

Popular Referendum and Electoral Accountability

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Abstract

This paper studies how the possibility of voters calling for a referendum affects the decision-making of elected representatives. In the absence of direct democracy, elected officials who do not share the preferences of the majority of the electorate may enact their preferred policies even at the cost of decreasing the likelihood of reelection. The introduction of direct democracy diminishes the policy benefits of doing that, as voters may now overturn some of the policy decisions. I show that, as a consequence, elected officials are induced to implement the policies preferred by the voters not only on the issue dimensions that are subject to a possible popular referendum, but also on those that are not. This result holds even when the voters' information about their true interests is limited. The true impact of popular referenda on decision-making may thus be much greater and much more pervasive than has previously been recognized in theoretical and empirical research. Moreover, the lack of expertise of the electorate may be a less severe problem than critics of the referendum argue. Further, the introduction of direct democracy changes the value of information to the voters: whereas in a strictly representative democracy, there are negative spillover effects of information in the sense that being more likely to become informed about some policy dimensions may undermine voters ability to control public officials on other dimensions, the possibility of popular referenda means that additional information improves voter control, including on issues that may be outside the direct democracy domain. JEL Codes: D72.

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1 Introduction

By and large, modern democracies are representative democracies, in the sense that citizens delegate, via elections, political decision-making powers to a small group of public officials. In some democracies, however, citizens retain the power to settle certain policy decisions directly, by holding referenda.¹ Such referenda may be constitutionally mandated, initiated by the government, or initiated by the citizens themselves.² This paper studies how the possibility of citizen-initiated referenda affects the ability of voters to hold elected officials accountable.

The premise behind representative democracy is that elected representatives, given their higher level of expertise and information, will be better able than the people to determine which course of action furthers the public good.³ The essential purpose of elections is to ensure that elected representatives indeed use their higher level of expertise and information to advance the public good, and not their own private interests. Formal models of electoral accountability suggest, however, that the electoral mechanism might fail to induce incumbents to act in accordance with voters' interests.⁴ First, elections may not provide sufficient incentives for elected officials to behave as faithful agents of the electorate if there is a sufficiently large divergence in preference between voters and representatives. Incumbents might then choose to implement policies that hurt the voters even at the cost of foregoing reelec-

¹I use the term 'referendum' to designate a direct vote of the general electorate on a single political question. For a similar definition, see Butler and Ranney (1994).

²For an overview of direct democracy institutions around the world see Butler and Ranney (1994); Uleri and Gallagher (1996).

³Consider the following quote by Madison from the Federalist number 10: The effect of delegation is "to refine and enlarge the public views, by passing them through the medium of a chosen body of citizens, whose wisdom may best discern the true interest of their country, and whose patriotism and love of justice, will be least likely to sacrifice it to temporary or partial considerations. Under such a regulation, it may well happen that the public voice pronounced by the representatives of the people, will be more consonant to the public good, than if pronounced by the people themselves convened for the purpose."

⁴See Ashworth (2012) for a review of the literature on electoral accountability.

tion (see Ferejohn, 1986; Seabright, 1996; Persson, Roland and Tabellini, 1997). Second, in a purely representative democracy, elected officials may choose to implement policies that are popular with the electorate, yet not in the public interest, in order to increase their chances of reelection (see Canes-Wrone, Herron and Shotts, 2001; Maskin and Tirole, 2004; Besley, 2006). In the first case, the electoral mechanism fails because elected officials do not value reelection enough; in the second, because they value reelection too much.

Proponents of direct democracy then argue that the popular referendum, by giving voters the power to overrule certain decisions of their representatives, may serve as a corrective when elected representatives fail to take the preferences of the electorate into account and may thereby improve the congruence between citizen preferences and enacted policies with respect to those policy dimensions about which a referendum may be held. Recent theoretical models give credence to this idea and argue that citizen-initiated referenda may improve congruence on those policy dimensions which may be subjected to a popular vote not only through the direct approval of certain measures by the voters but also indirectly: legislators may change their policy choices *ex ante* to avoid being overturned by a referendum (Gerber, 1996; Besley and Coate, 2008; Hug, 2004).

A common feature of these models is the assumption that the electorate is fully informed about which policies correspond to the public interest. This literature thus does not address the main criticism formulated against direct democracy, namely, that citizens lack the expertise to make wise decisions. This lack of expertise might result in policy decisions that, although popular, are not in the public interest, be it because voters enact such policies in a direct vote (Gerber and Lupia, 1995), or because elected officials follow public opinion in fear of being overturned in a popular referendum (Matsusaka, 2005).

Moreover, the formal theory literature has so far only considered the implications of

citizens' initiatives on policies that may be subjected to a popular vote and disregarded the possibility of broader implications of direct democracy on representative government. An important, informal, argument against direct democracy, however, is that it weakens the authority of elected officials and representatives and therefore undermines the process of representative government (Butler and Ranney, 1994; Mueller, 1996; Broder, 2000).

Using a formal model of electoral accountability, I show, however, that giving voters the power to call for a referendum on certain policy issues not only improves congruence between enacted policies and the true preferences of the citizens on those policy dimensions which can be subjected to a referendum, but, surprisingly, also on those which, perhaps for constitutional reasons, can not. Moreover, I show that this holds true, even when voters are potentially misinformed about their true interests. These results have broad implications for the analysis of the effects of direct democracy on the decision-making of elected representatives. Indeed, the true impact of popular referenda on decision-making may be much greater and much more pervasive than has previously been recognized. Empirical research, in particular, by focussing solely on those dimensions which can be subjected to a referendum (cf. Matsusaka, 1995, 2004; Gerber, 1999; Feld and Kirchgässner, 2001; Feld and Matsusaka, 2003), may have systematically underestimated the true effect of direct democracy. Moreover, the lack of expertise of the electorate may be a less severe problem than critics of the referendum argue.

The intuition behind these results is as follows. Consider an elected official who has to make multiple policy decisions in a given term. Suppose the official both cares about the policies he implements and about retaining office at the end of his term. Moreover, the official may share the preferences of the majority of the electorate, (that is, be *congruent*) or he may have policy preferences that differ from those (be *non-congruent*), while the electorate is uncertain as to the policy preferences of the elected representative. Going

forward the electorate would like to retain congruent representatives over non-congruent ones, and thus congruent incumbents have incentive to signal their policy preferences to the electorate, while non-congruent ones might want to conceal them to increase their chances of reelection. In such a situation non-congruent incumbents essentially face the following trade-off. On the one hand, they may choose the same policies as congruent incumbents thereby concealing their policy preferences and improving their chances of re-election. This, however, comes at the cost of choosing policies the non-congruent incumbent dislikes. On the other hand, non-congruent officials may choose their preferred policies. This, in turn, comes at the risk of revealing to the electorate their ideology and losing re-election. Which option non-congruent incumbents will choose, ultimately depends on the value of holding office relative to the policy costs non-congruent incumbents need to incur to mimic the behavior of congruent incumbents and get reelected. The key insight of my analysis is that these policy costs vary with the institutional setting and the information available to the electorate.⁵ In particular, giving the voters the power to call for a referendum reduces these costs and thus increases the incentive for non-congruent incumbents to seek re-election by adopting policies also chosen by congruent incumbents.

To make this logic fully apparent, consider the extreme case in which voters are fully informed about which policy choice is in their best interest on *all* policy dimensions and are given the power to hold a referendum on *some* policy dimensions. In the absence of the popular referendum, the non-congruent incumbent will need to go against his policy preferences on all policy dimensions in order to appear congruent and gain re-election. The

⁵In this, my argument is related to Fox and Stephenson (2013) who argue that pro-minority judicial review reduces the costs to the pro-minority leader of mimicking the pro-majority leader's behavior, as the pro-minority judiciary will bail out pro-minority leaders from the consequences of pro-majoritarian policy choices. In my model, popular referenda clearly do not bail out non-congruent incumbents. Moreover, Fox and Stephenson (2013) consider a single-dimensional policy space in which voters are fully informed about the policies that are in the public interest and therefore do not generate predictions about spill-over effects of popular referenda and information across policy issues as I do here.

policy costs that non-congruent incumbents must shoulder in order to be re-elected are thus likely to be quite substantial and may outweigh the benefits of holding office. When voters retain the right to amend policy decisions on some policy dimensions via a popular vote, elected officials lose the final say over these policy issues. Whether the non-congruent incumbent chooses his preferred policy or the policy preferred by the electorate does not matter policy wise, as voters will simply overturn, if needed, the policy decision of the elected representative. Hence, to secure re-election the non-congruent incumbent will only need to incur the policy costs of going against his preferences on those policy dimensions which cannot be subjected to a referendum. As a result the benefits of holding office may now be sufficiently high for the non-congruent incumbent to be willing to make this policy sacrifice. Thus, the introduction of the popular referendum strengthens the incentives for non-congruent incumbents to seek re-election by behaving as if congruent instead of implementing their preferred, and possibly unpopular, policies and thus potentially foregoing reelection. Of course, to avoid being distinguished from a congruent incumbent, a non-congruent one will need to mimic the congruent behavior across the full range of policy choices, including on policy dimensions that are not subject to popular referenda.

When the voters' information about optimal policy choices is limited, this stronger office-holding motive may in principle lead the incumbents to pander more often, as critics of the referendum argue. I show, however, that if the electorate is well informed about *some* of the policy issues, the popular referendum improves the congruence between enacted policies and the true preferences of the voters on *all* policy dimensions, even when the voters are very unlikely—even certain not to—acquire information about the optimal policies on other dimensions, in particular those on which they can call for a referendum. Indeed, upon observing that the incumbent behaved poorly on some policy dimension, the voters update unfavorably on the type of the incumbent and on the likelihood that the policy decision

on other dimensions corresponds to the voters' best interest. Despite not knowing which policy is actually optimal, voters may then decide to hold a referendum, thereby effectively constraining the policy benefits that a non-congruent incumbent can receive when choosing his preferred policies and inducing him to seek reelection instead.

