Mergers and Collusion with Horizontal Subcontracting

Preliminary version

Jan Bouckaert[1] and Geert Van Moer[2]

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Abstract

Horizontal subcontracting—outsourcing activities to competitors—stifles competition to the extent that subcontractors capture the subcontracting surplus. In our framework, price-competing firms choose the subcontracting terms to maximize profits. A merger does *not* create cost savings at the industry level because of efficient subcontracting before the merger. It does, however, reduce the merged parties' *in-house* costs. Merging is therefore always (i) profitable because it credibly limits the amount paid to subcontractors, and (ii) price-reducing as it decreases outsiders' profits from subcontracting. Furthermore, subcontracting makes collusion more difficult to sustain, because the static Nash-equilibrium profits after deviation are considerable.

Keywords: horizontal subcontracting, mergers, collusion.

JEL-code: D43, L13, L14, L41

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[1] Department of Economics, University of Antwerp. jan.bouckaert@uantwerpen.be

[2] PhD Fellow of the Flanders Research Foundation, Department of Economics, University of Antwerp. geert.vanmoer@uantwerpen.be

1. Introduction

Horizontal subcontracts are contracts between competing firms, where a firm (the contractor) outsources the production of goods or services to its competitors (the subcontractors). We study the incentives for horizontally subcontracting firms to merge and to engage in collusive practices.

Public procurements and concessions are prominent examples, because the successful tenderer frequently horizontally subcontracts part of the main contract to competing bidders.[3] As an example, for highway construction procurements in California, firms that were bidding as prime contractor have been selected by the winning bidder as their subcontractor (Huff (2013) and Marion (2015)). Public procurement is considerable and is estimated to represent 13-20% of GDP worldwide (OECD, 2013).

Many firms in the private sector engage in horizontal subcontracting as well. Businesses that compete with each other to serve final consumers often rely on subcontracts to cost-efficiently allocate the production *among* their downstream competitors. For example, power generating companies need not generate in-house if they can reduce costs by shifting generation from their expensive gas-fired plants towards their downstream rivals' cheap wind mills. Horizontal subcontracting is also common practice in the financial sector for underwriting services. These services offered by financial institutions guarantee borrowers to raise sufficient capital to fund e.g. loans, leveraged buy-outs or initial public offerings. Potential underwriters compete with each other to guarantee the borrower an amount of cash in a price range. Winning underwriters (lead arrangers) typically pass part of their underwriting commitment to multiple sub-underwriters (co-arrangers), including the non-winning underwriters, to attract enough investors. This practice increases the prospects for successful funding and, importantly, reduces the underwriting costs by diversifying risks. Syndicated loans e.g. are becoming increasingly important for non-financial businesses in the US and Europe and nowadays represent higher volumes than the public issuing of corporate debt and equity together (Drucker and Puri (2007), Sufi (2007), and Ferreira and Matos (2012)).

A number of theoretical papers explain horizontal subcontracting—the selling and buying positions between rivalry firms—by asymmetries or convexities in their cost functions. Two opposing findings have been distinguished. First, signing subcontracts shifts production from one firm to the other resulting in production efficiencies. Second, horizontal subcontracting may stifle competition. The

[3] Horizontal subcontracting should be distinguished from instances where the successful bidder is obliged to contract a certain minimum part of the main contract from *non*-competing bidders. For example, affirmative action programs with subcontractor goals towards non-competing disadvantaged business enterprises may contain an important part of public procurement rules. Such *vertical* subcontractor goals are e.g. implemented by the US Department of Transportation and by the European Commission for the award of contracts in the fields of defense and security (Directive 2009/81/EC). Marion (2009) estimates that California's Proposition 209, *prohibiting* affirmative action considerations, reduced prices for state-funded projects by 5.6% relative to Federal state-funded projects. Similarly, Moretti and Valbonesi (2015) show that for Italian public procurements, bidders with *optional* subcontracting submit lower prices as compared to firms with a subcontracting obligation.

reasoning is that serving the market by setting the most competitive price comes at the cost of foregoing subcontracting revenues as shown by Kamien, Li, and Samet (1989) and Spiegel (1993).[4] Marion (2015) finds empirical support for both effects. Production efficiencies can be observed because firms that also participate as subcontractors offer lower bids. Firms, however, bid less aggressively when the subcontracting opportunities improve. He finds that both effects balance each other, though concludes that total surplus could suffer from any restriction by antitrust authorities towards horizontal subcontracting.[5]

Horizontal subcontracting is regarded by competition authorities as one form of horizontal collaboration (US) or cooperative agreement (EU) between competing firms. In general, the forms of such *horizontal* collaboration are multiple and range from strategic alliances, joint ventures and information exchange, to joint production agreements like specialization, among many others. Antitrust agencies across the world provide businesses with safe harbors by devoting specific Guidelines regarding collaborations or agreements between competitors.[6] Our paper investigates mergers and collusion in industries with horizontal subcontracting. We present our main results and contributions to the literature by the following instructive example.

Example setup

There are four firms that can produce up to 2 units. Their cost to produce each unit is depicted in table 1. To produce the first unit, each firm i incurs zero costs. The marginal cost equals 3 for the second unit.[7]

Firm i	1st unit	2nd unit
Firm a	0	3
Firm b	0	3
Firm c	0	3
Firm d	0	3

Table 1: in-house marginal costs of four symmetric firms

[4] Other contributions are Gale et al. (2000) who study the bidding effects from sequential procurements and Haile (2003) who considers equilibrium bidding with option values to buy or sell in the secondary market.

[5] Related work by Huff (2012) shows for highway procurement auctions in California that horizontal subcontracts, by raising rival's opportunity costs, result in 6% higher prices compared to subcontracts that result from cost-efficiencies.

[6] See e.g. the US Antitrust Guidelines for Collaborations between competitors (2000), Collaboration Guidelines in Canada (2009), and the EC Guidelines on Horizontal Cooperation Agreements (2011).

[7] Our model presented in the following sections provides the results for any number of firms characterized by strictly convex costs, that compete for consumers with downward-sloping demand.

A procurer or consumer has a fixed demand for two units and organizes a first-price auction. The price refers to the price p *per unit*. The firm that charges the lowest price wins and sells two units at that price. In case of a tie, one of the lowest-price firms is selected, each with equal probability, to be the winner that serves the market.

Before the auction, each firm declares one subcontractor, i.e. the firm it will call upon for subcontracting services after winning the auction.[8] Since all firms are symmetric, we suppose that each of the four firms is a subcontractor for one of the other firms. So, for example, firm a subcontracts for firm b, firm b subcontracts for firm c, and so on. Suppose that the subcontracting terms maximally favor the subcontractors. To be precise, the subcontractor makes a take-it-or-leave-it offer to the winning firm. The subcontractor allows the winning firm to contract one unit in return for a payment equal to 3, the winner's opportunity cost of producing a second unit in-house.[9] Note that the subcontract is efficient in the sense that industry costs are minimized after subcontracting. The winner produces 1 unit at zero cost and the subcontractor also produces 1 unit at zero cost.

We will argue that the equilibrium is for each firm to charge the price p_i^* at which each firm i is indifferent between winning and losing. The winning firm gets a revenue of $2p_i^*$ in return for the commitment to deliver two units. The winner produces one unit in-house at zero costs, and pays 3 to call upon his subcontractor that delivers the second unit. The profits of the winner therefore equal $2p_i^* - 3$. The losers earn 3 in the event that they are called upon (with probability 1/3 conditional upon losing), i.e. if their declared contractor is chosen to win the auction. Otherwise they earn zero profits. The profits of losing therefore equal $\frac{1}{3}3 = 1$. The price at which each firm is indifferent between winning and losing satisfies $2p_i^* - 3 = 1$ and equals 2. If the price equals 2, each firm expects to earn a profit of 1.

We proceed by checking whether the price of $p_i^* = 2$ is indeed an equilibrium. By charging a higher price, a firm would lose the auction for sure and earn an expected profit of 1. Also, a firm cannot gain from charging a lower price. It would then win for sure but earn a price $p < 2$. Since its costs would equal 3, profits $2p - 3$ would be smaller than 1.[10]

[8] Miller (2014) reports that, in the Californian construction industry, the Subcontracting and Subletting Fair Practice Act requires that firms declare their subcontractor *before* competing for the contract.

[9] Our framework will analyze how firms choose the subcontracting terms endogenously so as to maximize profits.

[10] Note that, if firms would not be able to subcontract, the equilibrium price would equal the average cost, 1.5.

Profit-maximizing subcontracts

In the example, we only look at those subcontracts where the subcontractors are able to charge the contractor's opportunity cost of in-house production. In other words, all the *surplus from subcontracting*, i.e. the cost-reductions achieved by the subcontract, is captured by the subcontractors. Suppose more generally that, before competing, each subcontractor offers his contractor the option to buy 1 unit at the price of $\sigma 3$ with $0 \leq \sigma \leq 1$. Share σ is then interpreted as the share of surplus that goes to the subcontractor, whereas the remaining share $1-\sigma$ goes to the contractor. We will claim that the equilibrium price depends on share σ. Indeed, by winning, a firm now earns $2p-\sigma 3$. By losing, a firm earns $\frac{1}{3}\sigma 3$. The price at which each firm is indifferent between winning and losing satisfies $2p_i^* - \sigma 3 = \frac{1}{3}\sigma 3$ and equals $p_i^*(\sigma) = \sigma 2$. Firms want to set σ optimally at σ^* so as to maximize profits. Since the industry incurs zero production costs, this is achieved by maximally raising the equilibrium price. Firms implement $\sigma^* = 1$ by signing option contracts that allow the contractor to buy 1 unit from the subcontractor in return for a payment equal to 3.

More generally, the equilibrium price depends on how firms share the cost-reductions — the surplus from subcontracting — between the contractor and the subcontractors. Our analysis shows how firms use this share, i.e. the subcontracting terms, as a device to increase profits. In our example, firms can raise the equilibrium price by increasing subcontracting costs. Ideally, firms would like to implement subcontracting costs that outweigh the contractor's in-house opportunity cost. However, since the contractor would then prefer not to subcontract, firms choose to raise subcontracting payments up to the level of the contractor's opportunity costs.

The following sections will show that firms do not always want to raise the price as much as possible. Indeed, the industry may prefer an equilibrium price that is not *too* high, in order to preserve sufficient demand. By giving some surplus of subcontracting to the contractor, firms then realize an equilibrium price equal to the monopoly price.

The effects of a merger

Suppose there is a merger between firm c and d. We then have the following table 2. We make a distinction between the outside —*high-cost*— firms h, here firm a and b, and the merged —*low-cost*— firm l, here $\{c,d\}$.

		1st unit	2nd unit	3rd unit	4th unit
Firm h	Firm a	0	3		
	Firm b	0	3		
Firm l	Firm $\{c,d\}$	0	0	3	3

Table 2: in-house marginal costs after the merger

By merging, firm c and d bring together their production capabilities without changing them. In particular, both firms reduce costs by allocating production equally across both production plant portfolios. Firm l can now produce two units in-house at zero costs. Following Farrell and Shapiro's (1990, p.112) terminology, the merger reduces in-house costs by the possibility for l to rationalize production, but generates "no synergies" arising from e.g. management efficiencies or learning effects.[11]

We will argue that, in equilibrium, each firm h offers the price at which it is indifferent between winning and losing against l. We denote that price by p^*_{h-l}. Firm l wins in equilibrium by slightly undercutting p^*_{h-l}.

By winning, firm h earns $2p$ but incurs a cost of 3 because it subcontracts the second unit. So, profits from winning amount to $2p-3$. By losing against l, firm h earns zero profits. The reason is that l can produce both units in-house at zero cost, so that it does not call upon a subcontractor. The price at which h is indifferent between winning and losing against l satisfies $2p^*_{h-l}-3=0$ and therefore equals $3/2$. Firm l charges a price just below p^*_{h-l}. It wins and produces both units in-house at zero costs. Its profits are $2p^*_{h-l}=3$. Note that merging was profitable because pre-merger, firm c and d each earned only 1.

To see why this is an equilibrium, firm h cannot do better by charging a lower price $p<3/2$ because it would make a loss equal to $2p-3<0$. By charging a higher price, firm h would lose for sure, thereby earning the same profits. Also, the low-cost firm cannot do better by losing. In particular, firm h would win, and no matter who acts as a subcontractor for whom, firm l would then never be able to earn more than 3. Indeed, firm l could never charge more for subcontracting services than firm h's

[11] Synergies in Farrell and Shapiro's terminology would e.g. be reflected in a lower marginal cost to produce the third and fourth unit.

opportunity cost of in-house production. Finally, firm l can also not gain from charging a lower price of $p < 3/2$, because it would then earn only $2p < 3$.

We observe that, after the merger, the equilibrium price is lower than before the merger. Importantly, this is *not* the result of a cost reduction at the *industry* level. Firms already produce at zero costs before the merger because of the possibility to subcontract efficiently. By consolidating two production plant portfolios, the merger does not affect the cost function at the industry level.

The merger does, however, reduce the *in-house* costs of the merged firm, which has two important consequences. First, the merged firm no longer needs to rely on subcontracting to deliver the two units. The merger thereby achieves a substantial cost advantage for the merged firm, which explains why the merger is profitable. Second, since the outside firms can no longer profit from subcontracting after the merger, they are willing to compete more fiercely to conquer the market. The outsiders' profits *decrease* as a result of the merger, while the consumers benefit because the equilibrium price goes down.[12]

Without subcontracting, if insufficient synergies arise between the merging parties, prices are typically expected to raise, as shown by Davidson and Deneckere (1985) for price competition and by Farrell and Shapiro (1990) when firms compete in quantities. Consumer welfare, the typical standard used by antitrust authorities to judge mergers, then goes up only if the merging entities enjoy sufficient synergies.

Our results show that any merger in industries with efficient horizontal subcontracting is profitable, results in *lower* final prices and enhances total welfare. We do not require that a merger improves the production possibilities of the industry. Instead, the key reason is that mergers reduce the outsiders' profits from subcontracting, encouraging them to compete more fiercely for the market. Put differently, an increase in market concentration resulting from a merger improves competition by reducing the surplus from subcontracting.

Finally, it is instructive to consider the follow-up merger between a and b. There are then two firms in the market with zero in-house costs. Fierce price competition results in an equilibrium price of zero. This suggests that it is better for consumers to have a concentrated market with two large price-competing firms, each with low in-house costs, rather than a large number of small firms that rely heavily on subcontracting to accomplish their commitments.

[12] The reader may check that, since the merger has already fully eliminated the surplus from subcontracting in the example, a follow-up merger between $\{c,d\}$ and b would not further increase profits or decrease the price. This suggests that the profitability and the price-decreasing effect of the merger directly follow from the reduction of subcontracting surplus caused by the merger.

Tacit collusion

Let us now consider the possibility for firms to collude. Suppose there is a maximal willingness to pay for both units, e.g. a price above which demand is zero. Since industry costs are zero, the industry profits are maximized if all firms charge that price. We therefore denote that price by p^M, the monopoly price. Let us suppose here that p^M exceeds the Nash-equilibrium price of the static game, so $p^M > p_i^*(\sigma)\big|_{\sigma=\sigma^*=1} = 3$.

The collusive agreement is for each firm to charge p^M. The industry then earns $2p^M$ profits because industry costs are zero. If one firm deviates, firms revert to the Nash-equilibrium of the static game forever. Even though the industry profits of colluding firms do not depend on the subcontracting terms, they *do* affect the incentive for a firm to deviate. We will address the question how colluding firms choose the subcontracting terms, σ_c, as a device to maximally sustain collusion. So, firms choose $\sigma_c = \sigma_c^*$ so as to minimize the common critical discount factor.

