appendix Figure 1:

$$\Delta U(a, b; c) = \begin{cases} WP_q, & \text{if } c = C_l \\ W - C_m, & \text{if } c = C_m \\ WP_u, & \text{if } c = C_h \end{cases}, \text{ given } b < \Pi^* < a. \tag{21}$$

In a symmetrical manner, given $a < \Pi^* < b$, this incentive is negative for every human capital cost type and non-decreasing in c, as seen in Panel B of the same figure:

$$\Delta U(a, b; c) = \begin{cases} -WP_q, & \text{if } c = C_l \\ -W + C_m, & \text{if } c = C_m \\ -WP_u, & \text{if } c = C_h \end{cases}, \text{ given } a < \Pi^* < b. \tag{22}$$

Before we search for ESE, we further impose for the sake of simplicity that

K_h is sufficiently high that $K_h > WP_q$, while K_l is greater than $W - C_m$ but smaller than WP_q: $W - C_m < K_l < WP_q < K_h$. Then, when group B's believed investment rate is assumed to be less than the threshold Π^* (i.e., $b_1 < \Pi^*$), the actual skill investment rate for the endogenously constructed group A, denoted by $\phi(a; b_1)$, is $\Pi_l, \forall a \in [0, \Pi^*)$, because $\Delta U(a, b_1; c) = 0$ and $R(a) = W(1-P_q) < C_m$. It is $\Pi'_h, \forall a \in [\Pi^*, 1]$, in which $\Pi'_h = (0.5\Pi_h + \Pi_l\eta)/(0.5 + \Pi_l\eta)\,(> \Pi_h)$, because only those whose cost set is (C_l, K_l) will "switch" from his or her own natural orientation B to type-A and $R(a) = W(1 - P_u) \geq C_m$, as seen in Panel A of the figure.

On the other hand, when group B's believed investment rate is assumed to be greater than the threshold (i.e., $b_2 > \Pi^*$), the actual skill investment rate for the endogenously constructed group A, $\phi(a; b_2)$, is $\Pi'_l, \forall a \in [0, \Pi^*)$, in which $\Pi'_l = (0.5\Pi_l - \Pi_l\eta)/(0.5 - \Pi_l\eta)\,(< \Pi_l)$, because only the population aggregate with its cost set $(C_l, -K_l)$ will "switch" from its natural orientation A to type-B and $R(a) = W(1-P_q) < C_m$, as noted in Panel B of the figure. It is $\Pi_h, \forall a \in [\Pi^*, 1]$, because $\Delta U(a, b_2; c) = 0$ and $R(a) = W(1 - P_u) \geq C_m$. Hence, we achieve the step functions of $\phi(a; b_1)$ and $\phi(a; b_2)$, which are depicted in Panel C of the figure, together with their benchmark curve $\phi(a; a)$, in which the believed investment rates for the two groups are equal: $\phi(a; a)$ is $\Pi_l, \forall a \in [0, \Pi^*)$, and $\Pi_h, \forall a \in [\Pi^*, 1]$.

Using these actual skill investment rate functions $\phi(a; b)$, we can compute the correspondence $\Gamma(b)$, which is a set of group A's believed skill investment rates that are self-confirmed by its actual skill investment rates, given that the other group B's believed skill investment rate is fixed as b: $\Gamma(b) = \{a | a = \phi(a; b)\}$. From the functions $\phi(a; b_1)$ and $\phi(a; b_2)$ and a 45-degree line in Panel C, we infer that when $b < \Pi^*$, $\Gamma(b) = \{\Pi_l, \Pi'_h\}$, while $\Gamma(b) = \{\Pi'_l, \Pi_h\}$ when $b \geq \Pi^*$. By symmetry, we also have $\Gamma(a) = \{\Pi_l, \Pi'_h\}$ when $a < \Pi^*$, and $\Gamma(a) = \{\Pi'_l, \Pi_h\}$ when $a \geq \Pi^*$. A set of ESE (Ω_{KL}) is a set of (a, b)s that satisfy both $a \in \Gamma(b)$ and $b \in \Gamma(a)$. Using the two correspondences $\Gamma(b)$ and $\Gamma(a)$ overlapped in