The incentives of non-congruent incumbents to seek re-election over choosing their preferred policies also depend in non-trivial, and surprising, ways on the likelihood that voters become informed as to whether implemented policy choices are in their best interest. In the absence of the popular referendum, I show that being more likely to observe the consequences of the elected official's actions with respect to some policy issue A may diminish the ability of voters to hold elected officials accountable on some other policy issue B without improving accountability on policy issue A. In other words, there are negative spillover effects of information for voters. Indeed, when voters are fully informed, non-congruent incumbents need to go against their policy preferences on all policy dimensions and may thus prefer to forego re-election by choosing their preferred policies. On the contrary, when voters are likely to be well informed about some of the policy issues and likely to be poorly informed about others, non-congruent incumbents may conceal their true preferences by mimicking congruent behavior on those dimensions where the electorate is well informed and by choosing popular, yet not optimal for voters, policies on those dimensions about which the voters are less well informed. In that, the non-congruent incumbents may need to give up only some—rather than all—of the policies they prefer in order to get reelected. Interestingly, the introduction of the referendum significantly changes how electoral accountability relates to the probability that voters become informed. In particular, I show that when the benefits of holding office are relatively limited, the ability of the electorate to improve congruence on policy dimensions which cannot be subjected to the popular referendum depends non-monotonically on the likelihood that voters learn whether the policy choice was correct on those dimensions

which can. Moreover, when the benefits of holding office are sufficiently high, the negative spillover effects of information disappear when the possibility of popular referenda is introduced. That is, the ability of the electorate to hold elected officials accountable increases on all policy dimensions as voters become more likely to learn whether implemented policies are in the public interest or not.

The model also generates a rich set of empirical predictions and suggests the need for a shift in the focus of empirical research. First, empirical analyses of the effects of citizen initiatives on decision-making, by solely focusing on policy dimensions subjected to referenda, may have systematically underestimated the influence of direct democracy. Further empirical research on the effects of direct democracy should look beyond those policy fields which can be subjected to a referendum. Second, as non-congruent incumbents are more likely to adopt the behavior of congruent incumbents, states with the possibility of popular referenda should have less politician turnover. Third, in the model, the voters are more likely to hold a referendum when they believe it to be more likely that the incumbent is non-congruent. Hence, incumbents whose policy choice is overturned in a referendum should be less likely to be reelected at the end of their term. Fourth, referenda should be more likely when policy outcomes, on dimensions on which no referendum may be held, are bad. And fifth, referenda should only occur on policy fields where politicians are uncertain about the policy preferences of the electorate.

My paper is related to a number of literatures. First, a series of papers compares the respective benefits of representative democracy versus direct democracy (Persson and Tabellini, 1994; Redoano and Scharf, 2004; Maskin and Tirole, 2004; Binswanger and Prüfer, 2012). Instead, the focus here is on how institutions of direct democracy alter the decision-making process in a representative democracy. As noted by Hug (2009), only few papers study this question. Gerber (1996), Matsusaka and McCarty (2001), and Hug (2004) amend

the agenda setter model (see Romer and Rosenthal, 1978, 1979) by allowing other actors to decide whether a referendum should be held or not. An important feature of these models is that politicians do not have reelection concerns and only care about the policies implemented. In contrast, it is precisely the trade-off between policy preferences and office-motivation that lies behind the results in my model. Besley and Coate (2008) study a citizen-candidate model with a two-dimensional policy space and show that citizen initiatives may improve congruence on those policy dimensions which can be subjected to a referendum.⁶ Finally, Prato and Strulovici (2013) (see also Kessler, 2005) offer an interesting counterpoint and argue that citizen initiatives reduce the incentive for politicians to search for good policies and thereby may negatively affect voters' welfare. They consider a setting where politicians do not know *ex ante* which policy is in the voters' best interests, whereas voters, as in all papers previously cited, do. In contrast, I study the case where politicians are *ex ante* better informed than the electorate. Moreover, none of these papers consider the effects of the popular referendum on policy dimensions which cannot be subjected to a referendum.

My model is also related to the literature on multi-task issues in political agency settings (see Ashworth, 2005; Ashworth and Bueno de Mesquita, 2006; Gehlbach, 2006; Hatfield and Padro i Miquel, 2006; Bueno de Mesquita, 2007; Ashworth and Bueno de Mesquita, 2013; Bueno de Mesquita and Landa, 2013*b,a*; Morelli and Van Weelden, 2013). These papers focus on entirely different sets of issues, however. In particular, in none of these papers is it the case that an institution which, by design, only affects one dimension of policy-making directly, leads indirectly to changes in policy-making on other policy dimensions. One exception is the paper by Daley and Snowberg (2011) who show that campaign finance reform can lead politicians to signal their competence by expending more effort on improving the quality of

⁶In a related paper, Besley and Coate (2003) study a citizen-candidate model with two policy issues: regulating a monopoly and public spending. They show that when a single elected official, who appoints a regulator, is responsible for both policies, the implemented regulation can diverge from voters' preferences. Congruence can then be restored by having the citizens directly elect the regulator.

policy rather than through campaign expenditure. The mechanism is very different from the one in the present model in at least two ways. First, results in Daley and Snowberg (2011) are driven by the assumption that it is cheaper for the incumbent to signal ability through campaign expenditure than through improving policy.⁷ In my model, no such assumption is needed for the identified spillover effect of popular referenda. Second, campaign finance reform does not alter the ability of the electorate to distinguish good and bad types. In my model, however, the introduction of the popular referendum leads non-congruent incumbents to behave like congruent ones more often, thereby reducing the ability of the electorate to select good types.

My analysis is also related to the literature on transparency and accountability. While in classical moral hazard models of electoral agency (see Ferejohn, 1986; Austen-Smith and Banks, 1989; Seabright, 1996; Persson, Roland and Tabellini, 1997; Bueno de Mesquita, 2007), just as in contract theory (see Holmström, 1979; Holmström and Milgrom, 1991) accountability is improved when the politician's actions are observable by the electorate, two arguments against transparency of actions have been developed in the literature. First, a series of papers (see Prat, 2005; Fox, 2007; Ashworth and Shotts, 2010; Morelli and Van Weelden, 2013) has argued that when the electorate observes the incumbent's actions and those actions are in themselves informative as to the incumbent's type, incumbents may be lead to act according to how the principal expects a 'good' incumbent to act a priori, instead of choosing those actions which are in fact best for the electorate. Second, if an agent has to perform several tasks sequentially in order to reach a certain outcome and if the principal is unable to commit ex ante to a certain retention rule, the principal may end up using a reten-

⁷Consider the following summary of the mechanism provided by Daley and Snowberg (2011): 'When senders have multiple signaling channels at their disposal, and receivers cannot credibly commit to ignoring any of these channels, senders will use the least costly signals to reveal their types—even if receivers would prefer different signals were used. (...) Policies that restrict signaling channels that are less preferable to receivers can ... improv(e) receiver welfare at the expense of senders.' P. 342.

tion rule that increases effort in the late stages of the policy-making process at the expense of effort in earlier stages, thereby reducing allocative efficiency of effort across tasks (see Bueno de Mesquita and Landa, 2013*a*). In all these models, however, it is always beneficial for the electorate to obtain more precise measures of the consequences of the agent’s actions. Similarly, Lohmann (1998), Ashworth (2005), and Alesina and Tabellini (2007) show that, in career concerns models of electoral accountability, the incumbent exerts more effort as the policy outcome observed by the electorate is less due to noise. In contrast, in a multi-task signaling model with moral hazard and adverse selection, I show that being more likely to observe the outcomes of implemented policies may reduce the ability of the electorate to hold elected officials accountable. This stands in sharp contrast to results obtained in single-task signaling models of electoral accountability with moral hazard and adverse selection, (see Canes-Wrone, Herron and Shotts, 2001; Maskin and Tirole, 2004; Besley, 2006; Ashworth and Shotts, 2010; Fox and Van Weelden, 2012), where the incumbent is more likely to choose the policy that corresponds to the state of the world, the more likely the electorate is to learn the consequences of the incumbent’s action.⁸ Moreover, I show that there are negative spillover effects of information in that higher transparency of consequences on some policy issues may reduce accountability on other policy issues, even when the issues under consideration are not substitutes. This distinguishes my work from Holmström and Milgrom (1991) who find that, when tasks are substitutes, a lower transparency of performance on one task may decrease effort provision by the agent on this task, as well as on other tasks.

The paper is organized as follows. Section 2 presents the model. In section 3, I study this model in the case where the voters are not directly involved in policy-making. In the subsequent section, I characterize the equilibria when the voters are given the power to call for a referendum on certain policy dimensions. Comparing the equilibria of the model

⁸Note that Fox and Van Weelden (2012) show that this increased congruence between implemented policy and state of the world may decrease the expected welfare of the principal when costs are asymmetric.

presented in sections 3 and 4, I assess the impact of the popular referendum on representative democracy in section 5. In section 6, I show that the results are robust to a specification of the model which allows for incumbents to be congruent on some policy issues but not others. Section 8 concludes.

2 The Model

Consider a representative democracy consisting of a representative Voter (V) and an Incumbent (I). There are two periods, in which three policies, $p_1, p_2, p_3 \in \{-1, 1\}$, are enacted. Policies p_1 and p_2 are implemented in the first period, whereas policy p_3 is implemented in the second period. For each of these policy issues, the optimal action—the action that is in the public interest—depends on the state of the world, $\omega_1, \omega_2, \omega_3 \in \{-1, 1\}$, that may be unknown to the Voter.⁹ With respect to every policy, the Voter receives a payoff of 1 if the optimal action is implemented and 0 otherwise. The prior probability that $p_i = 1$ is the optimal action is $\alpha \in (\frac{1}{2}, 1]$, so that in the absence of additional information about the state of the world, the Voter would want $p_i = 1$ to be enacted for all the policy issues in the model.¹⁰ Unlike the Voter, the Incumbent is fully informed about the respective states of the world and thus knows which policies are best for him (and which policies are best for the Voter). Although the Voter does not know the states of the world ω_1 and ω_2 ex ante, she learns with certainty the state of the world on dimension p_2 whereas the state of the world ω_1 is revealed to her with probability $q_1 \in [0, 1]$. Henceforth, I refer to q_1 as the probability of feedback. The parameter q_1 can be conceived as the speed with which the consequences of the policies become apparent to the Voter. Alternatively, the probability q_1 can be thought

⁹The assumptions about the information available to the Voter will be detailed below.