Indeed, a deviating firm's profit depends on σ_c. In particular, if a firm deviates, it conquers the market by charging a price just below p^M, after which it contracts 1 unit. Hence, the deviating firm earns $2p^M - \sigma_c 3$. Colluding firms therefore maximally discourage deviation by setting $\sigma_c = \sigma_c^* = 1$, so that deviation is most costly. So, in our example, both colluding *and* competing firms have a preference for subcontracting terms that maximally favor the subcontractor.

Despite the possibility for firms to choose σ_c endogenously, our analysis will show that subcontracting makes collusion harder to sustain. Firms are not impressed by the threat of reverting to the Nash-equilibrium of the static game after deviation. Indeed, as we have seen, that Nash-equilibrium still yields considerable profits equal to 1 for each firm.

Section 2 describes the model and offers the equilibrium analysis for symmetric firms. Section 3 studies the competitive and welfare effects from merging. Section 4 investigates firms' incentives to collude. Finally, section 5 presents and discusses our concluding remarks.

2. The model and equilibrium analysis

The model – There are $N \geq 2$ risk-neutral, symmetric price-competing firms that produce homogeneous goods.[13] Market demand $Q(p)$ can originate from non-strategic final consumers, competitive retailers or a procurer and depends on p, the lowest price in the market. We assume that $Q(p)$ is differentiable, cuts both axes and is strictly downward sloping so that

$$\text{(A1)} \qquad Q'(p) < 0.$$

We denote the price at which demand is zero by $\bar{p}$ such that $Q(\bar{p}) = 0$.

Firms choose prices in stage one. The lowest-price firm must serve demand $Q(p)$. If several firms submit the lowest price, one of them is randomly selected, each with equal probability, to be the winner that serves the market.[14] After prices are chosen, stage-one outcomes are common knowledge.

Each firm i's cost function $C_i(Q)$ is twice differentiable and satisfies assumption

$$\text{(A2)} \qquad C_i(0) = 0,\ C_i'(0) < \bar{p},\ C_i'(Q) > 0,\ \text{and}\ C_i''(Q) > 0,$$

meaning that production is profitable and that marginal costs are positive and strictly increasing.[15] Costs are verifiable and common knowledge.[16]

Without subcontracting, the winning firm is the only firm that produces. Since all other firms earn zero, there is fierce competition to serve the market. The no-subcontracting equilibrium price, p^{ns}, equals the average cost

$$\text{(1)} \qquad p^{ns} = C_i\left(Q(p^{ns})\right) \big/ Q(p^{ns})$$

and all firms earn zero profits in equilibrium.[17]

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[13] The assumption of homogeneous goods is intuitive because subcontracting requires that producers are substitutable.

[14] Some settings are better described by the alternative assumption that each lowest-price firm serves an equal portion of market demand. We discuss the implications in the discussion section.

[15] Industries where horizontal subcontracting takes place are often characterized by convex variable costs. For instance, power generating firms first dispatch their lowest marginal cost units. If more generation is required, they must turn to more expensive generation units or buy from less expensive competitors. As another example, construction firms often lack the appropriate capacity or scale to manage very high workloads in a low-cost manner.

[16] Subcontracting is often studied in a complete information setting (see Kamien et al., 1989 and Spiegel, 1993). The advantage is that firms can then easily identify the subcontracting opportunities. Haile (2003) studies asymmetric information in auctions with resale. For incomplete contracts we refer to Miller (2014).

Of course, it may not be cost-efficient to have only one firm producing. Firms can reduce production costs by reallocating their production using subcontracts. The reason why is that subcontracts can shift production from one firm's most expensive production facilities to another firm's less expensive production facilities.

We therefore allow firms to implement subcontracts in stage two. The minimal industry's (sum of all firms) marginal costs after subcontracting, $C_{IND}^{'}(Q)$, is the horizontal summation of all firm-level marginal cost functions. It follows that $C_i(Q) > C_{IND}(Q)$ (a single firm's costs exceed the industry costs) and $C_i^{'}(Q) > C_{IND}^{'}(Q) > 0$ (a single firm's marginal costs exceed the industry marginal costs).[18]

If firm i would serve market demand Q, the surplus from subcontracting $S_i(Q)$ equals the cost-reductions made possible from subcontracting

(2) $$S_i(Q) \equiv C_i(Q) - C_{IND}(Q) > 0,$$

the difference between firm i's in-house costs and the minimal industry's production costs $C_{IND}(Q)$.[19] We assume that firms take advantage of all potential surplus from subcontracting. In other words, they use subcontracts to allocate production cost-efficiently, so that they only incur the minimal industry production cost.[20] The surplus from subcontracting is strictly positive, $S_i(Q) > 0$, because marginal costs are upward sloping (see (A2)). Since $C_i^{'}(Q) > C_{IND}^{'}(Q)$, we can write that the surplus from subcontracting is increasing, or

(3) $$S_i^{'}(Q) > 0.$$

We also assume that the average surplus from subcontracting is not decreasing, or

(A3) $$\frac{\partial\left[S_i(Q)/Q\right]}{\partial Q} \geq 0,$$

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[17] Note that (A1) and (A2) guarantee that p^{ns} exists and is unique.

[18] Note that, since all firms are characterized by the same upward sloping marginal cost function, it is efficient to allocate production equally across all firms. So, costs are minimized by letting each firm produce Q/N units. We then have $C_{IND}^{'}(Q) = C_i^{'}(Q/N)$ and $C_{IND}(Q) = NC_i(Q/N)$.

[19] The surplus from subcontracting is earlier modelled by e.g. Spiegel (1993) and Gale et al. (2000).

[20] If subcontracts are not cost-efficient, the industry's costs after subcontracting is not the minimal production cost possible. Such subcontracts can nonetheless be cost-reducing, though, so that surplus from subcontracting can still be positive. However, we would have the unattractive feature that firms could gain from renegotiating.

which is always satisfied if $C_i^{''}\left(Q\right) \geq C_{IND}^{''}\left(Q\right)$.[21]

The industry profits equal the difference between revenues and costs,

$$\pi_{IND}(p) \equiv \sum_{i \in N} \pi_i(p) = pQ(p) - C_{IND}\left(Q(p)\right), \tag{4}$$

where we denote industry profits by π_{IND} and each firm i's profits by π_i. We assume that π_{IND} is strictly concave by letting the second derivative with respect to price satisfy

$$2Q^{'}(p) + pQ^{''}(p) - C_{IND}^{''}(Q(p))\left(Q^{'}(p)\right)^2 - C_{IND}^{'}(Q(p))Q^{''}(p) < 0. \tag{A4}$$

There is then a unique price that maximizes industry profits, which we denote by monopoly price $p^M \equiv \arg\max_p \pi_{IND}\left(p\right)$ resulting in monopoly profits $\pi_{IND}^M \equiv \pi_{IND}\left(p^M\right) > 0$.[22]

There is a share σ of realized surplus from subcontracting that is appropriated by the selling firms (subcontractors), with $0 \leq \sigma \leq 1$. The remaining share $1-\sigma$ goes to the buyer (contractor).[23] We meet subgame perfectness by imposing that $0 \leq \sigma \leq 1$. To see why, in stage two, the contractor would never be willing to buy from the subcontractor if $\sigma > 1$. In-house production would then outperform subcontracting. Similarly, the subcontractor would not be willing to sell if $\sigma < 0$.

One possibility is that firms sign subcontracts *ex post* when they are implemented in stage two. Then, the terms of trade at which the contractor buys from the subcontractor would be determined by an *exogenous* share σ.

As we will see, the price chosen by firms in stage one depends on share σ. Firms can accordingly shape each other's behavior, and hence increase profits, by signing subcontracts *ex ante*, i.e., by

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[21] Examples include homogeneous cost functions of the form $C_i\left(Q\right) = \alpha Q^\beta$ or exponential costs like $C_i\left(Q\right) = \alpha\beta^Q$ with $\alpha > 0$ and $\beta > 1$.

[22] Since (A2) ensures that production is profitable, we know that π_{IND}^M must be strictly positive.

[23] Share σ captures a wide range of factors or institutions that determine the subcontractors' bargaining or market power. For duopoly competition, Kamien et al. (1989) interpret the subcontractor as a Stackelberg leader if $\sigma = 1$. The symmetric Nash-bargaining solution corresponds to $\sigma = 0.5$.

choosing how to share the surplus from subcontracting *endogenously* in stage zero.[24] We therefore consider *optimal* subcontracts, meaning that firms set $\sigma = \sigma^*$ so as to maximize industry profits.[25]

Firms can implement $\sigma = \sigma^*$ e.g. by setting up a subcontracting platform or exchange, in which they determine the rules of trading so as to favor the contractor —the buyer— or the subcontractors —the selling firms— as they wish. To reduce σ, firms can let the *buyer* move first by making a take-it-or-leave it offer. Instead, to increase σ, firms can give the first move to a *seller* who determines the subcontracting terms. Price caps or floors are other instruments that could shape the subcontracting terms in practice. Finally, ex ante, firms could also sign option contracts with each other that fix at what terms the contractor can buy from the subcontractors.

Remark that firms can *not* directly coordinate on prices. They can choose share σ in stage zero, but take into account that the stage-one price equilibrium must be subgame perfect.

Equilibrium analysis – The equilibrium analysis proceeds in three steps. Step 1 investigates at what prices firms prefer to win rather than to lose and vice versa. This is important because, in any equilibrium, a losing firm should prefer not to win instead and a winning firm should not prefer to lose instead. Step 2 studies under what circumstances the price at which firms are indifferent between winning and losing is an equilibrium. Step 3, finally, examines the equilibrium price if firms sign optimal ex ante subcontracts.

Step 1. The profits of the winning firm equal

$$W_i\left(p\right) \equiv pQ(p) - C_i\left(Q(p)\right) + (1-\sigma)S_i\left(Q(p)\right). \tag{5}$$

If a firm wins (by charging the lowest price or, in the event of a tie, by being randomly selected) it must serve the market. The firm's revenues from serving demand are captured by the first term. The second term reflects its costs without subcontracting. The last term represents the share $1-\sigma$ of the surplus from subcontracting that is appropriated by the winning firm.

If a firm loses (by not charging the lowest price or, in the event of a tie, by not being randomly selected) it does not serve the market. Nonetheless, the losing firm can earn positive profits because it can extract surplus from subcontracting. Since all $N-1$ subcontractors are symmetric, they all capture

[24] Ex ante contracts are often used in the contexts of procurements. Bidders are sometimes required to have contracts with subcontractors in order to guarantee that the winner will be able to deliver (see e.g. Marion, 2015).

[25] Firms that are not symmetric, e.g. as a result of a merger, may have different preferences with regard to σ. We believe it is most appropriate to assume that firms choose the σ^* that maximizes joint profits, because we use our framework to study mergers and collusion. We rule out the possibility for firms to transfer payments when setting σ, because this practice would probably not be allowed by competition authorities.

an equal portion $1/(N-1)$ of the surplus from subcontracting that goes to the subcontractors.[26] Therefore, the profits of a losing firm equal

(6) $$L_i(p) \equiv \frac{1}{N-1}\sigma S_i(Q(p)).$$

We denote the price at which firms are indifferent between winning and losing by $p_i^*(\sigma)$ such that

(7) $$W_i\left(p_i^*(\sigma)\right) = L_i\left(p_i^*(\sigma)\right)$$

By using (5), (6) and (2), we can write $W_i\left(p_i^*(\sigma)\right) - L_i\left(p_i^*(\sigma)\right) = 0$ as

(8) $$\underbrace{p_i^*(\sigma)Q(p_i^*(\sigma)) - C_{IND}\left(Q(p_i^*(\sigma))\right)}_{\pi_{IND}\left(p_i^*(\sigma)\right)} - \frac{N}{N-1}\sigma S_i\left(Q(p_i^*(\sigma))\right) = 0.$$

Notice from (4) that the first two terms equal $\pi_{IND}(p_i^*(\sigma))$. We get

(9) $$\pi_{IND}\left(p_i^*(\sigma)\right) = \frac{N}{N-1}\sigma S_i\left(Q(p_i^*(\sigma))\right).$$

By rearranging (8), we also know that $p_i^*(\sigma)$ should equal

(10) $$p_i^*(\sigma) = \underbrace{\frac{C_{IND}\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}}_{\text{avarage costs}} + \underbrace{\frac{1}{Q(p_i^*(\sigma))}\frac{N}{N-1}\sigma S_i\left(Q(p_i^*(\sigma))\right)}_{\text{markup}},$$

the sum of the average industry costs and a markup. We assume that, for a sufficiently high price $\lim_{p\to\bar{p}} p$ and hence sufficiently low market demand $\lim_{p\to\bar{p}} Q(p)$, firm i prefers to win rather than lose.

(A5) $$\lim_{p\to\bar{p}} W_i(p) > \lim_{p\to\bar{p}} L_i(p) \text{ for } 0 \le \sigma \le 1.$$

The purpose of (A5) is technical. It guarantees that, for any σ, the price given by (10) defines a unique price $p_i^*(\sigma) < \bar{p}$. Assumption (A5) ensures the existence of an equilibrium in pure strategies.

[26] The assumption here is that symmetric firms earn the same subcontracting revenues. The ex ante subcontract does not favor one subcontractor over the other.

Lemma 1. *For any* $0 \le \sigma \le 1$, *there exists a unique price* $p_i^*(\sigma) < \overline{p}$ *at which each firm is indifferent between winning and losing. A firm wants to win rather than to lose if and only if the price exceeds* $p_i^*(\sigma)$.

The proof is in the appendix.

The intuition behind Lemma 1 is as follows. By losing, a firm acts as subcontractor. As the price increases, demand goes down so that the amount that is subcontracted decreases. Hence, a firm profits less from losing –and therefore subcontracting—as the price goes up. By winning, in contrast, a firm obtains revenues from the price that consumers pay. As a result, a firm wants to win only if the price is sufficiently high. From Lemma 1, we know that profits from winning and losing intersect exactly once at a price strictly below $\overline{p}$.[27]

Step 2. In equilibrium, firms cannot profitably deviate by charging another price. Since the profits of a winning and losing firm are continuous functions of the price, any equilibrium price must satisfy that each firm i is indifferent between winning and losing. So, price $p_i^*(\sigma)$ is the only candidate equilibrium price. Since the equilibrium price must satisfy (10), we see that, if the number of firms N goes to infinity, the markup does *not* converge to zero whenever there is surplus from subcontracting and $\sigma > 0$. Lemma 2 checks that, if $p_i^*(\sigma) \le p^M$, it is indeed an equilibrium.

Lemma 2. *If* $p_i^*(\sigma) \le p^M$*, it is the unique equilibrium price.*

The proof is in the appendix.[28]

Step 3. The third step of the equilibrium analysis examines how firms sign those subcontracts that maximize industry profits. Since symmetric firms all have the same profit function, they all have the same preference with regard to σ. Hence, for symmetric firms, maximizing industry profits coincides with maximizing firm-level profits.

The following proposition characterizes the equilibrium price with optimal (profit-maximizing) ex ante subcontracts. In particular, we either have that

[27] Note that they also intersect at $\overline{p}$, the price at which firms sell zero so that the winner as well as the losers earn zero. Assumption (A2) guarantees that production is profitable, so that $\overline{p}$ is not an equilibrium, because a firm can profitably charge a lower price instead.

[28] If instead we would look at a *second*-price auction, it is easy to see that $p_i^*(\sigma)$ is always an equilibrium. In particular, from Lemma 1 we know that, at $p_i^*(\sigma)$, firms would not gain from winning for sure or losing for sure.

(i) the equilibrium price equals the monopoly price $p_i^*\left(\sigma^*\right)=p^M$ and firms allocate share $0<\sigma^*\leq 1$ to the subcontractors, or

(ii) the equilibrium price is lower than the monopoly price $p_i^*\left(\sigma^*\right)<p^M$ and firms allocate share $\sigma^*=1$ to the subcontractors.