Panel D, we can identify four ESE: two trivial ESE, (Π_l, Π_l) and (Π_h, Π_h), and two nontrivial ESE, (Π'_l, Π'_h) and (Π'_h, Π'_l). Thus, knowing that $\Pi'_h > \Pi_h$ and $\Pi'_l < \Pi_l$, we prove that the inequality between endogenously constructed social groups can be greater than the inequality that can emerge between exogenously given groups: $|\Pi'_h - \Pi'_l| > |\Pi_h - \Pi_l|$.

Appendix B: Proof of Lemmas

Proof of Lemma 2:

Consider a very small $\delta > 0$ such that $a = b + \delta$. We can denote $\sigma^A(\delta; b)$ and $\Sigma^A(\delta; b)$, which are functions of δ given b, and consequently $\sigma^{A'}(\delta; b)$ and $\Sigma^{A'}(\delta; b)$, which are the corresponding partial derivatives with respect to δ. The slope of the $\phi(a; b)$ curve at a=b can be expressed as follows, using $\sigma^A(\delta; b)$ and $\Sigma^A(\delta; b)$,

$$
\begin{aligned}
\left.\frac{\partial \phi(a;b)}{\partial a}\right|_{a=b} &= \lim_{\delta \to 0} \frac{\phi(b+\delta;b) - \phi(b;b)}{\delta} \\
&= \lim_{\delta \to 0} \frac{\sigma^A(\delta;b)/\Sigma^A(\delta;b) - \sigma^A(0;b)/\Sigma^A(0;b)}{\delta} \\
&= \lim_{\delta \to 0} \left[\frac{[\sigma^A(\delta;b) - \sigma^A(0;b)]\Sigma^A(0;b)}{\delta} - \frac{[\Sigma^A(\delta;b) - \Sigma^A(0;b)]\sigma^A(0;b)}{\delta} \right] \\
&\quad \cdot \frac{1}{\Sigma^A(\delta;b) \cdot \Sigma^A(0;b)} \\
&= \frac{\sigma^{A'}(0;b) \cdot \Sigma^A(0;b) - \sigma^A(0;b) \cdot \Sigma^{A'}(0;b)}{\lim_{\delta \to 0} \Sigma^A(\delta;b) \cdot \Sigma^A(0;b)}
\end{aligned}
\tag{23}
$$

To compute this outcome, first, define $\Delta(\delta; b)$ as $\Delta(\delta; b) \equiv R(b + \delta) - R(b)$, which is a function of δ given b: $\Delta' \left(\equiv \frac{\partial \Delta(\delta;b)}{\partial \delta}\right) = R'(b + \delta)$. We also know $H'(k) \approx H'(0)$ for small enough k. Then, using Lemma 1 and Panels A and B of Figure 2, the fraction of agents who elect to be A-type and decide to be skilled, $\sigma^A(\delta; b)$, and its derivative, $\sigma^{A'}(\delta; b)$, are approximated by

$$
\begin{aligned}
\sigma^A(\delta;b) &\approx G(R(b)+\Delta) \cdot [0.5 + H'(0)\,(V_1(b+\delta) - V_1(b))] - 0.5\,H'(0)\,g(R(b))\,\Delta^2, \\
\sigma^{A'}(\delta;b) &\approx g(R(b)+\Delta)\,R'(b+\delta)[0.5 + H'(0)\,(V_1(b+\delta) - V_1(b))] \\
&\quad + G(R(b)+\Delta)\,H'(0)\,V_1'(b+\delta) - H'(0)\,g(R(b))\,\Delta\,R'(b+\delta),
\end{aligned}
$$

in which the last terms that are related to a triangle area in the figure, $-0.5\,H'(0)\,R'(b)\,\Delta^2$ and $-H'(0)\,g(R(b))\,\Delta\,R'(b+\delta)$, are added when $R'(b) > 0$ (as in Panel A), and dropped when $R'(b) < 0$ (as in Panel B). Similarly, the fraction of agents who elect to be A-type, $\Sigma^A(\delta; b)$, and its derivative, $\Sigma^{A'}(\delta; b)$, are approximated by