¹⁰To simplify notation, I assume the same prior belief for all states of the world, i.e., $Pr(\omega_i = 1) = \alpha \in (\frac{1}{2}, 1]$ for all $i = 1, 2, 3$. Considering different values of α for each policy would not change how the introduction of the popular referendum affects decision-making.

of as a measure of the technicality of the policy issue in question.

An additional tension in the model arises from the fact that the Voter is uncertain about the preferences of the Incumbent. With probability $\pi > 1/2$ the Incumbent has the same preference ranking as the Voter would have if she was fully informed, in which case he is *congruent*. With probability $1 - \pi$ the policy preferences of the Incumbent differ from those of the Voter, in which case he is *non-congruent*.¹¹

In the first period, the Incumbent chooses two policies, p_1 and p_2 . Upon observing the policy choices p_1 and p_2 , and potentially their respective optimality, the Voter can choose to hold a referendum over policy p_1 at a cost $\kappa > 0$.¹² The assumption that the Voter cannot hold a referendum about policy p_2 excludes certain policy fields from the direct control of the Voter. This mirrors constitutional provisions in many countries. Article 75 of the Italian constitution for example specifies that a popular referendum can be requested by five hundred thousand voters. No referendum, however, ‘may be held on a law regulating taxes, the budget, amnesty or pardon, or a law ratifying an international treaty.’ Similar restrictions can be found, for example, in Austria (at the regional level), Brazil, Colombia, Germany (at the regional level), Hungary, Latvia, Slovakia, Serbia, Uruguay, etc. In the United States, among the 24 states that permit citizens’ initiatives, 12 have subject restrictions. For example, in Alaska, Massachusetts, Montana, and Wyoming initiatives may not make appropriations or dedicate revenues. In Arizona, Mississippi, Missouri, and Nevada appropriations require a funding source. Note further that in all the states, statutes are

¹¹One might consider the possibility that the Incumbent is congruent with respect to one of the two first period policies but not the other. As I show in section 6, introducing such semi-congruent types does not alter the fundamental incentive structure of the game and would thus not alter the substance of the results. Excluding semi-congruent types permits a much cleaner presentation and allows us to focus on the main trade-offs captured by the model.

¹²Note that the Voter can hold a referendum over the policy over which she might not be well informed. In the appendix I show that the results are robust to an alternate specification where the Voter learns ω_1 with certainty and ω_2 with probability $q_2 \in [0, 1]$. I focus on the case where $q_1 \in [0, 1]$, as the lack of expertise of the electorate is often presented as one of the main problems of direct democracy.

subject to judicial review which de facto creates certain restrictions on the legislation which can be passed through a referendum. Of course, there also exists jurisdictions where no restrictions apply, most notably the paragon of direct democracy Switzerland.¹³

After her decision to hold a referendum, the Voter chooses whether to reelect the Incumbent or not. I use the following notation to denote the probability with which the Voter reelects the Incumbent based on her observations. Let $r(p_1 = 1, p_2 = \omega_2)$ be the probability that the Voter reelects the incumbent upon observing $p_1 = 1$ and $p_2 = \omega_2$. This corresponds to the case where the Voter observes ω_2 and the Incumbent matched the policy p_2 to the state of the world, yet the Voter does not observe the state of the world ω_1 and thus only observes that the Incumbent set $p_1 = 1$. $r(p_1 = \omega_1, p_2 = \omega_2)$ and so forth are interpreted in the same way.

In the second period, the Incumbent, or the newly elected challenger, selects a third policy, p_3 , and then the game ends. The challenger is also congruent with probability π and non-congruent with probability $1 - \pi$. In the background I therefore assume that there is a mechanism that produces challengers such that the Voter always has a choice between the Incumbent and a challenger and the probability that the challenger is congruent is positive.¹⁴

To summarize, the sequence of the model is as follows:

1. *Period 1.* Nature determines states of the world $\omega_1, \omega_2 \in \{-1, 1\}$
2. Nature determines the Incumbent's type and the challenger's type.

¹³There is a further methodological reason for assuming that the Voter cannot call for a referendum on every policy dimension. Because some of the key aspects of the analysis presented below concern precisely the spillover effects of direct democracy, it is desirable to have a model in which referenda are possible on some but not all policy dimensions.

¹⁴Note that the second period is largely heuristic and makes sure that the Voter prefers to retain a congruent Incumbent and to dismiss a non-congruent one. As such, giving the Incumbent the power to make several policy decisions in the second period, as in the first, would not alter the results.

3. The Incumbent observes ω_1 and ω_2 and chooses policies $p_1, p_2 \in \{-1, 1\}$,
4. The Voter observes p_1, p_2, ω_2 , and, with probability $q_1 \in [0, 1]$, ω_1 and decides whether to hold a referendum over policy p_1 at cost $\kappa > 0$.
5. After its decision to hold a referendum, the Voter chooses whether to reelect the Incumbent.
6. *Period 2.* Nature chooses $\omega_3 \in \{-1, 1\}$.
7. The second-period office-holder observes ω_3 and chooses policy $p_3 \in \{-1, 1\}$.

The Voter receives a policy payoff of 1 for each policy that matches the state of the world ($p_i = \omega_i$) and 0 otherwise. The utility of the Voter can be written compactly as

$$U_V(\mathbf{p}, \boldsymbol{\omega}) = \sum_{i=1}^3 1_{\{p_i = \omega_i\}},$$

where $\mathbf{p} \equiv (p_1, p_2, p_3)$, $\boldsymbol{\omega} \equiv (\omega_1, \omega_2, \omega_3)$ and $1_{\{p_i = \omega_i\}}$ is an indicator function that takes on value 1 whenever $p_i = \omega_i$ and 0 otherwise.

A congruent Incumbent shares the policy preferences of the Voter. For each policy that does not match the state of the world ($p_i \neq \omega_i$) the policy payoff to the non-congruent Incumbent is 1, while it is 0 otherwise. However, the Incumbent does not care about policy when not in office. Thus, as in Maskin and Tirole (2004), elected officials want to leave a legacy and be remembered for their achievements.¹⁵ The Incumbent is also office-motivated receiving an additional benefit of holding office $B \in (0, 1)$ ¹⁶ if retained at the end of period

¹⁵I get substantially similar results in a model in which Incumbents also care about policy while not in office.

¹⁶In section 7, I consider the case where the value of holding office B is greater or equal to 1. I focus on $B < 1$ at first, as this is the more problematic case in the baseline model for solving the principal-agent problem. It is therefore particularly interesting to see that the introduction of the popular referendum improves congruence even in this hardest case.

1. The utility of a congruent Incumbent from implementing policy vector \mathbf{p} when the vector of states of the world is $\boldsymbol{\omega}$ can thus be written as

$$U_C(\mathbf{p}, \boldsymbol{\omega}, r) = \sum_{i=1}^2 1_{\{p_i=\omega_i\}} + q_1 r(p_1, p_2; \omega_1, \omega_2)(B + 1_{\{p_3=\omega_3\}}) + (1 - q_1) r(p_1, p_2; \omega_2)(B + 1_{\{p_3=\omega_3\}}),$$

while the utility of a non-congruent Incumbent is given by

$$U_N(\mathbf{p}, \boldsymbol{\omega}, r) = \sum_{i=1}^2 1_{\{p_i \neq \omega_i\}} + q_1 r(p_1, p_2; \omega_1, \omega_2)(B + 1_{\{p_3 \neq \omega_3\}}) + (1 - q_1) r(p_1, p_2; \omega_2)(B + 1_{\{p_3 \neq \omega_3\}}).$$

The solution concept I use is Perfect Bayesian Equilibrium. Informally, a PBE requires (1) that every player of the game chooses the strategy which maximizes her expected utility given her beliefs and the strategies of the remaining players and (2) that beliefs are computed using Bayes' rule whenever possible. Throughout, I assume that congruent types choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all (ω_1, ω_2) and solve for the non-congruent type's and the Voter's equilibrium strategies and beliefs. Equilibria with this specification of congruent types' behavior always exist.¹⁷

2.1 Preliminary Analysis

As the game ends with the second period, the election winner simply implements his preferred policy in the second period, i.e. the congruent Incumbent matches the second-period policy to the state of the world ($p_3 = \omega_3$), while the non-congruent one does not ($p_3 \neq \omega_3$). It follows

¹⁷ In the baseline model, an equilibrium survives a straightforward adaptation of criterion *D1* (Cho and Kreps, 1987) if, and only if, congruent types choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all (ω_1, ω_2) . In the model with referendum, an equilibrium survives *D1* if, and only if, congruent types choose $p_2 = \omega_2$. In the model with referendum there may be equilibria that survive *D1* in which congruent types choose $p_1 \neq \omega_1$. Such equilibria have the perverse, and empirically implausible, feature that congruent types strive to reveal to the Voter that they are congruent by choosing a non-congruent policy.

that the Voter prefers to retain a congruent Incumbent and to dismiss a non-congruent one. I next study when the Voter is going to decide to hold a referendum. Obviously, if the Voter observes that the policy p_1 does not match the state of the world, the payoff he receives when he chooses to hold a referendum to fix the ill-advised policy of the Incumbent is $1 - \kappa$, while refraining from using direct democracy yields a payoff of 0. It follows that the Voter calls for a referendum whenever $p_1 \neq \omega_1$ and the cost of holding a referendum is not too high ($\kappa \leq 1$). If the Voter does not observe whether the policy p_1 matches the state of the world or not, his payoff from holding a referendum to change the policy is given by $Pr(\omega_1 \neq p_1 | p_1, p_2; \omega_2) - \kappa$, where $Pr(\omega_1 \neq p_1 | p_1, p_2; \omega_2)$ is the belief of the Voter that $\omega_1 \neq p_1$ given that the Voter observes p_1, p_2 and ω_2 . If the Voter does not call for a referendum to change the policy he receives a payoff of $Pr(\omega_1 = p_1 | p_1, p_2; \omega_2)$. It follows that the Voter only holds a referendum upon not observing ω_1 , when $Pr(\omega_1 \neq p_1 | p_1, p_2; \omega_2) - \kappa \geq Pr(\omega_1 = p_1 | p_1, p_2; \omega_2)$. Clearly, the Voter never holds a referendum if the cost of holding a referendum is too high ($\kappa > 1$).