Proposition 1. *If symmetric firms sign optimal ex ante subcontracts, the unique equilibrium price is*

$$p_i^*\left(\sigma^*\right)=\min\left\{p_i^*(1), p^M\right\}.$$

We provide the proof in the appendix.

The intuition can be seen using (10), which shows that markups are high if the surplus from subcontracting is high. So, if the opportunity cost of in-house production is sufficiently expensive as compared to the efficient industry costs, the subcontracting terms can be chosen to let firms choose monopoly price levels in stage one. To be clear, the monopoly price equilibrium is not a special case, to the contrary; the monopoly price will always result whenever there is sufficient surplus from subcontracting.

Conversely, if the surplus from subcontracting is low, firms would like to implement subcontracting costs that outweigh the opportunity costs in order to maximize industry profits. However, since the contractor then prefers the "make"-option to the "buy"-option, firms optimally determine the subcontracting terms by setting $\sigma^*=1$. By doing so, the optimal ex ante subcontract raises the subcontracting payments to the level of the contractor's opportunity costs.

The effect of optimal subcontracts on consumers - We use the symmetric model to study under what circumstances subcontracts are good for consumers. The effect on consumers crucially depends on share σ, the share of the surplus from subcontracting that goes to the subcontractors.

Proposition 2*: subcontracts reduce prices if and only if sufficient surplus from subcontracting goes to the contractor, or*

$$p_i^*(\sigma)<p^{ns}\Leftrightarrow\sigma<\frac{N-1}{N}.$$

We provide the proof in the appendix.

The cutoff value for σ is $(N-1)/N$, which equals 0.5 for a duopoly and approaches 1 as the number of firms increases. The intuition of proposition 2 is consistent with the literature. The first observation relates closely to Kamien et al. (1989) and is that consumers are more likely to benefit from subcontracts if the subcontractors have little bargaining or market power. The intuition is that, then, subcontractors earn little profits and are therefore willing to compete fiercely to serve consumers, which leads to lower prices. The second observation relates to Haile (2003) and is that the requirement on σ becomes less stringent as the number of firms goes up. Each subcontractor then captures only a smaller part of the surplus from subcontracting. A subcontractor's profits decrease, so that he is more willing to compete for consumers by charging lower prices.

The implications of proposition 2 are striking if we relate them to our proposition 1 on *optimal* subcontracts. If firms optimally choose $\sigma^*=1$, we have $\sigma^*=1>(N-1)/N$ so that subcontracts always make consumers worse off. The only other possibility is that the price equals the monopoly level $p_i^*\left(\sigma^*\right)=p^M$ with $0<\sigma^*\leq 1$. More specifically, subcontracts are only good for consumers if $0<\sigma^*<(N-1)/N$. Then the scenario without subcontracting would have led to a price above p^M, because average in-house costs would have exceeded p^M.

3. The effects of a merger

This section takes the symmetric model from section 2 as a starting point and investigates (i) the effect of a merger on the equilibrium price and (ii) firms' incentives to merge.

Before the merger, there are $N\geq 3$ symmetric firms in the market. We use the analysis and notation of the symmetric model in section 2. Each firm i is characterized by cost function $C_i(Q)$. The unique equilibrium price equals $p_i^*\left(\sigma^*\right)$, satisfies (10) and proposition 1. Each firm earns profits $\pi_i\left(p_i^*\left(\sigma^*\right)\right)$ that satisfy (9).

After two firms merge, there are only $N-1$ firms in the market, one merged firm and $N-2$ outside firms. The merged firm has lower in-house costs because it has access to the production facilities of two pre-merger firms. Hence, we need to relax the assumption of symmetry. The post-merger analysis uses the character "$\tilde{\ }$" in order to distinguish the notation from the symmetric model that applies pre-merger.[29]

We first investigate firms' costs after the merger, after which we provide the equilibrium analysis.

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[29] To be clear, the post-merger analysis is asymmetric. We write that prices are $\tilde{p}$, profits are $\tilde{\pi}$, costs are $\tilde{C}(Q)$, the surplus from subcontracting is $\tilde{S}\left(Q\right)$ and the share that goes to the seller is $\tilde{\sigma}$.

Cost conditions after two firms merge – This subsection investigates the cost conditions of the outside firms, the merged firm and the industry.

The $N-2$ outside firms remain symmetric and have the same cost function as before the merger. Since they have high in-house costs as compared to the merged firm, we denote any of these outside firms by h. We denote h's in-house cost function by

$$\tilde{C}_h(Q) = C_i(Q).$$

There is one firm with low in-house production costs, the merged firm, which we label firm l. Firm l is characterized by in-house cost function

$$\tilde{C}_l(Q) < C_i(Q).$$

There are cost-efficiency gains at the level of the merged firm. By merging, the two firms combine their production facilities, creating extra opportunities to allocate production in-house so as to reduce costs. We suppose that the marginal cost function of the merged firm is simply the horizontal summation of two pre-merger marginal cost functions.[30] So, the in-house cost reduction from merging follows from scale economies only and exactly equals the cost reduction that both firms could have achieved before the merger by signing subcontracts with each other. This corresponds to Farrell and Shapiro's (1990, p. 112) terminology "no synergies" from merging. By not considering possible synergies, e.g. originating from learning effects or more efficient management, we isolate the strategic effects of merging.

Then, since we investigate the effect of a merger given that firms can sign efficient subcontracts, the merger does *not* lead to cost savings at the industry level. The reasoning is that firms already produce at the lowest possible cost pre-merger by using subcontracts. Industry costs result exclusively from the production facilities that are in the market, not on the names of their owners. We therefore have that the merger does not affect the minimal industry cost function, so that $\tilde{C}_{IND}(Q) = C_{IND}(Q)$ and hence $\tilde{p}^M = p^M$ and $\tilde{\pi}_{IND}(p) = \pi_{IND}(p)$.

To summarize, cost conditions satisfy

$$C_{IND}(Q) = \tilde{C}_{IND}(Q) < \tilde{C}_l(Q) < \tilde{C}_h(Q) = C_i(Q). \tag{11}$$

[30] In particular, the merged firm's costs are $\tilde{C}_l(Q) = 2C_i(Q/2)$. Its marginal costs are $\tilde{C}_l'(Q) = C_i'(2Q)$

So far we have argued that firms, by merging, reduce their in-house costs as compared to the outside firms. In particular, $\tilde{C}_l(Q) < \tilde{C}_h(Q)$. In order to prove our merger results, we now continue our cost analysis with the following lemma that further investigates the costs of the merged firm, $\tilde{C}_l(Q)$. The lemma states that in our setting, for any given quantity to be produced Q, the marginal cost-reduction from merging is decreasing, i.e. there are more cost-reductions if a firm joins a smaller group rather than a larger group of firms. Consider the following thought experiment. Each of the N firms has an in-house cost of $\tilde{C}_h(Q)$. Let one of them merge with a competitor, after which those two firms acquire a third competitor, and so on until there is a monopoly. Since before the merger, the industry consists of N firms, there are $N-1$ such sequential mergers possible. The last merger would result in a monopoly that is characterized by an in-house cost of $\tilde{C}_{IND}(Q)$. Hence, all $N-1$ mergers together would lead to a reduction of in-house costs by $\tilde{C}_h(Q) - \tilde{C}_{IND}(Q) = \tilde{S}_h(Q)$.

Lemma 3. *There are more in-house cost-reductions if a firm joins a smaller group rather than a larger group of firms.*

The proof is in the appendix.

It follows that, since the $N-1$ mergers of our thought experiment reduce in-house costs by $\tilde{S}_h(Q)$, the in-house cost reduction after the first merger must exceed $\frac{1}{N-1}\tilde{S}_h(Q)$. Formally, we have

(12)
$$\tilde{C}_h(Q) - \tilde{C}_l(Q) > \frac{1}{N-1}\tilde{S}_h(Q).$$

Lemma 3 is an important result because it verifies that a merger does not simply reduce in-house costs, but it reduces in-house costs *sufficiently*. As we will see, the in-house cost-reduction has a number of important consequences in terms of the equilibrium price and profits after the merger.

Equilibrium analysis after two firms merge – This subsection investigates the equilibrium price and profits after two firms merge. As in the previous section, the equilibrium analysis proceeds in three steps. Step 1 studies at what prices firm h and l prefer to win rather than to lose. Step 2 examines the equilibrium price by using that in any equilibrium, no loser should prefer to win instead, and no winner should prefer to lose instead. Step 3 provides the equilibrium price if firms sign optimal ex ante subcontracts.

Step 1 - Firm h

If firm h wins, it earns

(13) $$\tilde{W}_h(\tilde{p}) \equiv \tilde{p}Q(\tilde{p}) - \tilde{C}_h(Q(\tilde{p})) + (1-\tilde{\sigma})\tilde{S}_h(Q(\tilde{p})).$$

By winning, firm h earns $\tilde{p}Q(\tilde{p})$ revenues. Without subcontracting, it would incur in-house costs equaling $\tilde{C}_h(Q(\tilde{p}))$. As a contractor firm h captures share $1-\tilde{\sigma}$ of the surplus from subcontracting.

Firm h's profits if it loses depend on which firm wins, firm l or another firm h. If firm h loses and firm l wins, firm h earns

(14) $$\tilde{L}_{h-l}(\tilde{p}) \equiv \frac{1}{N-2}\tilde{\sigma}\tilde{S}_l(Q(\tilde{p})).$$

Firm l wins so that the surplus from subcontracting is $\tilde{S}_l(Q(p))$. Since all $N-2$ firms h are symmetric, and the ex ante subcontract does not favor one firm h over the other, it must be true that each firm h captures an equal portion $\tilde{\sigma}/(N-2)$ of the surplus from subcontracting. If another firm h wins and firm h loses, firm h earns profits given by

(15) $$\tilde{L}_{h-h}(\tilde{p}) \equiv \frac{1}{N-1}\tilde{\sigma}\tilde{S}_h(Q(\tilde{p})).$$

There are $N-1$ symmetric production plant portfolios available for subcontracting. Since the high-cost firm owns one of them, we assume that by losing, the high-cost firm captures share $\tilde{\sigma}/(N-1)$ of the surplus from subcontracting. In other words, firms sign ex ante subcontracts such that symmetric production plant portfolios yield the same subcontracting revenues.[31]

Define the price at which the *high-cost* firm is indifferent between winning and losing against l by $\tilde{p}^*_{h-l}$ such that

(16) $$\tilde{W}_h(\tilde{p}^*_{h-l}(\tilde{\sigma})) = \tilde{L}_{h-l}(\tilde{p}^*_{h-l}(\tilde{\sigma})).$$

We obtain that $\tilde{p}^*_{h-l}(\tilde{\sigma})$ satisfies

(17) $$\tilde{p}^*_{h-l}(\tilde{\sigma}) = \frac{\tilde{C}_{IND}(Q(\tilde{p}^*_{h-l}(\tilde{\sigma})))}{Q(\tilde{p}^*_{h-l}(\tilde{\sigma}))} + \frac{1}{N-2}\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}^*_{h-l}(\tilde{\sigma})))}{Q(\tilde{p}^*_{h-l}(\tilde{\sigma}))} + \frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}^*_{h-l}(\tilde{\sigma})))}{Q(\tilde{p}^*_{h-l}(\tilde{\sigma}))}.$$

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[31] In our model, the efficient subcontract lets each firm h produce Q/N and firm l produce $2Q/N$. So, we assume that firm l and h, on average, obtain the same revenue per unit of output that they subcontract. The merged firm, by subcontracting, then earns twice the profits of an outside firm that subcontracts. Put differently, the merger generates no synergies if the merged firm takes the role of subcontractor.

Define the price at which the *high-cost* firm is indifferent between winning and losing against h by $\tilde{p}^*_{h-h}$ such that

(18) $$\tilde{W}_h\left(\tilde{p}^*_{h-h}(\tilde{\sigma})\right)=\tilde{L}_{h-h}\left(\tilde{p}^*_{h-h}(\tilde{\sigma})\right).$$

We obtain

(19) $$\tilde{p}^*_{h-h}(\tilde{\sigma})=\frac{\tilde{C}_{IND}\left(Q(\tilde{p}^*_{h-h}(\tilde{\sigma}))\right)}{Q(\tilde{p}^*_{h-h}(\tilde{\sigma}))}+\frac{1}{Q(\tilde{p}^*_{h-h}(\tilde{\sigma}))}\frac{N}{N-1}\tilde{\sigma}\tilde{S}_h\left(Q(\tilde{p}^*_{h-h}(\tilde{\sigma}))\right).$$

From (11) we know that $\tilde{C}_h(Q)=C_i(Q)$, so that $\tilde{L}_{h-h}(\tilde{p})$ coincides with $L_i(p)$ given by (6), i.e. the loser's profits as analyzed in the symmetric model. From (11) we also observe that $\tilde{W}_h(\tilde{p})$ as defined in (13) coincides with $W_i(p)$ given by (5), the winner's profits we analyzed in the symmetric model. As a consequence, $\tilde{p}^*_{h-h}$ coincides with p^*_i given by (10) and inherits the properties of Lemma 1.

Step 1 - Firm l

If firm l wins it earns

(20) $$\tilde{W}_l(\tilde{p})\equiv\tilde{p}Q(\tilde{p})-\tilde{C}_l\left(Q(\tilde{p})\right)+(1-\tilde{\sigma})\tilde{S}_l\left(Q(\tilde{p})\right).$$

Firm l, by winning, earns revenues $\tilde{p}Q(\tilde{p})$. As compared to firm h, it has lower costs $\tilde{C}_l\left(Q(\tilde{p})\right)$ without subcontracting. As a contractor, firm l captures share $1-\tilde{\sigma}$ of the surplus from subcontracting.

Firm l's profits if it loses are

(21) $$\tilde{L}_l(\tilde{p})\equiv\frac{2}{N-1}\tilde{\sigma}\tilde{S}_h\left(Q(\tilde{p})\right).$$

By losing, the merged firm captures share $2\tilde{\sigma}/(N-1)$ of the surplus from subcontracting, twice the share of an outside firm.

Define the price at which the *low-cost* firm is indifferent between winning and losing by $\tilde{p}^*_l(\tilde{\sigma})$ such that

(22) $$\tilde{W}_l\left(\tilde{p}^*_l(\tilde{\sigma})\right)=\tilde{L}_l\left(\tilde{p}^*_l(\tilde{\sigma})\right).$$

We get

(23) $$\tilde{p}_l^*(\tilde{\sigma}) = \frac{\tilde{C}_{IND}\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))} + \frac{\tilde{\sigma}\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))} + \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}$$

As in the symmetric section, we assume that for a sufficiently high price $\lim_{\tilde{p}\to\bar{p}} \tilde{p}$ and hence sufficiently low market demand $\lim_{\tilde{p}\to\bar{p}} Q(\tilde{p})$, all firms prefers to win rather than lose.

(A6) $\lim_{\tilde{p}\to\bar{p}} \tilde{W}_h(\tilde{p}) > \lim_{\tilde{p}\to\bar{p}} \tilde{L}_{h-l}(\tilde{p})$, $\lim_{\tilde{p}\to\bar{p}} \tilde{W}_h(\tilde{p}) > \lim_{\tilde{p}\to\bar{p}} \tilde{L}_{h-h}(\tilde{p})$ and $\lim_{\tilde{p}\to\bar{p}} \tilde{W}_l(\tilde{p}) > \lim_{\tilde{p}\to\bar{p}} \tilde{L}_l(\tilde{p})$.

See from (11) that the second inequality is in fact redundant because it coincides with (A5).