$$\begin{aligned}
\Sigma^A(\delta; b) &\approx 0.5 + H'(0)\,(V_0(b+\delta) - V_0(b)) + G(R(b)+\Delta)\,H'(0)\,\Delta - 0.5\,H'(0)\,g(R(b))\,\Delta^2, \\
\Sigma^{A'}(\delta; b) &\approx H'(0)\,V_0'(b+\delta) + G(R(b)+\Delta)\,H'(0)\,R'(b+\delta) + g(R(b)+\Delta)\,R'(b+\delta)\,H'(0)\,\Delta \\
&\quad - H'(0)\,g(R(b))\,\Delta\,R'(b+\delta).
\end{aligned}$$

Using the above approximations, we achieve the following results when $\delta = 0$:

$$\begin{cases}
\sigma^A(0; b) & \approx 0.5\,G(R(b)) \\
\sigma^{A'}(0; b) & \approx 0.5\,g(R(b))\,R'(b) + G(R(b))\,H'(0)\,V_1'(b) \\
\Sigma^A(0; b) & \approx 0.5 \\
\Sigma^{A'}(0; b) & \approx H'(0)\,V_0'(b) + G(R(b))\,H'(0)\,R'(b)
\end{cases} \tag{24}$$

Applying these results and $\lim_{\delta \to 0} \Sigma^A(\delta; b) = 0.5$ to equation (23), we have the following approximation, noting $R'(b) \equiv V_1'(b) - V_0'(b)$:

$$\left.\frac{\partial \phi(a; b)}{\partial a}\right|_{a=b} \approx g(R(b))\,R'(b) + 2\,H'(0)\,R'(b)\,G(R(b)) \cdot [1 - G(R(b))]. \tag{25}$$

QED.

Proof of Lemma 3:

Because $R(\pi_h) > R(y)$ for any $y \notin [\pi_h, \pi_h']$, we know $\phi(\pi_h; y) > G(R(\pi_h)) = \pi_h$ from Proposition 2. Hence, the curve $\phi(x; y)$ intercepts the 45 degree line at $x > \pi_h$, $\forall y \notin [\pi_h, \pi_h']$, implying $\Gamma(y)^h > \pi_h$ (Refer to Figure 3). Since this curve intercepts the benchmark curve $\phi(x; x)(= G(R(x)))$ at $x = y'$ such that $R(y) = R(y')$, $\Gamma(y)^h$ should be less than 1. Similarly, because $R(y) > R(\pi_l)$ for any $y \in [\pi_l, \pi_l']$, we know $G(R(\pi_l)) = \pi_l > \phi(\pi_l, y)$ from the proposition. Hence, the curve $\phi(x, y)$ intercepts the 45 degree line at $x < \pi_l$, $\forall y \in [\pi_l, \pi_l']$, implying $\Gamma(y)^l < \pi_l$. Since we know $\phi(0, y) > 0$ owing to $G(0) > 0$, $\Gamma(y)^l$ should be greater than 0. QED.

Proof of Lemma 4:

We can find a correspondence value x' nearby $\hat{x}$ such that $x' = \phi(x'; \hat{x} + \Delta)$, which means $x' \in \Gamma(\hat{x} + \Delta)$, as displayed in Appendix Figure 2. Given the slope of $\phi(x; y)$ at $(\hat{x} + \Delta, \hat{x} + \Delta)$ denoted by $\frac{\partial \phi(x;y)}{\partial x}\big|_{x=y=\hat{x}+\Delta}$ and the slope of $\phi(x; x)$ at $(\hat{x}, \hat{x})$, which equals $g(R(\hat{x}))R'(\hat{x})$, the correspondence value x' approximately satisfies the following condition, as conjectured from the figure:

$$x' - [\hat{x} + g(R(\hat{x}))\, R'(\hat{x})\, \Delta] \approx \frac{\partial \phi(x; y)}{\partial x}\Big|_{x=y=\hat{x}+\Delta} \cdot [x' - (\hat{x} + \Delta)]\,. \qquad (26)$$