As a result the model presented above corresponds to two possible types of representative democracy: a fully representative democracy in which direct democracy at the initiative of the Voter is not an option and a representative democracy which includes democratic elements at the leisure of the Voter. To study how the introduction of the popular referendum alters the functioning of the fully representative democracy, I first describe the equilibria of the representative democracy game ($\kappa > 1$). In the subsequent section, I then study the equilibria of the game, when the Voter can decide to call for a referendum ($\kappa \leq 1$).

3 Baseline Model

In this section I present the equilibrium of the game when the Voter cannot call for a referendum and discuss some of its implications. I illustrate the equilibrium behavior of

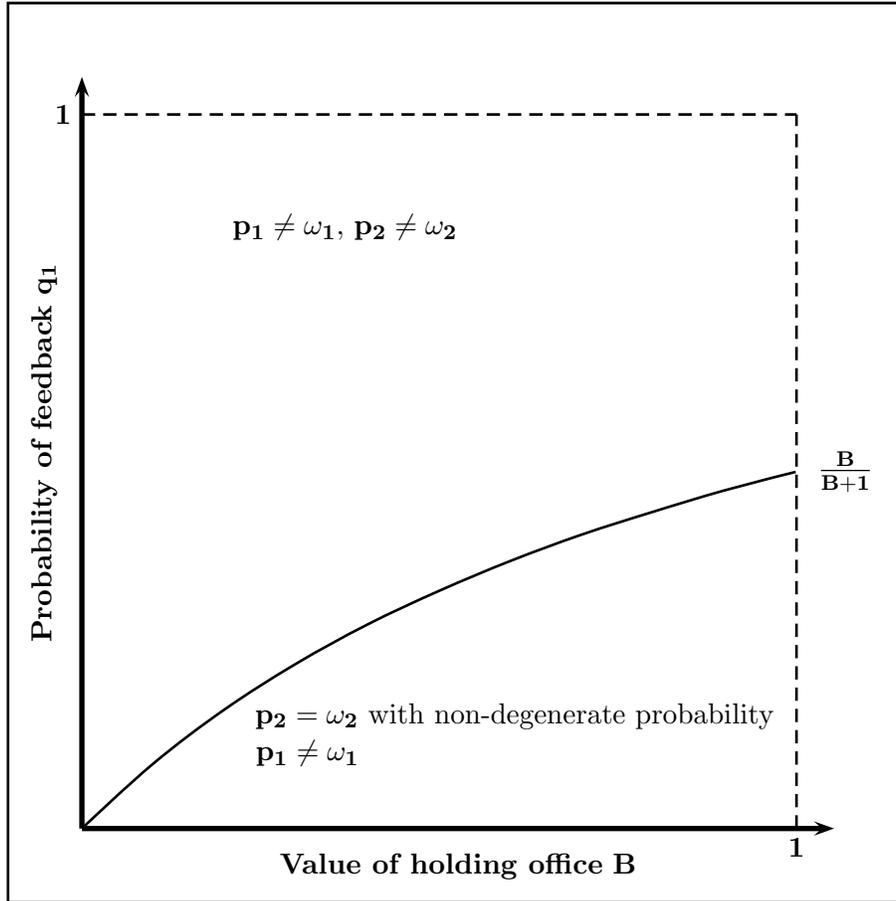
the non-congruent Incumbent in Figure 1. The value of holding office B is plotted on the horizontal axis, while the probability of feedback q_1 is on the vertical axis. As illustrated in Figure 1, the equilibrium then depends on the probability of feedback q_1 and the value of holding office B .

Proposition 1. *In equilibrium:*

1. *if the probability of feedback is high ($q_1 \geq \frac{B}{B+1}$), non-congruent incumbents choose their preferred first-period policies ($p_1 \neq \omega_1, p_2 \neq \omega_2$),*
2. *if the probability of feedback is low ($q_1 < \frac{B}{B+1}$), non-congruent Incumbents choose the Voter's preferred policy on the second dimension ($p_2 = \omega_2$) with non-degenerate probability. On the first dimension, however, they choose the policy preferred by themselves ($p_1 \neq \omega_1$) with certainty.¹⁸*

¹⁸A complete statement of equilibrium strategies and beliefs for this and other games analyzed below can be found in the appendix.

Figure 1: Equilibrium Behavior of non-congruent Incumbent in Baseline Model, $B < 1$



Note that as the Voter observes ω_2 , non-congruent Incumbents always separate from congruent ones when the probability of feedback q_1 is high and partially separate when it is low.

The intuition for this result is simple. For any (q_1, B) pair, the non-congruent Incumbent must decide whether he wants to masquerade as a congruent Incumbent or not. Obviously, mimicking the behavior of congruent Incumbents presents an upside and a downside. On the upside, it potentially improves the chances of reelection of the non-congruent Incumbent. On the downside, it requires the non-congruent Incumbent to choose first-period policies he dislikes. The first important step of our analysis is then to recognize that, as the value of

holding office is limited ($B < 1$), non-congruent Incumbents are never willing to behave in the public interest with respect to both first-period policy issues. If they were to do so, they would at most receive a payoff of $B + 1$, while implementing their preferred first-period policies yields a payoff of 2.

This does not imply, however, that non-congruent Incumbents always disregard the preferences of the public in the first period. Indeed, suppose the non-congruent Incumbent were to choose $(p_1 \neq \omega_1, p_2 = \omega_2)$ with positive probability. If the Voter does not observe the state of the world ω_1 , i.e. if the Voter only observes whether policy p_1 is equal to 1 or to -1 , the Voter will be uncertain as to the type of the Incumbent. For example, if the Voter observes $(p_1 = -1, p_2 = \omega_2)$ she might either be facing a congruent Incumbent who observes that $\omega_1 = -1$ or a non-congruent Incumbent who observes that $\omega_1 = 1$. If the non-congruent Incumbent chooses $(p_1 \neq \omega_1, p_2 = \omega_2)$ with sufficiently low probability, the Voter will then be compelled to reelect the Incumbent whenever she does not observe whether the implemented policy p_1 is in the public's interest. When choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$, the non-congruent Incumbent thus incurs a policy cost of 1 in the first period, as he is giving up policy p_2 , but may obtain a gain of $B + 1$ whenever he gets reelected. If the non-congruent Incumbent gets reelected sufficiently often when choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$ he will thus choose this policy vector over implementing his preferred first-period policies.

The probability of being reelected upon choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$ depends on the probability of feedback q_1 , however. Indeed, if uncertainty about ω_1 resolves, the Voter will infer from $p_1 \neq \omega_1$ that she is facing a non-congruent Incumbent and will vote the non-congruent Incumbent out of office. Hence, as q_1 increases, the probability with which the non-congruent Incumbent gets reelected upon choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$ decreases. If the probability of feedback is sufficiently high, i.e. $q_1 \geq \frac{B}{B+1}$, the non-congruent Incumbent is likely to get caught when choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$ and he thus opts to separate from

congruent types by choosing $(p_1 \neq \omega_1, p_2 \neq \omega_2)$. When q_1 is sufficiently low, the non-congruent Incumbent will have incentives to choose $(p_1 \neq \omega_1, p_2 = \omega_2)$ at least part of the time.

4 Introducing Direct Democracy

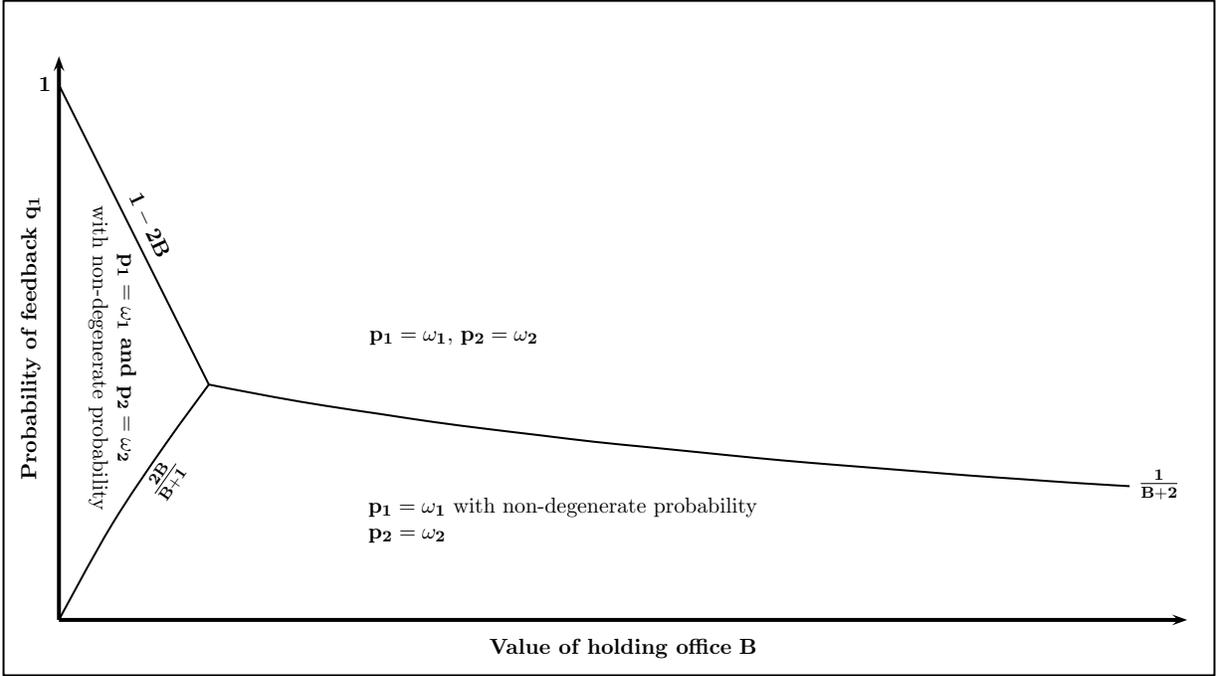
I now consider the case where the Voter is given the opportunity to call for a referendum on policy p_1 . Figure 2 illustrates the equilibria of the game in this parameter range. Here, the equilibrium depends on the probability of feedback q_1 and the value of holding office B . In particular, there are three regions which are referred to as q_1 high, q_1 low, and q_1 intermediate. These regions give rise to different equilibrium strategy profiles:

Proposition 2. *In equilibrium:*

1. *if the probability of feedback is high ($q_1 \geq \max\{1 - 2B, \frac{1}{B+2}\}$), non-congruent Incumbents implement the Voter's preferred policies with respect to all policy dimensions:
 $(p_1 = \omega_1, p_2 = \omega_2)$,*
2. *if the probability of feedback is low ($q_1 < \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}$), non-congruent Incumbents choose $p_1 = \omega_1$ with non-degenerate probability and $p_2 = \omega_2$ with certainty,*
3. *if the probability of feedback takes on intermediate values ($q_1 \in [\frac{2B}{B+1}, 1 - 2B]$), non-congruent Incumbents choose $p_1 = \omega_1$ and $p_2 = \omega_2$ with non-degenerate probability.¹⁹*

¹⁹Note that $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$ implies that $B < \underline{B}$, where \underline{B} denotes the value of B which solves $\frac{2B}{B+1} = 1 - 2B$.

Figure 2: Equilibrium Behavior of non-congruent Incumbent in Direct Democracy Model



When the probability of feedback q_1 is high, the non-congruent Incumbent has almost no control over p_1 as the Voter holds a referendum to set $p_1 = \omega_1$ whenever $p_1 \neq \omega_1$ is revealed to her. Thus, choosing $p_1 \neq \omega_1$ yields almost no policy gains over choosing $p_1 = \omega_1$ but is likely to cost the Incumbent reelection, as the Voter learns her true type whenever $p_1 \neq \omega_1$ is revealed to her. Therefore, the non-congruent Incumbent essentially has only to give up one policy, namely p_2 , to win reelection, which incentivizes him to choose $(p_1 = \omega_1, p_2 = \omega_2)$.

To understand what happens when q_1 takes on lower values, notice first that the payoff to a non-congruent Incumbent of choosing $p_2 \neq \omega_2$ depends on the decision of the Voter to hold a referendum when she observes $p_2 \neq \omega_2$. Suppose for example that the Voter always chooses to set $p_1 = 1$ when she observes $p_2 \neq \omega_2$ and uncertainty is not resolved about ω_1 . Then, the payoff to the non-congruent Incumbent of choosing $p_2 \neq \omega_2$ is 1 whenever he observes $\omega_1 = 1$ but $2 - q_1$ whenever $\omega_1 = -1$ is revealed to him.

Consider now what happens if the non-congruent Incumbent chooses $(p_1 = 1, p_2 = \omega_2)$ upon observing $\omega_1 = -1$ and the Voter does not observe ω_1 . In this case, as $p_1 = 1$ is the ex ante popular policy and Incumbents are ex ante more likely to be congruent ($\pi > 1/2$), the Voter chooses not to hold a referendum. Hence, when the probability of feedback q_1 is low, the non-congruent Incumbent can essentially get his preferred outcome with respect to policy dimension p_1 when he observes that the state of the world is $\omega_1 = -1$. Moreover, the Voter believes the Incumbent to be more likely to be congruent than a potential challenger and thus reelects. Hence, if q_1 is low, the non-congruent Incumbent only needs to give up policy p_2 in order to get reelected when $\omega_1 = -1$. This implies that for low values of q_1 , the non-congruent Incumbent is better off choosing $(p_1 = 1, p_2 = \omega_2)$ when $\omega_1 = -1$ independently of the decision of the Voter to hold a referendum or not upon observing $p_2 \neq \omega_2$. If the Voter then chooses to enforce $p_1 = 1$ whenever she observes $p_2 \neq \omega_2$ and ω_1 is not revealed to her, the non-congruent Incumbent only receives a payoff of 1 when choosing $p_2 \neq \omega_2$ upon observing $\omega_1 = 1$. But then, the non-congruent Incumbent prefers to masquerade as a congruent type by choosing $p_2 = \omega_2$ whenever $\omega_1 = 1$.

What happens if the probability of feedback q_1 takes on intermediate values? In such an instance, it might be that the non-congruent Incumbent does not get caught often enough to consider p_1 as a foregone policy, yet too often to consider p_1 as a policy over which the Incumbent has almost full control upon observing $\omega_1 = -1$. In such a case, the non-congruent Incumbent, upon observing $\omega_1 = -1$, essentially needs to give up policy p_2 but also to some extent policy p_1 in order to get reelected. If the benefit of holding office is sufficiently high, the non-congruent Incumbent is willing to do so. When the benefit is too low, however, he might be tempted to choose $p_2 \neq \omega_2$ with positive probability, unless the Voter holds a referendum with sufficiently high probability to set $p_1 = -1$ upon observing $p_2 \neq \omega_2$. Note however that when q_1 is low, the equilibrium is sustained by the decision of

the Voter to hold a referendum to set $p_1 = 1$ whenever needed, upon observing $p_2 \neq \omega_2$. If the Voter now holds a referendum to set $p_1 = -1$ with positive probability, when observing $p_2 \neq \omega_2$, the incentives for the non-congruent type of choosing $p_2 \neq \omega_2$ upon observing $\omega_1 = 1$ increase. Hence, to sustain an equilibrium in which the non-congruent Incumbent always chooses $p_2 = \omega_2$, the probability with which the Voter sets $p_1 = -1$ upon observing $p_2 \neq \omega_2$ has to be neither too high nor too low. As it turns out, when the benefit of holding office is sufficiently low, there exists a range of values for q_1 in which it is not possible for the Voter to set $p_1 = -1$ upon observing $p_2 \neq \omega_2$ with a probability sufficiently high and sufficiently low to deter the non-congruent Incumbent from choosing $p_2 \neq \omega_2$ upon observing $\omega_1 = -1$ and upon observing $\omega_1 = 1$ respectively. For this range of parameter values, there exists an infinity of equilibria. For any of these equilibria, non-congruent types will choose $p_2 \neq \omega_2$ and $p_1 = \omega_1$ with positive probability both upon observing $\omega_1 = 1$ and $\omega_1 = -1$.

5 The Impact of the Popular Referendum

5.1 Improved Congruence

Comparison of the equilibria in Proposition 1 to the equilibria in Proposition 2 illustrates the effects of the popular referendum on representative democracy when $B < 1$. An observation worth repeating is that congruent Incumbents behave optimally from the point of view of the Voter with regard to every single policy of the game, whether the Voter is given the ability to call for a referendum on policy p_1 or not. It is the behavior of non-congruent Incumbents which is affected by the popular referendum. Let us first consider the impact of the popular referendum on policy p_1 , the policy about which the Voter is given the power of direct control. In the baseline model, non-congruent Incumbents always disregard the

Voter’s preferences about policy p_1 , implementing $p_1 \neq \omega_1$ independently of the probability of feedback q_1 . In the model with direct democracy, non-congruent types always choose $p_1 = \omega_1$ with positive probability for all values of q_1 .

A more surprising aspect of the analysis is that the popular referendum not only improves the congruence with respect to policies on which a referendum may be placed, but also with respect to policy issues that are outside the reach of direct democracy. In the baseline model, non-congruent Incumbents never choose $p_2 = \omega_2$ when $q_1 > \frac{B}{B+1}$ and choose $p_2 \neq \omega_2$ with non-degenerate probability when $q_1 \leq \frac{B}{B+1}$. The introduction of direct democracy leads the non-congruent Incumbents to always choose $p_2 = \omega_2$ when $q_1 \leq \frac{B}{B+1}$. Moreover, when $q_1 > \frac{B}{B+1}$ there always exists equilibria in which $p_2 = \omega_2$ is implemented with positive probability by the non-congruent Incumbents.

Proposition 3. *For any probability of feedback q_1 , giving the Voter the power to call for a referendum on policy p_1 improves the congruence between enacted policies and Voter’s preferences with respect to p_1 as well as with respect to p_2 , the policy about which the Voter cannot hold a referendum.*

Interestingly, this improved congruence between enacted policies and the preferences of the Voter even occurs when the Voter is very unlikely to—even certain not to—observe the state of the world on dimension 1 and thus is barely able to correct the errors of non-congruent Incumbents. Even in the extreme case where the Voter is certain not to learn whether the implemented policy is in the public interest, i.e. $q_1 = 0$, the possibility that the Voter may call for a referendum increases the probability that the implemented policies will match the first-period policies to the respective states of the world.

It is instructive to compare these results to those found in models by Gerber (1996), Hug (2004), and Besley and Coate (2008). There, the introduction of the popular referendum

improves the congruence between enacted policies and voters' preferences, but only on those policy dimensions which can be subjected to direct democracy. Moreover, voters are always assumed to be fully informed about their true preferences in these models. Proposition 3 above shows, however, that the Voter is getting higher congruence even when she is not fully informed about her true interests. Proposition 3 also stands in contrast to Matsusaka and McCarty (2001) who find, in a model with policy-driven politicians, that initiatives may move the policy farther away from the voter's ideal point. In the present paper, the reelection concerns are precisely what motivates the Incumbent to alter his behavior and attend more closely to the Voter's preferences.