Lemma 4. *There exists a unique $\tilde{p}_l^*(\tilde{\sigma}) < \bar{p}$, a unique $\tilde{p}_{h-l}^*(\tilde{\sigma}) < \bar{p}$ and a unique $\tilde{p}_{h-h}^*(\tilde{\sigma}) < \bar{p}$. Firm l wants to win rather than to lose if and only if the price is larger than $\tilde{p}_l^*(\tilde{\sigma})$. Firm h wants to win rather than to lose against l if and only if the price is larger than $\tilde{p}_{h-l}^*(\tilde{\sigma})$. Firm h wants to win rather than to lose against h if and only if the price is larger than $\tilde{p}_{h-h}^*(\tilde{\sigma})$. For positive $\tilde{\sigma}$, we have that $\tilde{p}_l^*(\tilde{\sigma}) < \tilde{p}_{h-l}^*(\tilde{\sigma}) < \tilde{p}_{h-h}^*(\tilde{\sigma})$.*

The proof is in the appendix and uses that the cost conditions after the merger must satisfy Lemma 3 and hence (12). Figure 1 visualizes Lemma 4.

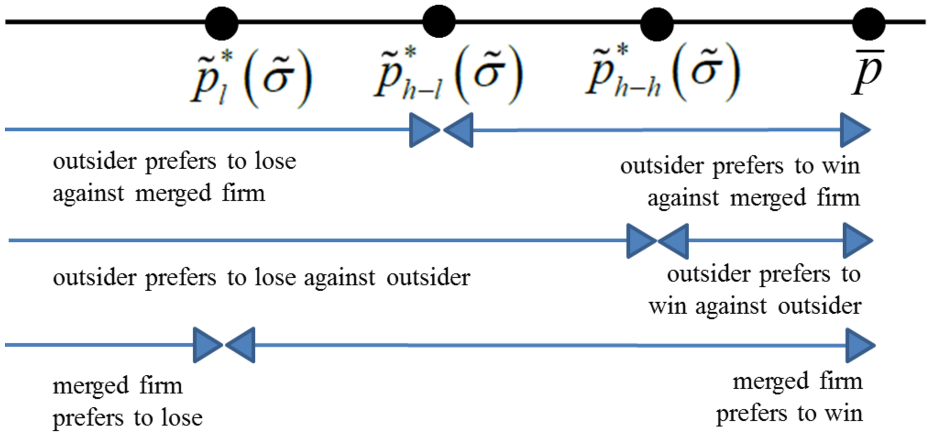

Figure 1. A visualization of lemma 4.

Being a subcontractor is less attractive for a firm if the price is high, because then demand is low so that there is little need for subcontracting. At the same time, winning is only attractive if it generates sufficient revenues from consumers, i.e. if the price is sufficiently high. Lemma 4 states that, for both h and l, profits from winning and losing intersect exactly once at a price below $\bar{p}$.

Step 2. Second, we examine under what circumstances firm h or l wins in equilibrium and what the candidate equilibrium prices are.

In any equilibrium, there are two possibilities. Either firm l wins, or a firm h wins. For $\tilde{\sigma} > 0$, we will rule out that h wins in equilibrium by contradiction. Suppose that a high-cost firm wins in equilibrium.[32] Then firm l should prefer not to win instead, meaning that the equilibrium price cannot exceed $\tilde{p}_l^*(\tilde{\sigma})$ (from lemma 4). It is then a weakly dominant strategy for l to offer $\tilde{p}_l^*(\tilde{\sigma})$. Also, the high-cost firm should prefer not to lose instead. Of course, firm h's profits from losing depend on who wins. If, by charging a higher price, firm l wins, the high-cost firm should prefer not to lose against l. Hence, the equilibrium price cannot be smaller than $\tilde{p}_{h-l}^*(\tilde{\sigma})$.[33] If instead, by charging a higher price, another high-cost firm wins, in equilibrium firm h should not prefer to lose against another firm h which implies that the equilibrium price cannot be smaller than $\tilde{p}_{h-h}^*(\tilde{\sigma})$. We can conclude that, in any equilibrium where a high-cost firm wins, we must either have $\tilde{p}_l^*(\tilde{\sigma}) \geq \tilde{p}_{h-l}^*(\tilde{\sigma})$ or $\tilde{p}_l^*(\tilde{\sigma}) \geq \tilde{p}_{h-h}^*(\tilde{\sigma})$. For strictly positive $\tilde{\sigma}$, these assertions contradict Lemma 4 so that firm h cannot win in equilibrium.

Lemma 5 shows that, if $\tilde{p}_{h-l}^*(\tilde{\sigma}) \leq p^M$, the equilibrium price is $\tilde{p}_{h-l}^*(\tilde{\sigma})$. More specifically, for $\tilde{\sigma} > 0$, in the unique equilibrium, firm l wins by slightly undercutting price $\tilde{p}_{h-l}^*(\tilde{\sigma})$ by an arbitrarily small amount, which we set equal to zero for simplicity.[34]

Lemma 5. *If $\tilde{p}_{h-l}^*(\tilde{\sigma}) \leq p^M$, the unique equilibrium price is $\tilde{p}_{h-l}^*(\tilde{\sigma})$.*

The proof is in the appendix.

Lemma 5 is intuitive. The low-cost firm charges a price just below the price at which the high-cost firm would prefer to win.

Step 3. The third step investigates how firms optimally sign ex ante subcontracts after the merger. Proposition 1 proves that we either have that

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[32] This possibility may seem counterintuitive. However, also in Spiegel's (1993) analysis, the low-cost firm acts as a subcontractor. Since having lower costs improves both the profits from winning *and* the profits from subcontracting, it is not trivial that the merged firm prefers to win in equilibrium.

[33] For example, if $N = 3$, there is only one firm h in the market after the merger. Then, if h loses, l wins.

[34] To avoid the open set problem, we could also adjust the tie-breaking rule in favor of the low-cost firm.

(i) the equilibrium price equals the monopoly price $\tilde{p}^M$ and firms allocate share $0 < \tilde{\sigma}^* \leq 1$ to the subcontractors, or

(ii) the equilibrium price is lower than the monopoly price $\tilde{p}^*_{h-l}(1) < \tilde{p}^M$ and firms allocate share $\tilde{\sigma}^* = 1$ to the subcontractors.

***Proposition 3**. After the merger, if firms sign optimal ex ante subcontracts, the unique equilibrium price is*

$$\tilde{p}^*_{h-l}\left(\tilde{\sigma}^*\right) = \min\left\{\tilde{p}^*_{h-l}(1), \tilde{p}^M\right\}.$$

The proof is in the appendix.

The intuition is analogous to that of proposition 1. Firms would like to have subcontracting costs that lead to an equilibrium price equal to the monopoly price. However, they are sometimes constrained by the fact that subcontracting costs cannot exceed the contractor's opportunity cost of in-house production. In that event, the equilibrium price is lower than the monopoly price and firms choose the subcontracting terms that maximally favor the subcontractors.

The effects of a merger on prices and profits – This subsection compares the symmetric equilibrium analysis that applies before the merger with the asymmetric equilibrium analysis that applies after two firms merge.

Proposition 4 studies the effect of a merger if firms earn monopoly profits pre-merger.

***Proposition 4.** If firms earn monopoly profits pre-merger ($p_i^*\left(\sigma^*\right) = p^M$), the merger is never harmful for consumers.*

Proposition 4 follows from proposition 3 which implies that the post-merger price is $\min\left\{\tilde{p}^M, \tilde{p}^*_{h-l}\left(\tilde{\sigma}^*\right)\right\}$ and thus never exceeds pre-merger price p^M (because $\tilde{p}^M = p^M$). Note that proposition 4 is not a special case. To see why, recall that with ex ante subcontracts, firms want to choose σ^* so as to exactly realize the monopoly price ($p_i^*\left(\sigma^*\right) = p^M$).

We now study what happens when firms do not earn monopoly profits before the merger. Proposition 5 describes the effect on the equilibrium price and proposition 6 investigates which mergers are profitable.

***Proposition 5.** If firms do not earn monopoly profits before the merger, the merger is good for consumers (reduces the equilibrium price).*

Proof. We prove that, if firms do not earn monopoly profits before the merger, the merger decreases the price.

From proposition 1, we know that the pre-merger price is $p_i^*(1)$. Proposition 3 gives us that post-merger, the price is $\min\left\{\tilde{p}_{h-l}^*(1), \tilde{p}^M\right\}$.

We prove by contradiction that the post-merger price is smaller. Suppose the post-merger price would be not be smaller. Then

$$\min\left\{\tilde{p}_{h-l}^*(1), \tilde{p}^M\right\} \geq p_i^*(1), \text{ which implies}$$

$$\tilde{p}_{h-l}^*(1) \geq p_i^*(1).$$

The price at which two pre-merger firms are indifferent between winning and losing against each other, $p_i^*(1)$, is exactly the same as the price at which to high-cost firms are indifferent between winning and losing against each other, $p_{h-h}^*(1)$. This is seen by plugging in $S_i(Q) = \tilde{S}_h(Q)$ and $C_{IND}(Q) = \tilde{C}_{IND}(Q)$ in expressions (10) and (19). We can therefore write

$$\tilde{p}_{h-l}^*(1) \geq p_{h-h}^*(1),$$

which contradicts Lemma 4. We conclude that the merger decreases the price. ■

Since the merged firm is characterized by lower in-house production costs, the surplus from subcontracting that can be shared by subcontractors decreases. Due to this effect, outside firms earn less profits by subcontracting, so that they are willing to compete more fiercely for the market by charging lower prices. In other words, the opportunity cost of winning goes down because of the merger, so that firms want to charge lower prices. As a result, a merger decreases the equilibrium price, despite the absence of cost savings at the industry level.

We proceed with proposition 6 to study the profitability of mergers.

Proposition 6. *The merger is profitable.*

The proof is in the appendix.

Before we provide the intuition behind proposition 6, we emphasize that the merger does *not* achieve cost savings at the industry level, because firms already take advantage of any possible cost-reduction by using subcontracts.

By merging, firms replace the profits of two pre-merger firms with the profits of only one merged firm. In other words, a profitable merger requires that the merged firm can earn at least double the profits of a pre-merger firm. This is satisfied because of a strategic effect. The merged firm is characterized by lower in-house opportunity costs and thereby caps the amount paid to the rivals for subcontracting. As we know from proposition 5, this effect happens to be good for consumers as well because it increases the outsider's willingness to compete fiercely. We thus find that the interests of merging firms and consumers are aligned. They each benefit from lower surplus from subcontracting. Concentration measures can be misleading to assess the competitiveness of industries with subcontracting.

Our merger results are summarized in figure 2, which shows the effects of the merger on prices and profits given that firms do not earn monopoly profits before the merger.

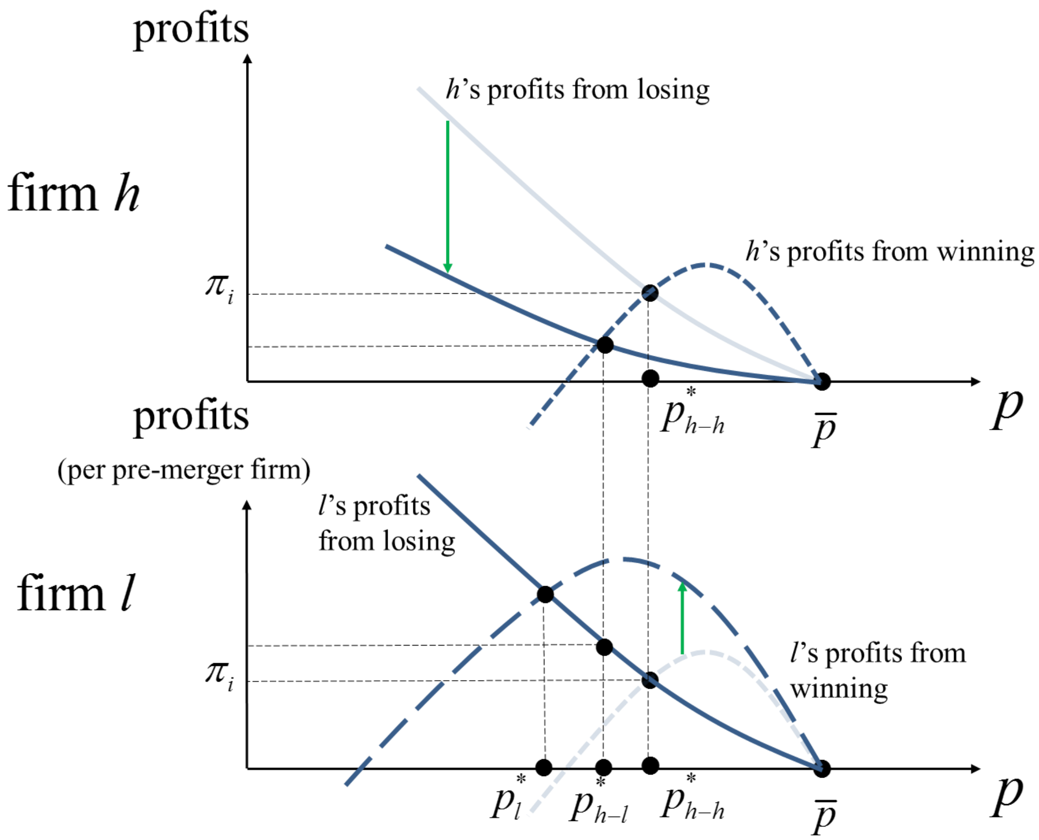

Figure 2. The effect of a merger on prices and profits.

The upper graph shows the effects for an outside firm. The lower graph shows the effects for the merged firm. The vertical axis shows profits and the horizontal axis shows the lowest price in the market. Before the merger, all firms are symmetric and indifferent between winning and losing at p^*_{h-h}.

After the merger, the outside firms h are harmed because they can profit less if they *lose*. Their opportunity cost of winning goes down so that they are indifferent between winning and losing at a lower price $p^*_{h-l} < p^*_{h-h}$. The story is different for the merged firm. Merging reduces in-house costs, which is profitable if the merged firm *wins* the market. The merged firm maximizes revenues by charging a price just below p^*_{h-l}, the price at which the high-cost firm no longer wants to win. The outsiders' profits go down from, whereas the merging firms increase their profits.

Social optimum – We have learned that a merger reduces the price. It remains to check whether a lower price raises total welfare. The socially optimal price, p^{SO}, sets price equal to the marginal industry cost

$$p^{SO} = C'_{IND}\left(Q(p^{SO})\right).$$

The intuition is that, then, consumers will buy an extra unit if and only if their willingness to pay is larger than the industry marginal cost.

The equilibrium price in the symmetric model is seen from (10). The equilibrium price in the asymmetric model is seen from (17).

Proposition 7. *The equilibrium prices before and after the merger,* $p_i^*\left(\sigma^*\right)$ *and* $\tilde{p}^*_{h-l}\left(\tilde{\sigma}^*\right)$ *respectively, are never smaller than the socially optimal price* p^{SO}.

The proof is in the appendix.

Proposition 7 together with proposition 4 and 5 implies that a profitable merger brings the equilibrium price closer to the socially optimal price, thereby increasing the sum of consumer and producer surplus.

Proposition 7 is not trivial. To see why, the equilibrium price consists of an *average* cost term plus a markup, while the socially optimal price consists of a *marginal* cost term. While marginal costs exceed average costs in our framework, proposition 7 nevertheless finds that the equilibrium price exceeds the socially optimal price.

Discussion — Our analysis suggests that, with subcontracting, it is better to have a market in which two large firms compete in prices, than a market where production capacity is fragmented across many small firms. In particular, with many small firms, the winning firm would rely heavily on the subcontracting services of the losing firms, which discourages firms to conquer the market by charging low prices.

Also, remark that consolidating production capabilities to reduce in-house costs is only profitable for firms if they serve consumers. So, merging is only profitable if a firm wins. This suggests that, in a market with small and large firms, the largest firm has the strongest incentive to merge because it serves all consumers. We should expect the largest firm to successively acquire other firms until the industry converges to a monopoly.