The slope of the *correspondence curve* at a trivial ESE $(\hat{x}, \hat{x})$, denoted by $\Gamma'(\hat{\pi})$, is approximately equal to $\lim_{\Delta \to 0} \frac{x' - \hat{x}}{\Delta}$:

$$\Gamma'(\hat{x}) \approx \left[g(R(\hat{x}))\, R'(\hat{x}) - \lim_{\Delta \to 0} \frac{\partial \phi(x; y)}{\partial x}\Big|_{x=y=\hat{x}+\Delta} \right] \Big/ \left[1 - \lim_{\Delta \to 0} \frac{\partial \phi(x; y)}{\partial x}\Big|_{x=y=\hat{x}+\Delta} \right]. \qquad (27)$$

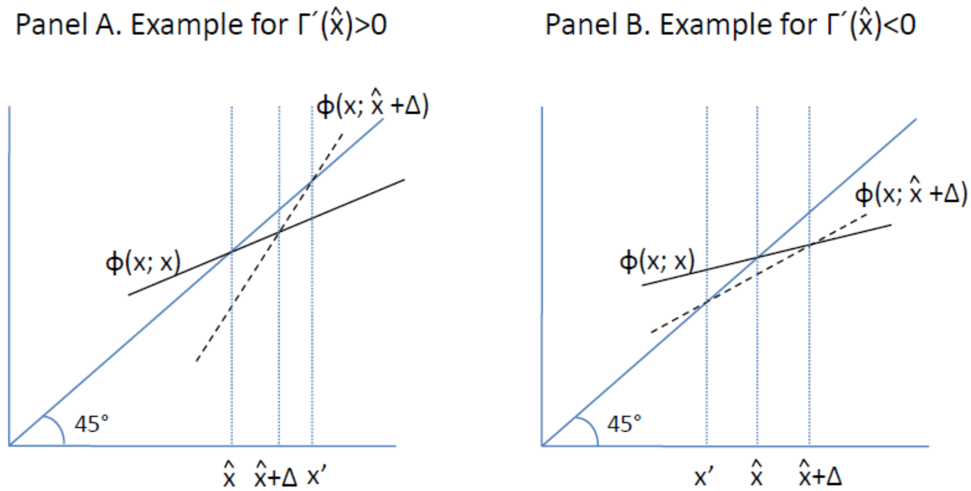
From Lemma 2 and $G(R(\hat{x})) = \hat{x}$, we have

$$\lim_{\Delta \to 0} \frac{\partial \phi(x; y)}{\partial x}\Big|_{x=y=\hat{x}+\Delta} = g(R(\hat{x}))R'(\hat{x}) + 2H'(0)R'(\hat{x})\hat{x}(1 - \hat{x}). \qquad (28)$$

Applying this result to equation (27), we achieve the given result for $\Gamma'(\hat{x})$. The examples for the positive $\Gamma'(\hat{x})$ and the negative $\Gamma'(\hat{x})$ are depicted separately in Panels A and B of Appendix Figure 2. QED.

Proof of Lemma 5:

Based on the following three elementary facts, we can directly derive the results from Lemma 4: (1) $R'(\hat{x})$ is positive for any PSE $\hat{x}$ because we assume $\pi_i < \bar{\pi}, \forall i \in \{l, m, h\}$; (2) The slope of the $\phi(a; a)$ curve at $a = \pi_m$ is always greater than one: $g(R(\pi_m))R'(\pi_m) > 1$; (3) The slope of the $\phi(a; a)$ curve at $a = \pi_h$ (or π_l) is smaller than one: $0 < g(R(\hat{x}))R'(\hat{x}) < 1, \forall \hat{\pi} \in \{\pi_h, \pi_l\}$. (You



Appendix Figure 2. Slope of Correspondence at Trivial ESE

may refer these facts quickly from Figure 3.) QED.