A related question is whether the introduction of direct democracy increases or decreases pandering. At least two understandings of the concept of pandering can be found in the literature. In the family of pandering models in which the Voter is uncertain about the competence of elected officials (Canes-Wrone, Herron and Shotts, 2001), pandering refers to the situation in which an incumbent 'implement(s) a policy that voters think is in their best interest, even though the policy maker knows that a different policy is actually better for the voters' (Ashworth and Shotts, 2010). When the Voter's uncertainty is over the policy preferences of the Incumbent (see Maskin and Tirole, 2004), pandering corresponds, however, to the case in which an elected official chooses a popular policy in order to get reelected, although this policy does not conform with the policy preferences of the official. When an Incumbent is congruent, the two notions coincide, i.e. a congruent Incumbent panders if he chooses the popular policy although he knows that this policy is not in the public's interest. In the case of non-congruent incumbents however, the two interpretations differ. Under the first interpretation, one might consider that a non-congruent incumbent panders when he chooses $p_1 = 1$ although he observes $\omega_1 = -1$ for example. Under the second interpretation however, pandering corresponds to the case, where in order to get reelected, a non-congruent

Incumbent chooses $p_i = \omega_i$, with $i \in \{1, 2\}$. If he chooses $p_i = \omega_i = 1$, he simply panders to the beliefs of the electorate, whereas when he chooses $p_i = \omega_i = -1$ he essentially panders to the beliefs about the optimal policy that the Voter is likely to have in the future. Maskin and Tirole (2004) call this last behavior ‘forward-looking pandering’.

In this model, an equivalent interpretation of the fact that direct democracy improves the congruence of implemented policies with the true interests of the electorate is thus that direct democracy decreases pandering according to the interpretation of Canes-Wrone, Heron and Shotts (2001) yet increases pandering according to Maskin and Tirole (2004). Note, however, that as congruent Incumbents match the first-period policies to their respective states of the world, under representative democracy as well as under direct democracy, the introduction of the popular referendum does not lead congruent Incumbents to choose $p_1 = 1$ when the state of the world is $\omega_1 = -1$ and this independently of the probability of feedback q_1 . In other words, the introduction of the popular referendum affects the pandering behavior of non-congruent Incumbents but not of congruent ones.

A related aspect of the analysis is that the introduction of the referendum increases the likelihood that the popular policies, namely $p_1 = 1$ and $p_2 = 1$, are implemented. In the baseline model non-congruent Incumbents choose $p_1 = 1$ only when the state of the world is $\omega_1 = -1$. Hence, they implement $p_1 = 1$ with probability $1 - \alpha$. Under direct democracy, when q_1 is high, non-congruent Incumbents choose $p_1 = 1$ whenever $\omega_1 = 1$. The popular policy is therefore enacted with probability $\alpha > 1 - \alpha$. When q_1 is low, non-congruent Incumbents choose $p_1 = 1$ whenever $\omega_1 = -1$ but also with positive probability whenever $\omega_1 = 1$. Similarly, in the baseline model, non-congruent Incumbents choose $p_2 = 1$ with probability $1 - \alpha$ when q_1 is high ($q_1 \geq \frac{B}{B+1}$). When q_1 is low ($q_1 < \frac{B}{B+1}$) they choose $p_2 = 1$ with probability $2(1 - \alpha)\alpha$. Once direct democracy is introduced they choose $p_2 = 1$ with a probability superior to $1 - \alpha$ when $q_1 \geq \frac{B}{B+1}$ and with probability $\alpha > 2(1 - \alpha)\alpha$ when

$q_1 < \frac{B}{B+1}$. Therefore, the introduction of the referendum increases the congruence between enacted policies and the voters' ex ante beliefs about what is in their best interests.

The prediction that the popular policy $p_1 = 1$ is implemented more often with the introduction of a popular referendum is familiar and conforms to empirical results. Several studies find for example that the implemented policies regarding the death penalty, abortion and same sex marriage more closely reflect public opinion in states that permit voters to call for a referendum (see Gerber, 1996, 1999; Hug, 2004; Matsusaka, 2010). In a recent book, Matsusaka (2004) (see also Matsusaka, 1995) finds that the initiative process has lead over the last thirty years to (1) lower public spending, (2) more decentralized spending decisions, and (3) a lower reliance on taxes rather than user fees for services. He then finds that these three policy changes brought about by the initiative reflect the will of the majority of the population as expressed via opinion data. The prediction that the the introduction of the popular referendum also increases the likelihood that the popular policy $p_2 = 1$ is implemented, although no referendum can be held about p_2 , is new to the literature and suggests a more pervasive effect of referenda as well as the need for additional empirical analysis.

5.2 The effect of additional information

The introduction of the popular referendum also changes how the behavior of the Incumbent relates to the information available to the Voter. In fact, whereas in the baseline model, the control the Voter exerts over policy-making may decrease—rather than increase—when the Voter is more likely to observe the states of the world, this relationship is reversed for a large range of the parameter space when the Voter is given the power to call for a referendum. Indeed, in the baseline model, the level of control which the Voter is able to exert over

policy dimension p_2 depends on the probability that the Voter will learn the state of the world on dimension p_1 . There, non-congruent Incumbents always disregard the Voter's true preferences on policy dimension p_2 by implementing $p_2 \neq \omega_2$ when the probability of feedback q_1 is high, but choose $p_2 = \omega_2$ with positive probability when q_1 is low. Hence, the Voter may be able to exert more control over policy p_2 , when the probability of feedback on dimension p_1 is lower rather than higher. This holds true although an increase in the probability of feedback q_1 does not increase the probability that the Incumbent implements $p_1 = \omega_1$. Indeed, in the absence of the referendum, the non-congruent Incumbent chooses $p_1 \neq \omega_1$ for all $q_1 \in [0, 1]$. Moreover, it is interesting to note that this negative spillover effect of information occurs even though the two policy dimensions are entirely independent.

Once the popular referendum is introduced, the ability of the electorate to control the behavior of elected officials increases with the probability of feedback, for a large range of the parameter space. If the value of holding office is sufficiently high, $B \geq \underline{B}$, non-congruent Incumbents choose the policy that hurts the Voter on dimension p_1 , when q_1 is low, but always match the policy to the state of the world, $p_1 = \omega_1$, when q_1 is high.

Note, however, that when the value of holding office is low, $B < \underline{B}$, and uncertainty about policy p_1 may not resolve, a higher probability of feedback may reduce the control that the Voter exerts over elected officials. In this range of the parameter space, the relationship between the probability that a non-congruent Incumbent chooses $p_2 = \omega_2$ and the probability of feedback q_1 is non-monotonic. More precisely, a non-congruent Incumbent sets $p_2 = \omega_2$ if $q_1 < \frac{2B}{B+1}$ or if $q_1 > 1 - 2B$, but chooses $p_2 \neq \omega_2$ with positive probability when $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$. Similarly, the relationship between the probability that a non-congruent Incumbent sets $p_1 = \omega_1$ and q_1 is ambiguous. Remember that when $B < \underline{B}$ and $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$, there exists an infinity of equilibria. In any of these equilibria, non-congruent Incumbents choose $p_1 = \omega_1$ with positive probability. Depending on the specific equilibrium

that the Incumbents coordinate on, the probability with which $p_1 = \omega_1$ may go up or go down compared to the case where q_1 is low.

Proposition 4. *Assume $B \geq \underline{B}$. Whereas in the baseline model the control the Voter exerts over policy p_2 is weakly decreasing in the probability of feedback q_1 , once direct democracy is introduced the control the Voter exerts over policies p_1 and p_2 is increasing in q_1 .*

Allowing citizen-initiated referenda has a similar impact on the relationship between α , the probability that the state of the world ω_1 is equal to 1, and the probability that the non-congruent Incumbent implements $p_2 = \omega_2$. In fact, in the baseline model when q_1 is low ($q_1 < \frac{B}{B+1}$) the probability with which the non-congruent Incumbent chooses $p_2 = \omega_2$ in equilibrium decreases in α . In this setting, α can represent how certain the Voter is that $p_1 = 1$ is the optimal policy, when ω_1 is not revealed to her. Hence, as the uncertainty about the optimal policy p_1 increases, i.e. as α decreases, the Voter improves his control over policy p_2 . The logic behind this result is as follows. In equilibrium, the non-congruent Incumbent chooses $(p_1 = 1, p_2 = \omega_2)$ upon observing $\omega_1 = -1$ and mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 \neq \omega_2)$ upon observing $\omega_1 = 1$. For the non-congruent Incumbent to be willing to choose $(p_1 = -1, p_2 = \omega_2)$, it must be the case that the Voter reelects the Incumbent with positive probability upon observing $(p_1 = -1, p_2 = \omega_2)$. This, in turn, means that the Voter must believe that the Incumbent is at least as likely to be congruent as a random replacement. As α increases, the congruent Incumbent becomes less likely to choose $p_1 = -1$, and thus the Voter is more inclined to believe that the Incumbent is non-congruent upon observing $p_1 = -1$. Thus to maintain the Voter's willingness to reelect upon observing $(p_1 = -1, p_2 = \omega_2)$, the non-congruent Incumbent needs to choose $(p_1 = -1, p_2 = \omega_2)$ with lower probability.

Again, introducing popular referenda reverses this relationship for a large range of the parameter space. Indeed, when $q_1 < \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}$ the non-congruent Incumbent chooses

$(p_1 = 1, p_2 = \omega_2)$ with probability $2 - \frac{1}{\alpha}$, whenever he observes $\omega_1 = 1$. In other words, the probability that the non-congruent Incumbent chooses $p_2 = \omega_2$ increases in α . Hence, as the Voter becomes more confident that $\omega_1 = 1$ is indeed the right policy, i.e. as α increases, so does the control the Voter exerts over policy p_2 .