These insights hold as long as there are *at least two* firms that compete in prices. Indeed, we have studied the effect of a merger in an industry with at least $N \geq 3$ firms. If instead there is a duopoly ($N = 2$) that merges to a monopoly, there are no outside firms that can change their equilibrium behavior as a result of the merger. The merger is therefore never unprofitable. From proposition 1 we know that pre-merger, the price is not larger than the monopoly price. After the merger, there is a monopoly that charges the monopoly price. So, a merger from duopoly to monopoly never decreases the price, and is therefore never good for consumers.

4. Collusion

This section investigates the incentive to engage in tacit collusion for N symmetric firms. We apply the analysis of section 2 and denote each firm by i.

We study symmetric collusive agreements where colluding firms maximize industry profits. In particular, in stage one, all colluding firms charge price p^M, so that each firm i earns an profit of π_{IND}^M / N because of symmetry.

Moreover, colluding firms sign ex ante subcontracts that apply share σ_c to allocate the surplus from subcontracting. Subscript c distinguishes share σ_c applied by colluding firms from share σ applied in the static equilibrium analyzed in section 2. Notice that share σ_c does not affect collusive profits π^M. To see why, since colluding firms charge p^M, they always earn monopoly profits, regardless of share σ_c. Share σ_c does, however, affect each firm's incentives to deviate by not charging the collusive price p^M. Specifically, the critical discount factor $\delta^C(\sigma_c)$, the minimal discount factor required to sustain collusion, depends on share σ_c. We suppose that colluding firms want to choose σ_c optimally, meaning that

$$\sigma_c^* \equiv \arg\min_{0 \leq \sigma_c \leq 1} \delta^C(\sigma_c)$$

is chosen to minimize the critical discount factor.

Deviation profits are denoted by $\pi^{D}\left(\sigma_{c}\right)$. Despite the fact that deviation will be detected after stage one, firms are contractually committed to share the surplus from subcontracting in accordance with σ_{c}. Hence, deviation profits depend on the binding ex ante subcontract σ_{c}.

We analyze colluding firms that employ grim-trigger strategies, so that starting from the period after deviation, firms revert to the equilibrium of the static game forever. Firms then apply σ^{*} as in section 2, charge equilibrium price $p_{i}^{*}(\sigma^{*})$ accordingly and each earn profits $\pi_{i}\left(p_{i}^{*}(\sigma^{*})\right)$.

Proposition 8. *The optimal ex ante subcontracting terms also minimize the critical discount factor.*

$$\sigma^{*}=\sigma_{c}^{*}.$$

We provide the proof in appendix.

Clearly, if firms can realize the monopoly price in the static game by choosing σ^{*}, any discount factor sustains collusion. Firms do not need to collude in order to earn monopoly profits.

If firms cannot realize the monopoly price in the static game, colluding firms are tempted to conquer the market by undercutting the monopoly price. A deviating firm serves the market after which it contracts from the other firms. Of course, colluding firms want to choose σ_{c}^{*} so as to discourage deviation. In particular, by choosing $\sigma_{c}^{*}=1$, they make sure that the deviating firm would earn zero surplus from subcontracting.

Proposition 9. *Firms either*

(i) *attain monopoly profits without collusion ($\pi_{i}\left(p_{i}^{*}(\sigma^{*})\right)=\pi_{IND}^{M}/N$), or*

(ii) *horizontal subcontracts do not facilitate collusion.*

We provide the proof in the appendix. The result follows from the fact that, after deviation, firms can still charge substantial markups because there is surplus from subcontracting (see 10). Therefore the collusive agreement is more difficult to sustain.

5. Conclusion and discussion

Our paper studies mergers and collusion in industries with horizontal subcontracting. Horizontal subcontracts enable firms to allocate production efficiently, not only in public procurement, but also e.g. in the construction industry, the financial sector and the power generation industry.

The effects of a merger with horizontal subcontracting differ from the traditional merger analysis in several respects. In particular, firms do not need to merge in order to achieve cost-reductions by reallocating production efficiently across firms. Indeed, firms can already sign efficient subcontracts before the merger. So, if the benchmark is not to consider synergies from outside the model like learning effects, management efficiencies, mergers in industries with horizontal subcontracting do *not* reduce industry costs. The merger only improves the merged firm's *in-house* production possibilities. It is exactly this in-house cost-reduction that drives our two main results. First, a merger is profitable. The reason is that the merged firm needs to rely less on subcontracting to serve the market. Moreover, better in-house production opportunities guarantee more favorable subcontracting terms. Second, a merger never harms consumers. The outside firms profit less from subcontracting, so that they are willing to compete more fiercely to serve consumers. This result has important implications for competition authorities that evaluate the effects of a merger. In particular, in industries with subcontracting, a merger does not need to reduce costs in order to reduce the equilibrium price. Increased concentration can be pro-competitive without reference to an efficiency defense. Similarly, this insight is also reflected in agreements falling short of a merger. If bidding consortia are assessed as not restricting competition, article TFEU 101(1) does not apply and there is no need to refer to the efficiency defense exemption provided by TFEU 101(3). Our analysis also suggests that the market structure favored by consumers is the one where two large firms compete in prices, because then none of the firms can profit much from subcontracting.

Our motivation for subcontracting arises from *strictly* convex costs and the fact that exactly one firm is chosen to serve market demand. However, we do not need strictly convex costs if we allow that firms draw their cost function from some ex ante symmetric distribution. To see why, such a model would include the setting where firms draw a constant marginal cost from some distribution. If there is a losing firm that draws a lower marginal cost than the winner, firms can reduce costs by subcontracting. We could include the possibility of having more than one state of nature, i.e. more than one combination of cost functions (one for each firm). The interpretation of a subcontract would then complicate. In particular, for *each* possible state of nature, there is a state-specific share σ that would apply. Our findings on this possible extension indicate that no extra insights are added to the results. The advantage of our tie-breaking rule is that (i) it is often used and (ii) it explains subcontracting. In particular, there is only one possible contractor, the winning firm, and all other firms exclusively subcontract. If another tie-breaking rule applies, or if firms compete in quantities, several firms may each be serving some portion of market demand. To model the myriad of subcontracting flows between firms, we would require a specific framework. We see no reason, however, why the key driver of our merger results, namely that with subcontracting the merger improves the merged firm's in-house production possibilities, would no longer hold. Implementing another tie-breaking rule may, importantly, impact the subcontracting surplus, thereby affecting the equilibrium price. This suggests that consumers or procurers may have a preference for mechanisms that minimize or eliminate the subcontracting surplus. Our winner-takes-all framework then applies well for settings where consumers or procurers rather coordinate with just one supplier so as to avoid asymmetric information issues among multiple contractors (see e.g. Sufi, 2007 for an application to the syndicated loans market). Our collusion results have intuitive appeal in other settings as well. Colluding firms can discourage deviation by increasing subcontracting costs, but are confronted with the fact that subcontracting surplus generates considerable profits for firms, even in the static Nash-equilibrium. It follows that, with subcontracting, colluding firms find it harder to resist the temptation to deviate.

Finally, in a setting where firms are asymmetrically sized, the effects of a merger between the two largest firms should be carefully assessed for the following reason. As in many settings, the equilibrium price follows crucially from the firm that is willing to offer the second-lowest price. In particular, the largest firm wins in equilibrium, and makes sure to offer a sufficiently low price such that the *second*-largest firm is not willing to undercut. So, after a merger between the two largest firms, the equilibrium price would crucially follow from the *third*-largest firm that was originally in the market. Of course, the merger will cause that firm to compete more fiercely for consumers, for the reasons we studied. However, since the third-largest firm is characterized by higher costs than the second-largest firm originally in the market, it is also less capable of charging a low price, which may counteract the price-reducing effect.

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Appendix

Proof of Lemma 1.

We first show that price $p_i^*(\sigma) < \bar{p}$ always exists and is unique. We follow the techniques used in Kamien et al. (1989).

We use (5), (6) and (2) to write the difference between the winning firm's profits and the losing firms' profits as

$$W_i(p) - L_i(p) = Q(p)\left(p - \frac{C_{IND}(Q(p))}{Q(p)} - \frac{N}{N-1}\frac{\sigma S_i(Q(p))}{Q(p)} \right).$$

We define $H(p) \equiv \frac{W_i(p) - L_i(p)}{Q(p)} = p - \frac{C_{IND}(Q(p))}{Q(p)} - \frac{N}{N-1}\frac{\sigma S_i(Q(p))}{Q(p)}$. So, firms prefer to win rather than to lose if and only if $H(p) > 0$. We know that

$$H^{'}(p)=1-\underbrace{\frac{\partial\left[\frac{C_{IND}(Q(p))}{Q(p)}\right]}{\partial Q}Q^{'}(p)}_{<0}-\frac{N}{N-1}\underbrace{\frac{\partial\left(\frac{\sigma S_i(Q(p))}{Q(p)}\right)}{\partial Q}Q^{'}(p)}_{\leq 0}>0,$$

where we sign the second and last term by using (A1), (A2) and (A3).

We also know that $H(0)=-\frac{C_{IND}(Q(0))}{Q(0)}-\frac{N}{N-1}\frac{\sigma S_i(Q(0))}{Q(0)}<0$, and that $\lim_{p\to\bar{p}}H(p)=\lim_{p\to\bar{p}}\frac{W_i(p)-L_i(p)}{Q(p)}>0$ because of (A5).

Since $H(0)<0$, $\lim_{p\to\bar{p}}H(p)>0$ and $H^{'}(p)>0$, there must exists a unique price $0<p_i^*(\sigma)<\bar{p}$ for which $H(p_i^*(\sigma))=0$. See that $H(p_i^*(\sigma))=0$ coincides with (10). Hence, firms want to win rather than to lose if and only if the price exceeds $p_i^*(\sigma)$.■

Proof of Lemma 2.

We show that, if $p_i^*(\sigma)\leq p^M$, it is the equilibrium. To be precise, we show that no other price would lead to higher profits. Clearly, charging a higher price, and hence losing for sure, keeps a firm's profits equal since we know from (7) that at $p_i^*(\sigma)$ firms are indifferent between winning and losing. Also, a firm cannot profitably charge a lower price $p<p_i^*(\sigma)$ if

$$W_i^{'}(p)\Big|_{p<p_i^*(\sigma)\leq p^M}\geq 0,$$

which we prove by contradiction. Suppose otherwise, then (i) we would have $W_i^{'}(p)\Big|_{p<p_i^*(\sigma)\leq p^M}<0$ and (ii) from (3) and (A1) we always have $L_i^{'}(p)<0$. As a consequence, we would have $\pi_{IND}^{'}(p)\Big|_{p<p_i^*(\sigma)\leq p^M}=\left[W_i^{'}(p)+(N-1)L_i^{'}(p)\right]\Big|_{p<p_i^*(\sigma)\leq p^M}<0$, which contradicts (A4). In particular, (A4) guarantees that $\pi_{IND}^{'}(p)>0$ for prices below the monopoly level. ■

Proof of proposition 1.

To prove proposition 1, we investigate the three possible scenarios: $p_i^*(\sigma^*)<p^M$, $p_i^*(\sigma^*)>p^M$ and $p_i^*(\sigma^*)=p^M$.

1. Suppose $p_i^*\left(\sigma^*\right) < p^M$. From Lemma 1 and 2 we know that $p_i^*\left(\sigma^*\right)$ is the unique equilibrium. Since the profit function is strictly concave, see (A4), we must have that $\pi_{IND}^{'}\left(p_i^*\left(\sigma^*\right)\right) > 0$. Proposition 1 claims that then $\sigma^* = 1$. We prove by contradiction. Suppose otherwise, namely that $\sigma^* < 1$. Then from (8), the implicit function theorem and (4) we know that the effect of σ on industry profits would be

$$\pi_{IND}^{'}\left(p_i^*\left(\sigma^*\right)\right)p_i^{*'}\left(\sigma^*\right) = \underbrace{\pi_{IND}^{'}\left(p_i^*\left(\sigma^*\right)\right)}_{>0}\frac{\frac{N}{N-1}S_i(Q(p_i^*\left(\sigma^*\right)))}{\underbrace{\pi_{IND}^{'}\left(p_i^*\left(\sigma^*\right)\right)}_{>0}\underbrace{-\frac{N}{N-1}\sigma^* S_i^{'}(Q(p_i^*\left(\sigma^*\right)))Q^{'}(p_i^*\left(\sigma^*\right))}_{>0}} > 0,$$

where the denominator is positive because of (A1), (3), $p_i^*\left(\sigma^*\right) < p^M$ and (A4). Hence, firms could do better by choosing a higher σ, so that $\sigma^* < 1$ would not be optimal, a contradiction.

2. Suppose $p_i^*\left(\sigma^*\right) > p^M$. Proposition 1 claims that this never occurs. We show by contradiction and proceed in two steps.

Step one proves that $p_i^*(0) < p^M$. We show by contradiction. Suppose otherwise, namely that $p_i^*(0) \geq p^M$.

We know that monopoly profits are strictly positive ($\pi_{IND}^M > 0$) so that the monopoly price p^M must exceed the average industry variable cost

$$p^M > \frac{C_{IND}\left(Q(p^M)\right)}{Q(p^M)}.$$

We also know from (10) that if $\sigma = 0$, firms choose

$$p_i^*(0) = \frac{C_{IND}\left(Q(p_i^*(0))\right)}{Q(p_i^*(0))}.$$

If $p_i^*(0) \geq p^M$, we would have that $\frac{C_{IND}\left(Q(p_i^*(0))\right)}{Q(p_i^*(0))} > \frac{C_{IND}\left(Q(p^M)\right)}{Q(p^M)}$. But this would contradict (A2), because $Q(p_i^*(0)) < Q(p^M)$ from $p_i^*(0) > p^M$ and (A1). Hence, step one concludes that we must have $p_i^*(0) < p^M$.

Step two contradicts that, if $p_i^*\left(\sigma^*\right) > p^M$, σ^* is optimal. By using step one, we would have $p_i^*(0) < p^M < p_i^*\left(\sigma^*\right)$. But since $p_i^*\left(\sigma\right)$ is continuous, there must then exist a $0 < \sigma^{'} < \sigma^*$ for which firms charge the equilibrium price $p_i^*\left(\sigma^{'}\right)=p^M$ such that firms earn monopoly profits. From strict concavity of the profit function, we know that the only possible equilibrium, $p_i^*\left(\sigma^*\right)$, would yield smaller profits $\pi_{IND}\left(p_i^*\left(\sigma^*\right)\right)<\pi_{IND}^M$. Hence, firms could increase profits by choosing $\sigma^{'}$ instead of σ^*, so that σ^* would not be optimal, a contradiction. We conclude that we never have $p_i^*\left(\sigma^*\right)>p^M$.

3. The final possibility is that the optimal ex ante subcontract realizes the monopoly price $p_i^*\left(\sigma^*\right)=p^M$ with $0<\sigma^*\le 1$. Note that, from step one of scenario 2, we can exclude the possibility that $\sigma^*=0$.■

Proof of proposition 2.

We first show that $p_i^*\left(\sigma\right)<p^{ns}\Rightarrow\sigma<\frac{N-1}{N}$. Second, we show that $p_i^*\left(\sigma\right)\ge p^{ns}\Rightarrow\sigma\ge\frac{N-1}{N}$. Hence, we must have that $p_i^*\left(\sigma\right)<p^{ns}\Leftrightarrow\sigma<\frac{N-1}{N}$.

1. Suppose $p_i^*\left(\sigma\right)<p^{ns}$. By using (1) and (10) we can write the equivalent condition

$$\frac{C_{IND}\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}+\frac{N}{N-1}\frac{\sigma S_i\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}<C_i\left(Q(p^{ns})\right)\Big/Q(p^{ns}).$$

We multiply by $Q(p_i^*(\sigma))$ and rearrange to get the equivalence

$$p_i^*\left(\sigma\right)<p^{ns}\Leftrightarrow\sigma<\frac{N-1}{N}\frac{Q(p_i^*(\sigma))\dfrac{C_i\left(Q(p^{ns})\right)}{Q(p^{ns})}-C_{IND}\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)}.$$

From $p_i^*\left(\sigma\right)<p^{ns}$, (A1) and (A2) we have $\frac{C_i\left(Q(p^{ns})\right)}{Q(p^{ns})}<\frac{C_i\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}$, so that $p_i^*\left(\sigma\right)<p^{ns}$ implies

$$\sigma<\frac{N-1}{N}\frac{Q(p_i^*(\sigma))\dfrac{C_i\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}-C_{IND}\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)}=\frac{N-1}{N}\frac{S_i\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)}=\frac{N-1}{N}.$$

2. From our above equivalence, we can also write

$$p_i^*\left(\sigma\right) \geq p^{NS} \Leftrightarrow \sigma \geq \frac{N-1}{N} \frac{Q(p_i^*(\sigma)) \dfrac{C_i\left(Q(p^{ns})\right)}{Q(p^{ns})} - C_{IND}\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)}.$$

If $p_i^*\left(\sigma\right) \geq p^{ns}$, from (A1) and (A2) we have $\dfrac{C_i\left(Q(p^{ns})\right)}{Q(p^{ns})} \geq \dfrac{C_i\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))}$, so that $p_i^*\left(\sigma\right) \geq p^{ns}$ implies

$$\sigma \geq \frac{N-1}{N} \frac{Q(p_i^*(\sigma)) \dfrac{C_i\left(Q(p_i^*(\sigma))\right)}{Q(p_i^*(\sigma))} - C_{IND}\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)} = \frac{N-1}{N} \frac{S_i\left(Q(p_i^*(\sigma))\right)}{S_i\left(Q(p_i^*(\sigma))\right)} = \frac{N-1}{N}. \blacksquare$$

Proof of Lemma 3.

We are interested in the in-house cost-reduction of a joint entity as a result of merging with firm h. In particular, we investigate to what extent firm h would reduce the joint entity's in-house cost of producing Q units. We claim that, if the joint entity is smaller, the merger with h achieves larger in-house cost reductions.

The first merger we consider is the merger between h and a joint entity n that consists of $1 \leq n \leq N-2$ other firms.

Before the merger, the entity n has in-house costs equal to $C_n\left(Q\right)$. After the merger, the merged entity $\left\{n,h\right\}$ can allocate its production Q cost-efficiently across its units. We denote the number of units of production allocated to firm h by $0 \leq q_h^{\{n,h\}} \leq Q$.[35] The superscript denotes the merged entity $\left\{n,h\right\}$. The sum of all firms' production meets market demand such that entity n produces $Q - q_h^{\{n,h\}}$ .We can write that the in-house cost-reduction thanks to the first merger equals

$$C_n\left(Q\right) - C_n\left(Q - q_h^{\{n,h\}}\right) - C_h\left(q_h^{\{n,h\}}\right).$$

[35] In our framework, since all firms have the same upward sloping marginal costs, the efficient allocation for each firm within the entity $\left\{n,h\right\}$ is to produce $Q/\left(n+1\right)$, so that $q_h^{\{n,h\}} = Q/\left(n+1\right)$.

The second merger we consider is the merger between h and a joint entity $\{n+1\}$ that consists of the same n other firms plus one.

Before the merger, the entity $\{n+1\}$ has an in-house cost equal to $C_{n+1}(Q)$. After the merger, again, the merged entity $\{n+1,h\}$ can allocate production Q cost-efficiently over firms. We denote the number of units of production allocated to firm h by $0 \le q_h^{\{n+1,h\}} \le Q$. Note that $q_h^{\{n+1,h\}} \le q_h^{\{n,h\}}$, meaning that firm h produces less in the larger entity as compared to the smaller entity.[36] The sum of all firms' production meets market demand such that entity $n+1$ produces $Q-q_h^{\{n+1,h\}}$. We can write that the in-house cost-reduction from the second merger equals

$$C_{n+1}(Q)-C_{n+1}\left(Q-q_h^{\{n+1,h\}}\right)-C_h\left(q_h^{\{n+1,h\}}\right).$$

We first prove that the in-house cost-reduction resulting from the first merger is never smaller than the in-house cost-reduction thanks to the second merger.

We write the in-house cost-reduction from the first merger as

$$\overbrace{C_n(Q)\underbrace{-C_n\left(Q-q_h^{\{n+1,h\}}\right)}^{\text{term 1}}\underbrace{+C_n\left(Q-q_h^{\{n+1,h\}}\right)}_{=0}\overbrace{-C_n\left(Q-q_h^{\{n,h\}}\right)-C_h\left(q_h^{\{n,h\}}\right)+C_h\left(q_h^{\{n+1,h\}}\right)}^{\text{term 2}}\overbrace{-C_h\left(q_h^{\{n+1,h\}}\right)}^{\text{term 3}}.$$

For easy exposition, we label three terms, term 1, term 2 and term 3. We now use that for all Q the marginal costs of entity n are strictly higher than the marginal costs of entity $n+1$, so that term 1 satisfies $C_n(Q)-C_n\left(Q-q_h^{\{n+1,h\}}\right) > C_{n+1}(Q)-C_{n+1}\left(Q-q_h^{\{n+1,h\}}\right)$. By rearranging we obtain that the in-house cost-reduction from the first merger *strictly* exceeds

$$\underbrace{C_{n+1}(Q)-C_{n+1}\left(Q-q_h^{\{n+1,h\}}\right)\overbrace{-C_h\left(q_h^{\{n+1,h\}}\right)}^{\text{term 3}}}_{\text{cost-reduction second merger}}\overbrace{+C_n\left(Q-q_h^{\{n+1,h\}}\right)-C_n\left(Q-q_h^{\{n,h\}}\right)-C_h\left(q_h^{\{n,h\}}\right)+C_h\left(q_h^{\{n+1,h\}}\right)}^{\text{term 2}}.$$

Notice that, by shifting term 3 to the left, we get the expression for the cost-reduction of the second merger. So, the in-house cost-reduction from the first merger is larger than the in-house cost reduction from the second merger if term 2 is positive, or

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[36] The efficient allocation for each firm within the entity $\{n+1,h\}$ is to produce $Q/(n+2)$, so that $q_h^{\{n+1,h\}} = Q/(n+2)$.

$$C_n\left(Q-q_h^{\{n+1,h\}}\right)-C_n\left(Q-q_h^{\{n,h\}}\right)-C_h\left(q_h^{\{n,h\}}\right)+C_h\left(q_h^{\{n+1,h\}}\right)\geq 0\,.$$

We also know from (A2) that marginal costs are upward sloping so that $C_n\left(Q-q_h^{\{n+1,h\}}\right)-C_n\left(Q-q_h^{\{n,h\}}\right)>C_n^{'}\left(Q-q_h^{\{n,h\}}\right)\left(q_h^{\{n,h\}}-q_h^{\{n+1,h\}}\right)$ and $-C_h\left(q_h^{\{n,h\}}\right)+C_h\left(q_h^{\{n+1,h\}}\right)>-C_h^{'}\left(q_h^{\{n,h\}}\right)\left(q_h^{\{n,h\}}-q_h^{\{n+1,h\}}\right)$. By rearranging, we know that it suffices to show that

$$\left(C_n^{'}\left(Q-q_h^{\{n,h\}}\right)-C_h^{'}\left(q_h^{\{n,h\}}\right)\right)\left(q_h^{\{n,h\}}-q_h^{\{n,+1h\}}\right)\geq 0\,.$$

Since in our framework, the efficient allocation is interior, $0<q_h^{\{n,h\}}<Q$, we know that then marginal costs are equalized at $C_n^{'}\left(Q-q_h^{\{n,h\}}\right)=C_h^{'}\left(q_h^{\{n,h\}}\right)$, so that the inequality holds. We proved that the in-house cost-reduction resulting from the first merger is larger than the in-house cost-reduction thanks to the second merger. In other words, the in-house cost-reduction resulting from the merger of h with the joint entity n is larger than the merger with joint entity $\{n+1\}$. Since all firms are symmetric, the in-house cost-reduction resulting from the merger of h with the joint entity n is the same for any firm h that merges with any n or $n+1$ of the other firms. So, if the joint entity is smaller, a merger with a firm always achieves larger in-house cost reductions.■

Proof of Lemma 4.

We split up Lemma 4 in the following 5 statements, which we prove successively.

1. There exists a unique $\tilde{p}_l^*\left(\tilde{\sigma}\right)<\overline{p}$. Firm l wants to win rather than to lose if and only if the price is larger than $\tilde{p}_l^*\left(\tilde{\sigma}\right)$.
2. There exists a unique $\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)<\overline{p}$. Firm h wants to win rather than to lose against l if and only if the price is larger than $\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$.
3. There exists a unique $\tilde{p}_{h-h}^*\left(\tilde{\sigma}\right)<\overline{p}$. Firm h wants to win rather than to lose against h if and only if the price is larger than $\tilde{p}_{h-h}^*\left(\tilde{\sigma}\right)$.
4. For positive $\tilde{\sigma}$, we have that $\tilde{p}_l^*\left(\tilde{\sigma}\right)<\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$.
5. For positive $\tilde{\sigma}$, we have that $\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)<\tilde{p}_{h-h}^*\left(\tilde{\sigma}\right)$.

1.

We first show that $\tilde{p}^*_{h-l}(\tilde{\sigma}) < \overline{p}$ always exists and is unique. We again follow the techniques used in Kamien et al. (1989). Consider a high-cost firm h. We use (13), (14) and (2) to write the difference between winning and losing against l as

$$\tilde{W}_h(\tilde{p}) - \tilde{L}_{h-l}(\tilde{p}) = Q(\tilde{p})\left(\tilde{p} - \frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{1}{N-1}\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})}\right).$$

We define $G(\tilde{p}) \equiv \tilde{p} - \frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{1}{N-1}\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})}$. Then

$$G'(\tilde{p}) = 1 - \underbrace{\frac{\partial\left[\frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})}\right]}{\partial Q}Q'(\tilde{p})}_{<0} - \underbrace{\frac{\partial\left(\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})}\right)}{\partial Q}Q'(\tilde{p})}_{\leq 0} - \frac{1}{N-1}\underbrace{\frac{\partial\left(\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})}\right)}{\partial Q}Q'(\tilde{p})}_{\leq 0} > 0\cdot$$

where we sign the last three terms by using (A1), A2) and (A3).

We also know that $G(0) = -\frac{\tilde{C}_{IND}(Q(0))}{Q(0)} - \frac{\tilde{\sigma}\tilde{S}_h(Q(0))}{Q(0)} - \frac{1}{N-1}\frac{\tilde{\sigma}\tilde{S}_l(Q(0))}{Q(0)} < 0$ and that

$\lim_{\tilde{p}\to\overline{p}} G(\tilde{p}) = \lim_{\tilde{p}\to\overline{p}} \frac{\tilde{W}_h(\tilde{p}) - \tilde{L}_{h-l}(\tilde{p})}{Q(\tilde{p})} > 0$, where we use (A6).

Since $G(0) < 0$, $\lim_{\tilde{p}\to\overline{p}} G(\tilde{p}) > 0$ and $G'(\tilde{p}) > 0$, there must exists a unique price $0 < \tilde{p}^*_{h-l}(\tilde{\sigma}) < \overline{p}$ for which $G(\tilde{p}^*_{h-l}(\tilde{\sigma})) = 0$. See that $G(\tilde{p}^*_{h-l}(\tilde{\sigma})) = 0$ coincides with (17). Firm h wants to win rather than to lose against l if and only if the price exceeds $\tilde{p}^*_{h-l}(\tilde{\sigma})$.

2.

Next, we show that $\tilde{p}^*_l(\tilde{\sigma}) < \overline{p}$ always exists and is unique. Consider a low-cost firm l. We use (20), (21) and (2) to write the difference between winning and losing as

$$\tilde{W}_l(\tilde{p}) - \tilde{L}_l(\tilde{p}) = Q(\tilde{p})\left(\tilde{p} - \frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})}\right).$$

We define $F(\tilde{p}) \equiv \tilde{p} - \frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})} - \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})}$. Then

$$F'(\tilde{p}) = 1 - \underbrace{\frac{\partial \left[\frac{\tilde{C}_{IND}(Q(\tilde{p}))}{Q(\tilde{p})}\right]}{\partial Q}Q'(\tilde{p})}_{<0} - \underbrace{\frac{\partial \left(\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}))}{Q(\tilde{p})}\right)}{\partial Q}Q'(\tilde{p})}_{\leq 0} - \frac{2}{N-1}\underbrace{\frac{\partial \left(\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}))}{Q(\tilde{p})}\right)}{\partial Q}Q'(\tilde{p})}_{\leq 0} > 0.$$

where we sign the last three terms by using (A1), (A2) and (A3).

We also know that $F(0) = -\frac{\tilde{C}_{IND}(Q(0))}{Q(0)} - \frac{\tilde{\sigma}\tilde{S}_l(Q(0))}{Q(0)} - \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h(Q(0))}{Q(0)} < 0$ and that

$$\lim_{\tilde{p}\to\bar{p}} F(\tilde{p}) = \lim_{\tilde{p}\to\bar{p}} \frac{\tilde{W}_l(\tilde{p}) - \tilde{L}_l(\tilde{p})}{Q(\tilde{p})} > 0$$

where we use (A6).

Since $F(0) < 0$, $\lim_{\tilde{p}\to\bar{p}} F(\tilde{p}) > 0$ and $F'(\tilde{p}) > 0$, there must exists a unique price $0 < \tilde{p}_l^*(\tilde{\sigma}) < \bar{p}$ for which $F(\tilde{p}_l^*(\tilde{\sigma})) = 0$. See that $F(\tilde{p}_l^*(\tilde{\sigma})) = 0$ coincides with (23).

3.

Third, we know that $\tilde{p}_{h-h}^*(\tilde{\sigma}) < \bar{p}$ always exists and is unique because $\tilde{p}_{h-h}^*(\tilde{\sigma})$ inherits the properties of $p_i^*(\sigma)$ given by (10) and lemma 1.

4.