5.3 Reelection strategy

In the baseline model, as well as in the direct democracy game, the reelection strategy used by the Voter has aspects of a retrospective rule. The Voter always dismisses the Incumbent when it is revealed that he did not match the policy to the state of the world. Similarly, when the Voter does not observe the state of the world on dimension 1, she is more likely to reelect the Incumbent when he chooses the popular policy $p_1 = 1$. Moreover, the model is consistent with empirical findings by Hugh-Jones (2010) and Bali and Davis (2007) which suggest that incumbents are more likely to be reelected in states that permit the initiative. Indeed, as non-congruent Incumbents are more likely to mimic the behavior of congruent ones when the popular referendum is introduced, the Voter is often indifferent between reelecting and dismissing the Incumbent. Hence, there always exist equilibria in which the Incumbent is reelected more often under direct than under representative democracy.

The reelection strategy also presents more surprising aspects. In the baseline model, when q_1 is low and non-congruent Incumbents thus only partially separate from congruent ones, the probability with which the Voter reelects the Incumbent is increasing in the probability of feedback q_1 and decreasing in the value of holding office B . As the non-congruent Incumbent mixes between $(p_1 = -1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 \neq \omega_2)$ upon observing $\omega_1 = 1$, he must be indifferent between these two policy vectors in equilibrium. As the probability of feedback q_1 increases, the type of the non-congruent Incumbent is more likely to be re-

vealed to the Voter which reduces the incentives of the non-congruent Incumbent to choose $(p_1 = -1, p_2 = \omega_2)$. On the contrary, an increase in the value of holding office strengthens the incentives of a non-congruent Incumbent to choose $(p_1 = -1, p_2 = \omega_2)$. Therefore, to maintain indifference between $(p_1 = -1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 \neq \omega_2)$, the probability with which the Voter reelects the Incumbent upon observing $(p_1 = -1, p_2 = \omega_2)$ must increase when q_1 increases and decrease when B increases.

The same relationship holds between the probability of reelection and the value of holding office in the direct democracy game. The introduction of the popular referendum alters the relationship between the probability of reelection and the probability of feedback, however. In fact, when q_1 is low, the probability that the Voter reelects the Incumbent upon observing $(p_1 = 1, p_2 = \omega_2)$ is decreasing in q_1 , while it is weakly increasing upon observing $(p_1 = -1, p_2 = \omega_2)$. When q_1 is low, the non-congruent Incumbent mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ upon observing $\omega_1 = 1$. As the Voter always reelects upon observing $p_1 = \omega_1$, the incentives for the non-congruent Incumbent to choose $(p_1 = 1, p_2 = \omega_2)$ increase when q_1 increases. Hence, to maintain indifference, the Voter must reelect the Incumbent less often upon observing $(p_1 = 1, p_2 = \omega_2)$.

5.4 The Frequency of Referenda

I now discuss some of the predictions generated by the model as they relate to the frequency of referenda. The first lesson is that for wide swaths of the model's parameter space, the Voter does not hold a referendum in equilibrium. Indeed, when $q_1 \geq \max\{1 - 2B, \frac{1}{B+2}\}$ all the types of the Incumbent implement $p_1 = \omega_1$ in equilibrium. Therefore, the Voter never has an incentive to correct the decision of the elected representative when q_1 is high. For the remaining range of the parameter space, non-congruent Incumbents enact $p_1 \neq \omega_1$

with positive probability and the Voter holds a referendum in equilibrium whenever it is revealed to her that the Incumbent chose the wrong policy on dimension p_1 . Note, however, that there is a striking difference between the Voter's behavior when q_1 is low and when q_1 is intermediate. When q_1 is low, the Voter only holds a referendum in equilibrium upon observing $p_1 \neq \omega_1$. When q_1 takes on intermediate values, however, the Voter calls for a referendum with positive probability upon observing $p_2 \neq \omega_2$, even when the state of the world ω_1 is not revealed to her. Note that in this case the Voter, however, never calls for a referendum when she observes $p_2 = \omega_2$ and she remains uncertain about the state of the world ω_1 .

- Proposition 5.** 1) *If the probability of feedback q_1 is high, i.e. $q_1 \geq \max\{1 - 2B, \frac{1}{B+2}\}$, then the Voter does not hold a referendum in equilibrium.*
- 2) *If the probability of feedback q_1 is low, i.e. $q_1 < \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}$, then the Voter holds a referendum in equilibrium if, and only if, the Voter observes $p_1 \neq \omega_1$.*
- 3) *If the probability of feedback q_1 takes intermediate values, i.e. $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$, then the Voter holds a referendum in equilibrium with certainty when she observes $p_1 \neq \omega_1$ and with positive probability when she observes $p_2 \neq \omega_2$.*

This behavior of the Voter presents several interesting features. First of all, as in Matsusaka and McCarty (2001) and Hug (2004), for a referendum to occur in equilibrium, the Incumbent needs to be uncertain about the true preferences of the Voter on the policy dimension on which a referendum may be placed. Secondly, the Voter only holds a referendum when it is revealed to her that the Incumbent is non-congruent, being because she observes $p_1 \neq \omega_1$ or $p_2 \neq \omega_2$. This has the implication that the Voter never re-elects the Incumbent when she holds a referendum in equilibrium. Holding a referendum is thus effectively akin to recalling the Incumbent. Thirdly, when the value of holding office is sufficiently high $B > \underline{B}$,

the relationship between the frequency of referenda and the probability that uncertainty about ω_1 resolves is non-monotonic. More precisely, the frequency of referenda is increasing in q_1 on $[0, \frac{1}{B+2}]$ before remaining constant at 0 on $[\frac{1}{B+2}, 1]$. This stems from the fact that the probability with which non-congruent Incumbents choose $p_1 \neq \omega_1$ in equilibrium does not depend on the probability of feedback q_1 , when $q_1 < \frac{1}{B+2}$. However, as q_1 increases, the Voter is more likely to learn that the Incumbent implemented $p_1 \neq \omega_1$ and thus to hold a referendum.

Corollary 6. *If $B \geq \underline{B}$, then the probability that the Voter holds a referendum increases in the probability of feedback q_1 on $[0, \frac{1}{B+2}]$ before remaining constant at 0 on $[\frac{1}{B+2}, 1]$.*

Most interestingly, for some parameter values, the decision of the Voter to hold a referendum about policy p_1 depends on the observed policy decision on dimension p_2 . More precisely, once the Incumbent is revealed as non-congruent to the Voter via his decision about policy p_2 , the Voter is more likely to try to constrain his decision making on dimension p_1 by using the referendum. There seems to be some empirical support for this prediction as Matsusaka (1992) finds that citizen initiatives play a more important role in periods when elected representatives appear to be corrupt. This result is also interesting in so far as one objection formulated against referenda is that the people sometimes vote against certain policies simply to punish the government without consideration as to whether the policy is sensible or not. Whether this is indeed correct or not is a contentious empirical question (see for example in the context of referenda on European integration Hobolt (2009)). An implication of this model, however, is that, even if it is correct, this fact does not necessarily imply that the institution of the referendum does not improve decision-making, as such a behavior may force elected officials to attend more closely to voters' wishes on other policy dimensions.

Moreover, when $B < \underline{B}$, the Voter sometimes holds a referendum to set $p_1 = -1$

although ω_1 is not revealed to her. Hence, as in Besley and Coate (2008), the fact that a referendum does not conform with the ex ante majority opinion of the electorate does not imply that the institution of the referendum does not improve congruence.

5.5 Voter Welfare

I now consider the effect of the popular referendum on ex ante Voter welfare. In the present setup, as shown above, the popular referendum improves the congruence between enacted policies and the Voter's true preferences in the first period. In this sense, direct democracy increases the first-period welfare of the Voter. Note, however, that this improved congruence results from the decision of non-congruent Incumbents to mimic the behavior of congruent Incumbents more often under direct democracy than in a purely representative environment. Therefore, the popular referendum decreases the ability of the Voter to select good types at the election booth and thus reduces the second-period welfare of the Voter. The introduction of the popular referendum thus represents a trade-off for the Voter. Given the heuristic nature of the second period I remain agnostic as to the relative importance of these two effects.

6 Semi-congruent Types

In the model, an incumbent is either congruent or non-congruent. Note, however, that in principle an incumbent may be congruent with respect to some policy dimensions and not others. In this section, I provide an argument for why including semi-congruent types does not alter fundamentally the logic of the model. Assume that π now represents the probability that the incumbent is congruent with respect to policy dimension p_i . In such a setting, the incumbent is congruent with respect to both first period policies with probability

π^2 , congruent with respect to policy p_1 but not congruent with respect to policy p_2 with probability $\pi(1 - \pi)$ and so forth. Assume further that the elected official has to choose two policies in the second period as well, i.e. p_3 and p_4 and that the policy preferences over p_1 and p_3 (respectively p_2 and p_4) are identical. In such a case, a congruent type matches both second-period policies to their respective states of the world, a semi-congruent type only matches one of the two policies to the state of the world, while a non-congruent type does not match any of the second period policies to the respective states of the world. The expected utility to the Voter of electing a challenger is thus 2π . The utility that the Voter receives in the second period when electing an incumbent which has been revealed to be semi-congruent or non-congruent, but not congruent, is at most 1. As $\pi > 1/2$, we have the following:

Proposition 7. *If the Voter observes a first-period policy vector which is never chosen by a congruent Incumbent in equilibrium, she does not reelect.*

An implication of this proposition is that a non-congruent Incumbent, in order to get re-elected, needs to enact a first-period policy vector that is also enacted by a congruent type, even when semi-congruent types exist. In particular, this implies that a non-congruent Incumbent needs to choose $p_2 = \omega_2$ in order to get reelected. Similarly, when the probability of feedback q_1 is sufficiently high the non-congruent Incumbent essentially needs to choose $p_1 = \omega_1$ in order to get reelected with a substantial probability. Correspondingly, the incentives for non-congruent types to separate in the baseline model and to pool in the model with the popular referendum remain similar.