We prove that $\tilde{p}_l^*(\tilde{\sigma}) < \tilde{p}_{h-l}^*(\tilde{\sigma})$ by contradiction. If we would have $\tilde{p}_l^*(\tilde{\sigma}) \geq \tilde{p}_{h-l}^*(\tilde{\sigma})$, we know from (17) and (23) that

$$\begin{aligned} &\frac{\tilde{C}_{IND}(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))} + \frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))} + \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))} \\ &\geq \frac{\tilde{C}_{IND}(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} + \frac{1}{N-2}\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} + \frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} \end{aligned}$$

We rearrange and obtain

$$\begin{aligned} &\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))} + \frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))} - \frac{1}{N-2}\frac{\tilde{\sigma}\tilde{S}_l(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} - \frac{\tilde{\sigma}\tilde{S}_h(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} \\ &\geq \frac{\tilde{C}_{IND}(Q(\tilde{p}_{h-l}^*(\tilde{\sigma})))}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))} - \frac{\tilde{C}_{IND}(Q(\tilde{p}_l^*(\tilde{\sigma})))}{Q(\tilde{p}_l^*(\tilde{\sigma}))}. \end{aligned}$$

The right hand side exceeds zero because $\tilde{p}_l^*(\tilde{\sigma}) \geq \tilde{p}_{h-l}^*(\tilde{\sigma})$, (A1) and (A2). We would thus have

$$\frac{\tilde{\sigma}\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}+\frac{2}{N-1}\frac{\tilde{\sigma}\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}-\frac{1}{N-2}\frac{\tilde{\sigma}\tilde{S}_l\left(Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))}-\frac{\tilde{\sigma}\tilde{S}_h\left(Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))}\geq 0.$$

If $\tilde{\sigma}>0$, we can divide both sides by $\tilde{\sigma}$. Also, from $\tilde{p}_l^*\left(\tilde{\sigma}\right)\geq \tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$, (A1) and (A3), we know that, by interchanging $\tilde{p}_l^*\left(\tilde{\sigma}\right)$ and $\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$ in the last term, the previous inequality must imply

$$\frac{\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}+\frac{2}{N-1}\frac{\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}-\frac{1}{N-2}\frac{\tilde{S}_l\left(Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))}-\frac{\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}\geq 0.$$

We rearrange and would obtain

$$\frac{\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}-\frac{1}{N-2}\frac{\tilde{S}_l\left(Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_{h-l}^*(\tilde{\sigma}))}-\frac{N-3}{N-1}\frac{\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}\geq 0.$$

From $\tilde{p}_l^*\left(\tilde{\sigma}\right)\geq \tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$, (A1) and (A3), we also know that we can interchange $\tilde{p}_l^*\left(\tilde{\sigma}\right)$ and $\tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$ in the middle term. We would have

$$\frac{\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}-\frac{1}{N-2}\frac{\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}-\frac{N-3}{N-1}\frac{\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{Q(\tilde{p}_l^*(\tilde{\sigma}))}\geq 0,$$

which can be written as

$$\frac{\tilde{S}_l\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}{\tilde{S}_h\left(Q(\tilde{p}_l^*(\tilde{\sigma}))\right)}\geq\frac{N-2}{N-1}.$$

We rearrange the inequality and obtain that there must exist a quantity Q for which $-\frac{\tilde{S}_l(Q)}{\tilde{S}_h(Q)}\leq\frac{-N+2}{N-1}$, or equivalently $1-\frac{\tilde{S}_l(Q)}{\tilde{S}_h(Q)}\leq\frac{1}{N-1}$. By multiplying both sides by $\tilde{S}_h\left(Q\right)$ we obtain

$$\tilde{S}_h(Q)-\tilde{S}_l(Q)\leq\frac{1}{N-1}\tilde{S}_h(Q).$$

From (2) we know that the inequality equals

$$\tilde{C}_h(Q)-\tilde{C}_l(Q)\leq\frac{1}{N-1}\tilde{S}_h(Q),$$

which contradicts (12). So $\tilde{p}_l^*\left(\tilde{\sigma}\right)\geq \tilde{p}_{h-l}^*\left(\tilde{\sigma}\right)$ cannot hold if $\tilde{\sigma}>0$.

5.

Finally, we show that $\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right) < p^*_{h-h}\left(\tilde{\sigma}\right)$ by contradicting that $\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right) \geq p^*_{h-h}\left(\tilde{\sigma}\right)$.

We use (17) and (19) to write

$$\begin{aligned}&\frac{\tilde{C}_{IND}\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}+\frac{1}{N-2}\tilde{\sigma}\frac{\tilde{S}_l\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}+\tilde{\sigma}\frac{\tilde{S}_h\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}\\ &\geq\frac{C_{IND}\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}+\frac{N}{N-1}\tilde{\sigma}\frac{S_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}.\end{aligned}$$

We rearrange the inequality to get

$$\begin{aligned}&\overbrace{\frac{\tilde{C}_{IND}\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}-\frac{\tilde{C}_{IND}\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}}^{\leq 0}+\overbrace{\tilde{\sigma}\frac{\tilde{S}_h\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}-\tilde{\sigma}\frac{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}}^{\leq 0}\\ &\geq\frac{1}{N-1}\tilde{\sigma}\frac{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}-\frac{1}{N-2}\tilde{\sigma}\frac{\tilde{S}_l\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}.\end{aligned}$$

The left hand side is negative because $\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right) \geq p^*_{h-h}\left(\tilde{\sigma}\right)$, (A1), (A2) and (A3). Hence, we would have

$$\frac{1}{N-1}\tilde{\sigma}\frac{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{Q(p^*_{h-h}\left(\tilde{\sigma}\right))}-\frac{1}{N-2}\tilde{\sigma}\frac{\tilde{S}_l\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}\leq 0.$$

For positive $\tilde{\sigma}$, we can rewrite the expression as

$$\frac{\tilde{S}_l\left(Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))\right)\big/Q(\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right))}{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)\big/Q(p^*_{h-h}\left(\tilde{\sigma}\right))}\geq\frac{N-2}{N-1}.$$

From $\tilde{p}^*_{h-l}\left(\tilde{\sigma}\right) \geq p^*_{h-h}\left(\tilde{\sigma}\right)$, (A1) and (A3), this also implies that

$$\frac{\tilde{S}_l\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)\big/Q(p^*_{h-h}\left(\tilde{\sigma}\right))}{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)\big/Q(p^*_{h-h}\left(\tilde{\sigma}\right))}\geq\frac{N-2}{N-1} \text{ and hence}$$

$$\frac{\tilde{S}_l\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}{\tilde{S}_h\left(Q(p^*_{h-h}\left(\tilde{\sigma}\right))\right)}\geq\frac{N-2}{N-1}.$$

We rearrange the inequality and obtain that there must exist a quantity Q for which $-\frac{\tilde{S}_l(Q)}{\tilde{S}_h(Q)} \leq \frac{-N+2}{N-1}$, or equivalently $1-\frac{\tilde{S}_l(Q)}{\tilde{S}_h(Q)} \leq \frac{1}{N-1}$. By multiplying both sides by $\tilde{S}_h(Q)$ we obtain $\tilde{S}_h(Q)-\tilde{S}_l(Q) \leq \frac{1}{N-1}\tilde{S}_h(Q)$. From (2) we know that the inequality equals $\tilde{C}_h(Q)-\tilde{C}_l(Q) \leq \frac{1}{N-1}\tilde{S}_h(Q)$, which contradicts (12). We conclude that $\tilde{p}^*_{h-l}(\tilde{\sigma}) < p^*_{h-h}(\tilde{\sigma})$. ■

Proof of Lemma 5.

We investigate the two possible scenarios, $\tilde{\sigma}=0$ or $0<\tilde{\sigma}\leq 1$. We prove that, if $\tilde{p}^*_{h-l}(\tilde{\sigma}) \leq p^M$, it is the unique equilibrium.

If $\tilde{\sigma}=0$.

Since $\tilde{\sigma}=0$, firms earn zero profits by subcontracting. Hence, they compete fiercely for the market. Since each firm can contract at the contractor's marginal cost, in equilibrium, each firm charges the average industry costs, which are given by $\tilde{p}^*_{h-l}(0)$. Note that $\tilde{p}^*_{h-l}(0)$ coincides with $\tilde{p}^*_l(0)$. Firms do not prefer to charge a higher price because it leads to zero profits as well. Charging a lower price results in losses.

From Lemma 4, we know that any higher price cannot be an equilibrium because firms would then prefer to win rather than to lose. Any lower price can also not be an equilibrium because firms would then prefer to lose rather than to win.

If $0<\tilde{\sigma}\leq 1$.

The main text proves that, if $\tilde{\sigma}>0$, in any equilibrium, firm l wins. We further prove that, if $\tilde{\sigma}>0$ and $\tilde{p}^*_{h-l}(\tilde{\sigma}) \leq \tilde{p}^M$, the unique equilibrium is where h charges $\tilde{p}^*_{h-l}(\tilde{\sigma})$ and l undercuts $\tilde{p}^*_{h-l}(\tilde{\sigma})$ by an infinitesimally small amount.

We first check that our equilibrium is indeed an equilibrium. The high-cost firm does not want to deviate from charging $\tilde{p}^*_{h-l}(\tilde{\sigma})$. Clearly, the firm cannot gain from losing for sure because firm h is indifferent between winning and losing at $\tilde{p}^*_{h-l}(\tilde{\sigma})$. Also, firm h cannot profitably reduce its price below $\tilde{p}^*_{h-l}(\tilde{\sigma})$ if $\tilde{W}_h^{'}(\tilde{p})\big|_{\tilde{p}<\tilde{p}^*_{h-l}(\tilde{\sigma})\leq\tilde{p}^M} \geq 0$, which we prove by contradiction. Suppose otherwise, then (i) we would have $\tilde{W}_h^{'}(\tilde{p})\big|_{\tilde{p}<\tilde{p}^*_{h-l}(\tilde{\sigma})\leq\tilde{p}^M} < 0$ and (ii) from (3) and (A1) we always have $\tilde{L}_l^{'}(\tilde{p})<0$ and

$\tilde{L}_{h-h}^{'}\left(\tilde{p}\right)<0$. As a consequence, we would have $\left.\tilde{\pi}_{IND}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}=\left.\left[\tilde{W}_{h}^{'}\left(\tilde{p}\right)+\left(N-3\right)\tilde{L}_{h-h}^{'}\left(\tilde{p}\right)+\tilde{L}_{l}^{'}\left(\tilde{p}\right)\right]\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}<0$, which contradicts (A4).

Firm l undercuts $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)$ by an infinitesimally small amount in equilibrium. Firm l does not want to deviate by charging a higher price because $\tilde{W}_{l}\left(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)\right)\geq\tilde{L}_{l}\left(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)\right)$ from $\tilde{p}_{l}^{*}\left(\tilde{\sigma}\right)<\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)$ and lemma 4. Charging a lower price is also not profitable because $\left.\tilde{W}_{l}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}\geq0$, which we prove by contradiction. Suppose otherwise, then (i) we would have $\left.\tilde{W}_{l}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}<0$ and (ii) from (3) and (A1) we always have $\tilde{L}_{h-l}^{'}\left(p\right)<0$. As a consequence, we would have $\left.\tilde{\pi}_{IND}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}=\left.\left[\tilde{W}_{l}^{'}\left(\tilde{p}\right)+\left(N-2\right)\tilde{L}_{h-l}^{'}\left(\tilde{p}\right)\right]\right|_{\tilde{p}<\tilde{p}_{h-l}^{*}(\tilde{\sigma})\leq\tilde{p}^{M}}<0$, which contradicts (A4).

Second, we check that our equilibrium is the only equilibrium. A higher price equilibrium would not be possible because then firm h would prefer to win instead. In principle, lower price equilibria, i.e. any price between $\tilde{p}_{l}^{*}\left(\tilde{\sigma}\right)$ and $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)$, are also possible. But charging $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)$ weakly dominates charging a lower price for firm h, because at lower prices firm h always prefers to lose rather than to win (see Lemma 4).■

Proof of proposition 3.

We investigate the three possible scenarios: $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}$, $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)>\tilde{p}^{M}$ and $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)=\tilde{p}^{M}$.

1. Suppose $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}$. From Lemma 5 we know that $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)$ is the equilibrium price of the price-setting subgame. From concavity of the profit function, see (A4), we then know that $\left.\tilde{\pi}_{IND}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}}>0$. Proposition 3 claims that then $\tilde{\sigma}^{*}=1$. We prove by contradiction. Suppose otherwise, namely that $\tilde{\sigma}^{*}<1$. From (17) we get

$$\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)-\frac{\tilde{C}_{IND}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))}-\frac{1}{N-2}\frac{\tilde{\sigma}\tilde{S}_{l}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))}-\frac{\tilde{\sigma}\tilde{S}_{h}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))\right)}{Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right))}=0.$$

We apply the implicit function theorem to obtain the effect of $\tilde{\sigma}$ on profits

$$\left.\tilde{\pi}_{IND}^{'}\left(\tilde{p}\right)\right|_{\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}}\ \tilde{p}_{h-l}^{*}{}^{'}\left(\tilde{\sigma}^{*}\right)$$

$$= \underbrace{\tilde{\pi}_{IND}^{'}\left(\tilde{p}\right)\Big|_{\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}}}_{>0} \frac{\overbrace{\tilde{S}_{h}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right))\right)+\frac{1}{N-1}\tilde{S}_{l}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right))\right)}^{>0}}{\underbrace{\tilde{\pi}_{IND}^{'}\left(p\right)\Big|_{\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}}}_{>0} \underbrace{-\tilde{\sigma}^{*}\tilde{S}_{h}^{'}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right))\right)Q^{'}\left(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)\right)-\frac{1}{N-1}\tilde{\sigma}^{*}\tilde{S}_{l}^{'}\left(Q(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right))\right)Q^{'}\left(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)\right)}_{>0}} > 0,$$

where the denominator is positive because of (A1), (3), (A4) and $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)<\tilde{p}^{M}$, firms could do better by choosing a higher $\tilde{\sigma}$, so that $\tilde{\sigma}^{*}<1$ would not be optimal, a contradiction.

2. Suppose $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)>\tilde{p}^{M}$. Proposition 3 claims that this never occurs. We show by contradiction and proceed in two steps.

Step one proves that $\tilde{p}_{h-l}^{*}\left(0\right)<\tilde{p}^{M}$. We show by contradiction. Suppose otherwise, namely that $\tilde{p}_{h-l}^{*}\left(0\right)\geq\tilde{p}^{M}$.

We know that monopoly profits are strictly positive ($\tilde{\pi}_{IND}^{M}>0$), so that the monopoly price must exceed the average industry variable cost

$$\tilde{p}^{M} > \frac{\tilde{C}_{IND}\left(Q(\tilde{p}^{M})\right)}{Q(\tilde{p}^{M})}.$$

We also know from (17) that if $\tilde{\sigma}=0$, firms choose

$$\tilde{p}_{h-l}^{*}\left(0\right)=\frac{\tilde{C}_{IND}\left(Q(\tilde{p}(0))\right)}{Q(\tilde{p}(0))}.$$

If $\tilde{p}_{h-l}^{*}\left(0\right)\geq\tilde{p}^{M}$, we would have that $\dfrac{\tilde{C}_{IND}\left(Q(\tilde{p}_{h-l}^{*}\left(0\right))\right)}{Q(\tilde{p}_{h-l}^{*}\left(0\right))}>\dfrac{\tilde{C}_{IND}\left(Q(\tilde{p}^{M})\right)}{Q(\tilde{p}^{M})}$. But this would contradict (A2), because $Q(\tilde{p}_{h-l}^{*}\left(0\right))<Q(\tilde{p}^{M})$ from $\tilde{p}_{h-l}^{*}\left(0\right)>\tilde{p}^{M}$ and (A1). Step one concludes that $\tilde{p}_{h-l}^{*}\left(0\right)<\tilde{p}^{M}$.

Step two contradicts that, if $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)>\tilde{p}^{M}$, $\tilde{\sigma}^{*}$ is optimal. From step one, we know that we would have $\tilde{p}_{h-l}^{*}\left(0\right)<\tilde{p}^{M}<\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)$. But since $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}\right)$ is continuous, there must then exist a $0\leq\tilde{\sigma}^{'}<\tilde{\sigma}^{*}$ for which $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{'}\right)=\tilde{p}^{M}$ and firms earn monopoly profits. From strict concavity of the profit function, we know that the only candidate equilibrium $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)$ yields smaller profits

$\tilde{\pi}_{IND}\left(\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)\right)<\tilde{\pi}_{IND}\left(\tilde{p}^{M}\right)$. Hence, firms could increase profits by choosing $\tilde{\sigma}'$ instead of $\tilde{\sigma}^{*}$, so that $\tilde{\sigma}^{*}$ would not be optimal, a contradiction. Hence, we never have $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)>\tilde{p}^{M}$.

3. The final possibility is that the optimal ex ante subcontract results in the monopoly price $\tilde{p}_{h-l}^{*}\left(\tilde{\sigma}^{*}\right)=\tilde{p}^{M}$. There is a share $0<\tilde{\sigma}^{*}\leq 1$. Note from step one of scenario 2 that we can exclude the possibility that $\tilde{\sigma}^{*}=0$.■

Proof of proposition 6.

We first prove that, if the pre-merger price equals the monopoly price, the merger must be profitable.

From proposition 3, we know that there are two possible scenarios after the merger. The equilibrium price equals the monopoly price $\tilde{p}^{M}$, or the equilibrium price is lower and equals $\tilde{p}_{h-l}^{*}(1)$.

Suppose the equilibrium price equals the monopoly price $\tilde{p}^{M}$ after the merger. Then, since we know from proposition 4 and 5 that the merger does not increase the price, the equilibrium price was the monopoly price before the merger as well. Industry profits are therefore the same before and after the merger. As a consequence, to prove that the merger is profitable, it suffices to show that after the merger, the merged firm earns more than two outside firms. Formally we can write that the merged firm earns

$$\underbrace{W_l\left(\tilde{p}^{M}\right)}_{\text{profits merged firm}}>L_l\left(\tilde{p}^{M}\right)=2L_{h-h}\left(\tilde{p}^{M}\right)>2\underbrace{L_{h-l}\left(\tilde{p}^{M}\right)}_{\text{profits outside firm}}.$$

The first inequality follows from the fact that firm l prefers to win rather than to lose in equilibrium. The next equality represents that firm l, from losing against h, earns twice the profits of an outside firm that would lose against h. The final inequality captures that firm h prefers to lose against h rather than lose against l, because the surplus from subcontracting would be lower if l wins.

Suppose the equilibrium price equals $\tilde{p}_{h-l}^{*}(1)$ after the merger. The merger is then profitable because

$$\underbrace{W_l\left(\tilde{p}_{h-l}^{*}(1)\right)}_{\text{profits merged firm}}>L_l\left(\tilde{p}_{h-l}^{*}(1)\right)=2L_{h-h}\left(\tilde{p}_{h-l}^{*}(1)\right)=2\frac{1}{N-1}S_h\left(Q\left(\tilde{p}_{h-l}^{*}(1)\right)\right)\geq\underbrace{2\frac{1}{N-1}\sigma^{*}S_h\left(Q\left(\min\left\{p^{M},p_i^{*}(1)\right\}\right)\right)}_{\text{pre-merger profits}}.$$

The first inequality follows from the fact that firm l prefers to win rather than to lose in equilibrium. The next equality represents that firm l, from losing against h, earns twice the profits of an outside firm that would lose against h. Since $\tilde{\sigma}^*=1$, these profits equal $2\frac{1}{N-1}S_h\left(Q\left(\tilde{p}_{h-l}^*(1)\right)\right)$. These profits exceed the last expression, which represents twice the pre-merger profits, because

(i) $0\le\sigma^*\le 1$, and

(ii) from proposition 4 and 5 we know that the merger decreased the price ($\tilde{p}_{h-l}^*(1)\le\min\left\{p^M, p_i^*(1)\right\}$), so that $S_h\left(Q\left(\tilde{p}_{h-l}^*(1)\right)\right)\ge S_h\left(Q\left(\min\left\{p^M, p_i^*(1)\right\}\right)\right)$ from (A1) and (3).■

Proof of proposition 7

From proposition 1 and 3 we know that, either firms give all surplus to the subcontractors ($\sigma^*=1$ or $\tilde{\sigma}^*=1$), or firms charge the monopoly price p^M in equilibrium. Clearly, the socially optimal price p^{SO} is smaller than the monopoly price p^M because the industry maximizes profits by applying a positive markup above the industry marginal cost. So the remainder of the proof considers $\sigma^*=1$ for the symmetric model and $\tilde{\sigma}^*=1$ for the asymmetric model.

Symmetric model. We start by considering the symmetric model. We prove by contradiction. Suppose $p^{SO}-p_i^*(1)>0$. We plug in the expressions for p^{SO} and $p_i^*(1)$ to obtain

$$C_{IND}^{'}\left(Q(p^{SO})\right)-\frac{C_{IND}\left(Q(p_i^*(1))\right)}{Q(p_i^*(1))}-\frac{N}{N-1}\frac{S_i\left(Q(p_i^*(1))\right)}{Q(p_i^*(1))}>0.$$

From $p_i^*(1)<p^{SO}$, (A1) and (A2), we can replace p^{SO} by $p_i^*(1)$ in the first term so that

$$C_{IND}^{'}\left(Q(p_i^*(1))\right)-\frac{C_{IND}\left(Q(p_i^*(1))\right)}{Q(p_i^*(1))}-\frac{N}{N-1}\frac{S_i\left(Q(p_i^*(1))\right)}{Q(p_i^*(1))}>0,$$

so there must exist a quantity Q for which

$$C_{IND}^{'}(Q)-\frac{C_{IND}(Q)}{Q}-\frac{N}{N-1}\frac{C_i(Q)}{Q}+\frac{N}{N-1}\frac{C_{IND}(Q)}{Q}>0.$$

We rearrange and obtain that

$$p^{SO}-p_i^*(1)>0\Rightarrow C_{IND}^{'}(Q)-\frac{N}{N-1}\frac{C_i(Q)}{Q}+\frac{1}{N-1}\frac{C_{IND}(Q)}{Q}>0.$$

The following expressions are useful for the remainder of the proof.

In our model, subcontracts allocate production Q cost-efficiently over firms. We denote the number of units of production allocated to firm i by $q_i = Q/N$. The sum of all firms' production meets market demand such that $\sum q_i = Q$. The sum of all firm's production costs is the minimal industry cost $C_{IND}(Q) = \sum_{i\in N} C_i(q_i)$. Since marginal costs are increasing, see (A2), we know that we must have

$$C_i(Q) \geq C_i(q_i) + C_i^{'}(q_i)(Q - q_i).$$

Also, since after subcontracting, each symmetric firm produces $q_i = Q/N$ at the same industry marginal cost so that $C_i(q_i) = \frac{1}{N} C_{IND}(Q)$ and $C_i^{'}(q_i) = C_{IND}^{'}(Q)$. We get

$$C_i(Q) \geq \frac{1}{N} C_{IND}(Q) + C_{IND}^{'}(Q)\frac{N-1}{N}Q$$

Hence, $p^{SO} - p_i^*(1) > 0$ implies that

$$C_{IND}^{'}(Q) - \frac{N}{N-1}\frac{\frac{1}{N}C_{IND}(Q) + C_{IND}^{'}(Q)\frac{N-1}{N}Q}{Q} + \frac{1}{N-1}\frac{C_{IND}(Q)}{Q} > 0,$$

which is rearranged to get $0 > 0$, a contradiction. We conclude that $p^{SO} \leq p_i^*(1)$.

Asymmetric model. The proof after the merger is analogous. We prove that $p^{SO} - \tilde{p}_{h-l}^*(1) \leq 0$ by contradiction. Suppose $p^{SO} - \tilde{p}_{h-l}^*(1) > 0$. We plug in the expressions for p^{SO} and $\tilde{p}_{h-l}^*(1)$ to obtain.

$$C_{IND}^{'}\left(Q(p^{SO})\right) - \frac{C_{IND}\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} - \frac{1}{N-2}\frac{S_l\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} - \frac{S_h\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} > 0.$$

From $\tilde{p}_{h-l}^*(1) < p^{SO}$, (A1) and (A2), we can replace p^{SO} in the first term by $\tilde{p}_{h-l}^*(1)$. We get

$$C_{IND}^{'}\left(Q(\tilde{p}_{h-l}^*(1))\right) - \frac{C_{IND}\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} - \frac{1}{N-2}\frac{S_l\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} - \frac{S_h\left(Q(\tilde{p}_{h-l}^*(1))\right)}{Q(\tilde{p}_{h-l}^*(1))} > 0.$$

As a consequence, there must exist a quantity Q for which

$$C'_{IND}(Q)-\frac{1}{N-2}\frac{C_l(Q)}{Q}-\frac{C_h(Q)}{Q}+\frac{1}{N-2}\frac{C_{IND}(Q)}{Q}>0.$$

The following expressions are useful for the remainder of the proof.

In our model, subcontracts allocate production Q cost-efficiently over firms. We denote the number of units of production allocated to firm l by $q_l = 2Q/N$. Each firm h produces $q_h = Q/N$. The sum of all firms' production meets market demand such that $q_l + \sum_h q_h = Q$. The sum of all firm's production costs is the minimal industry cost $C_{IND}(Q) = C_l(q_l) + \sum_h C_h(q_h)$. Since marginal costs are increasing, see (A2), we know that we must have

$$C_h(Q) \geq C_h(q_h) + C'_h(q_h)(Q-q_h) \text{ and}$$

$$C_l(Q) \geq C_l(q_l) + C'_l(q_l)(Q-q_l).$$

Also, since after subcontracting, h produces $q_h = Q/N$ and l produces $2Q/N$, all at the same industry marginal cost so that $C_h(q_h) = \frac{1}{N}C_{IND}(Q)$, $C_l(q_l) = \frac{2}{N}C_{IND}(Q)$ and $C'_h(q_h) = C'_l(q_l) = C'_{IND}(Q)$. We get

$$C_h(Q) \geq \frac{1}{N}C_{IND}(Q) + C'_{IND}(Q)\frac{N-1}{N}Q \text{ and}$$

$$C_l(Q) \geq \frac{2}{N}C_{IND}(Q) + C'_{IND}(Q)\frac{N-2}{N}Q.$$

Hence, $p^{SO} - \tilde{p}^*_{h-l}(1) > 0$ implies that there exists a Q for which

$$C'_{IND}(Q)-\frac{1}{N-2}\frac{\frac{2}{N}C_{IND}(Q)+C'_{IND}(Q)\frac{N-2}{N}Q}{Q}-\frac{\frac{1}{N}C_{IND}(Q)+C'_{IND}(Q)\frac{N-1}{N}Q}{Q}+\frac{1}{N-2}\frac{C_{IND}(Q)}{Q}>0$$

,

which can be rearranged to get $0 > 0$, a contradiction. We conclude that $p^{SO} \leq \tilde{p}^*_{h-l}(1)$.■

Proof of proposition 8.

From proposition 1, we know that either (i) $p_i^*(\sigma^*) = p^M$ and $0 \le \sigma^* \le 1$ or (ii) $p_i^*(\sigma^*) < p^M$ and $\sigma^* = 1$. We investigate both scenarios.

(i) $p_i^*(\sigma^*) = p^M$

If firms set $\sigma_c^* = \sigma^*$, the best response in the one-shot game is to charge the monopoly price. Any discount factor then sustains collusion.

(ii) $p_i^*(1) < p^M$.

We proceed in two steps.

The first step shows that firms deviate by just undercutting the collusive price. From lemma 1 we know that firms want to win rather than lose if and only if the price exceeds $p_i^*(\sigma_c^*)$. Since $p^M > p_i^*(1) \ge p_i^*(\sigma_c^*)$, firms want to deviate from the collusive price p^M by winning.

Moreover, they just undercut the collusive price by an arbitrarily small amount. In particular, a firm cannot profitably charge a lower price $p < p^M$ because

$$W_i^{'}(p)\big|_{p<p^M} \ge 0,$$

which we prove by contradiction. Suppose otherwise, then (i) we would have $W_i^{'}(p)\big|_{p<p^M} < 0$ and (ii) from (3) and (A1) we always have $L_i^{'}(p) < 0$. As a consequence, we would have $\pi_{IND}^{'}(p)\big|_{p<p^M} = \left[W_i^{'}(p) + (N-1)L_i^{'}(p)\right]\big|_{p<p^M} < 0$, which contradicts (A4).

The second step shows that

$$\sigma_c^* = 1 = \arg\min_{0 \le \sigma_c \le 1} \delta^C(\sigma_c),$$

meaning that colluding firms minimize the critical discount factor by choosing $\sigma_c^* = 1$.

The critical discount factor equals

$$\delta^C(\sigma_c^*) = \frac{\pi^D(\sigma_c^*) - \pi_{IND}^M / N}{\pi^D(\sigma_c^*) - \pi_i(p_i^*(\sigma^*))}.$$

Since share σ_c^* only affects deviation profits, firms minimize the critical discount factor by minimizing deviation profits. From step one and (5), we know that

$$\pi^D(\sigma_c^*) = p^M Q(p^M) - C_i\left(Q(p^M)\right) + (1-\sigma_c^*)S_i\left(Q(p^M)\right) = \pi_{IND}^M - \sigma_c^* S_i\left(Q(p^M)\right),$$

which is minimized by choosing $\sigma_c^* = 1$. We therefore have $\sigma_c^* = \sigma^*$. ■

Proof of proposition 9.

From proposition 1, we know that either (i) $p_i^*(\sigma^*) = p^M$ and $0 \le \sigma^* \le 1$ or (ii) $p_i^*(1) < p^M$. We investigate both scenarios.

(i) $p_i^*(\sigma^*) = p^M$

Since the best response in the one shot game is to charge the monopoly price, firms do not want to deviate. Firms do not need to collude to attain monopoly profits. Put differently, here, any discount factor sustains collusion.

(ii) $p_i^*(1) < p^M$.

We show that the critical discount factor exceeds the no subcontracting discount factor. It can easily be checked that, without subcontracts, our model coincides with standard Bertrand competition so that the critical discount factor equals $\left(N-1\right)/N$. We know from proposition 6 that firms choose $\sigma_c^* = 1$. Hence, we want to show that $\delta^C(1) > \left(N-1\right)/N$. This is satisfied if and only if

$$\frac{\pi^D(1) - \pi_{IND}^M / N}{\pi^D(1) - \pi_i(p_i^*(\sigma^*))} > \frac{N-1}{N}.$$

We can write deviation profits as

$\pi^D(\sigma_c^*) = p^M Q(p^M) - C_i\left(Q(p^M)\right) + (1-\sigma_c^*)S_i\left(Q(p^M)\right) = \pi_{IND}^M - S_i\left(Q(p^M)\right)$ and competition profits as $\pi_i(p_i^*(1)) = \dfrac{S_i\left(Q(p_i^*(1))\right)}{N-1}$ (from equation 9). We get

$$\delta^C(1) > \left(N-1\right)/N \Leftrightarrow \frac{\pi_{IND}^M - S_i\left(Q(p^M)\right) - \pi_{IND}^M/N}{\pi_{IND}^M - S_i\left(Q(p^M)\right) - S_i\left(Q(p_i^*(1))\right)/N-1} > \frac{N-1}{N},$$ which is equivalent to

$$\frac{\left(N-1\right)\pi_{IND}^M / N - S_i\left(Q(p^M)\right)}{\pi_{IND}^M - S_i\left(Q(p^M)\right) - S_i\left(Q(p_i^*(1))\right)/\left(N-1\right)} > \frac{N-1}{N}.$$

We multiply the numerators by N and the denominators by $N-1$. We get

$$\frac{\left(N-1\right)\pi_{IND}^{M}-NS_i\left(Q(p^M)\right)}{\left(N-1\right)\pi_{IND}^{M}-\left(N-1\right)S_i\left(Q(p^M)\right)-S_i\left(Q(p_i^*(1))\right)}>1.$$

We can write the equivalent condition

$$\left(N-1\right)\pi_{IND}^{M}-NS_i\left(Q(p^M)\right)>\left(N-1\right)\pi_{IND}^{M}-\left(N-1\right)S_i\left(Q(p^M)\right)-S_i\left(Q(p_i^*(1))\right).$$

We can drop the first terms and obtain

$$-NS_i\left(Q(p^M)\right)>-\left(N-1\right)S_i\left(Q(p^M)\right)-S_i\left(Q(p_i^*(1))\right).$$

We multiply the left and right hand side by -1 and get

$$NS_i\left(Q(p^M)\right)<\left(N-1\right)S_i\left(Q(p^M)\right)+S_i\left(Q(p_i^*(1))\right).$$

We can equivalently write $S_i\left(Q(p^M)\right)<S_i\left(Q(p_i^*(1))\right)$, which is satisfied because $p_i^*\left(1\right)<p^M$, (A1) and (3). ■