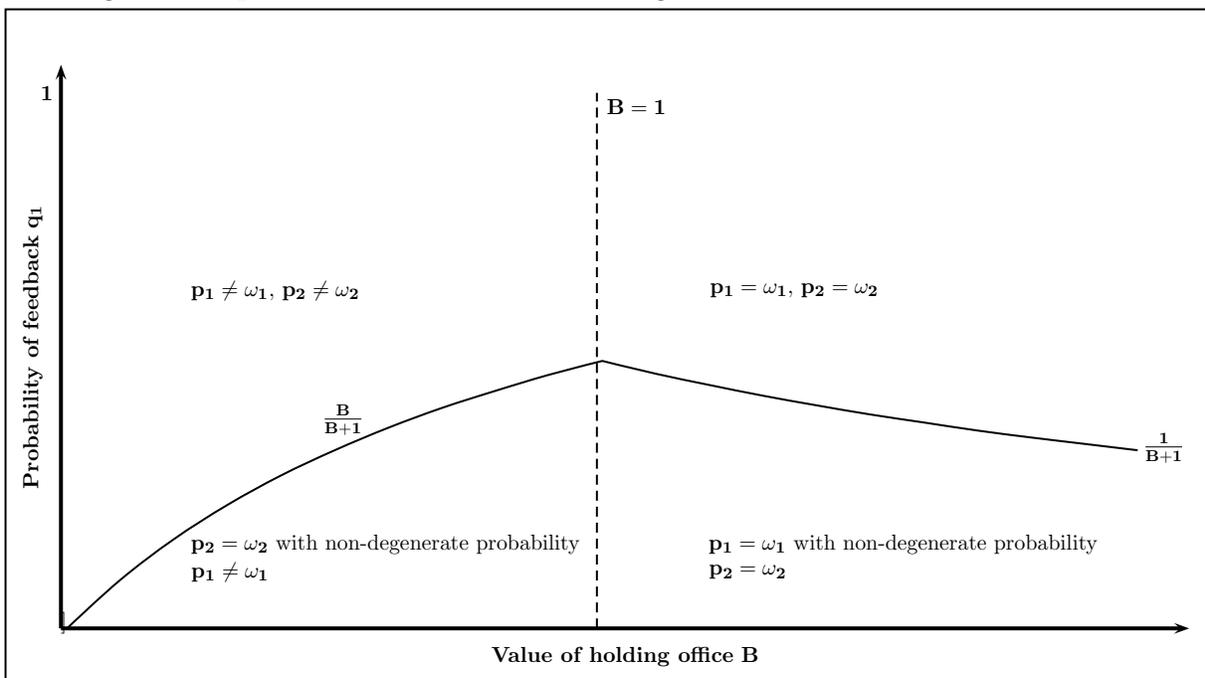
7 Strong office-holding motive, $B \geq 1$

In this section I study what happens when the Incumbent values staying in office over choosing his preferred first-period policies, i.e. $B \geq 1$. Figure 3 illustrates the equilibrium of the baseline model in this case. As before, the equilibrium depends on the probability of feedback q_1 and the value of holding office B .

Proposition 8. *Assume the value of holding office is high ($B \geq 1$). In equilibrium:*

1. *if the probability of feedback is high ($q_1 \geq \frac{1}{B+1}$), non-congruent incumbents choose the same first-period policies as congruent ones ($p_1 = \omega_1, p_2 = \omega_2$),*
2. *if the probability of feedback is low ($q_1 < \frac{1}{B+1}$), non-congruent Incumbents choose $p_1 = \omega_1$ with non-degenerate probability and $p_2 = \omega_2$ with certainty.*

Figure 3: Equilibrium Behavior of non-congruent Incumbent in Baseline Model



Proposition 9. *1. Increasing the value of holding office from $B < 1$ to $B \geq 1$ has a similar effect on the decision making of public officials as the introduction of the popular referendum.*

- 1. When $B \geq 1$ and $q_1 \in (\frac{1}{B+2}, \frac{1}{B+1})$, the introduction of the popular referendum strictly improves congruence with respect to p_1 .*

Interestingly, increasing the value of holding office has a similar effect as the introduction of the popular referendum. First of all, the congruence with respect to policies p_1 and p_2 in the baseline model improves when $B \geq 1$ compared to a situation in which $B < 1$. Similarly, the control the Voter exerts over elected representatives increases with the level of information available to the Voter when $B \geq 1$. This is interesting in so far as it clarifies the mechanism through which the introduction of the referendum affects the decision-making of public officials. Indeed, by constraining the policy options of the Incumbent, the popular referendum essentially increases the value of retaining office, if only in relative terms.

Remark also that the introduction of the popular referendum still improves congruence even when Incumbents have a strong office holding motive. In particular, when $q_1 \in [\frac{1}{B+2}, \frac{1}{B+1}]$, non-congruent Incumbents always choose $p_1 = \omega_1$ once the possibility of a popular referendum exists, whereas they choose $p_1 = \omega_1$ with non-degenerate probability in the absence of direct democracy. This stems from the fact that the costs a non-congruent Incumbent incurs when $p_1 \neq \omega_1$ is revealed to the Voter increase with the introduction of direct democracy, as the Voter now holds a referendum to set $p_1 = \omega_1$. As a result the Incumbent then not only loses re-election but also the policy benefit associated with $p_1 \neq \omega_1$. The probability of feedback above which the non-congruent Incumbent prefers to play $p_1 = \omega_1$ with certainty is thus lower under direct democracy than under representative democracy.

8 Conclusion

This paper has examined how the possibility of voters calling for a referendum affects electoral accountability. I have shown that the introduction of the popular referendum, by limiting the policy benefits that a non-congruent official can receive from choosing his (as opposed to the voters) preferred policies, increases the incentives of non-congruent public officials to mimic the behavior of congruent ones. A main implication of this result is that non-congruent elected officials are more likely to enact policies that are in the public interest once the popular referendum is introduced. Unlike the previous literature I have shown that this improved congruence between enacted policies and voters' preferences also concerns policy dimensions on which no referendum may be placed. In this sense, direct democracy may have a much more pervasive effect on representative democracy than previously thought. Moreover, I have shown that direct democracy improves congruence even when the electorate is unlikely to be fully informed about its true interests. The lack of expertise of voters, which opponents of direct democracy often present as a major concern, may thus be a less severe problem than heretofore acknowledged. Furthermore, under direct democracy the value to the electorate of being informed may be higher than under representative democracy. Indeed, under a strictly representative democracy, there are negative spillover effects of information in the sense that voters being more likely to become informed about some policy dimensions may lead non-congruent officials to disregard the preferences of the electorate more often on other policy dimensions. In contrast, there are conditions under which the possibility of popular referenda implies that being more informed improves voter control over public officials, including on policy dimensions on which no referendum may be placed.

The analysis suggests that empirical research on the impact of direct democracy on congruence has to some extent been looking for the keys under the lamp post. Testing

whether the popular referendum also improves congruence on policy dimensions on which no referendum is placed represents an interesting agenda for future empirical research.

References

- Alesina, Alberto and Guido Tabellini. 2007. "Bureaucrats or politicians? Part I: a single policy task." *The American Economic Review* 97(1):169–179.
- Ashworth, S. 2012. "Electoral accountability: Recent theoretical and empirical work." *Annual Review of Political Science* 15:183–201.
- Ashworth, S. and E Bueno de Mesquita. 2006. "Delivering the Goods: Legislative Particularism in Different Electoral and Institutional Settings." *Journal of Politics* 68(1):168–179.
- Ashworth, Scott. 2005. "Reputational dynamics and political careers." *Journal of Law, Economics, and Organization* 21(2):441–466.
- Ashworth, Scott and Ethan Bueno de Mesquita. 2013. "Multitask, Accountability, and Institutional Design."
- Ashworth, Scott and Kenneth W. Shotts. 2010. "Does Informative Media Commentary Reduce Politicians' Incentives to Pander?" *Journal of Public Economics* 94(11-12):838–847.
- Austen-Smith, D. and J. Banks. 1989. "Electoral accountability and incumbency." *Models of strategic choice in politics* pp. 121–150.
- Bali, V.A. and B.C. Davis. 2007. "One More Piece to Make Us Puzzle." *Political Research Quarterly* 60(2):215–229.

- Besley, T. 2006. *Principled agents?: The political economy of good government*. Oxford University Press, USA.
- Besley, T. and S. Coate. 2008. “Issue Unbundling via Citizens’ Initiatives.” *Quarterly Journal of Political Science* 3:379–397.
- Besley, Timothy and Stephen Coate. 2003. “Elected versus appointed regulators: Theory and evidence.” *Journal of the European Economic Association* 1(5):1176–1206.
- Binswanger, Johannes and Jens Prüfer. 2012. “Democracy, populism, and (un) bounded rationality.” *European Journal of Political Economy* 28(3):358–372.
- Broder, D.S. 2000. *Democracy Derailed: How Millionaires and Special Interest Groups Have Usurped the Initiative Process and Endangered the Government the Founders Envisioned*. Vol. 1 Harcourt (San Diego).
- Bueno de Mesquita, Ethan. 2007. “Politics and the suboptimal provision of counterterrorism.” *International Organization* 61(1):9.
- Bueno de Mesquita, Ethan and Dimitri Landa. 2013a. “Does Clarifying Responsibility Always Improve Policy?” *Working Paper* .
- Bueno de Mesquita, Ethan and Dimitri Landa. 2013b. “Moral Hazard with Sequential Policy Making.” .
- Butler, D. and A. Ranney. 1994. *Referendums around the world: The growing use of direct democracy*. Aei Press.
- Canes-Wrone, Brandice, Michael C. Herron and Kenneth W. Shotts. 2001. “Leadership and Pandering: A Theory of Executive Policymaking.” *American Journal of Political Science* 45(3):532–550.

- Cho, In-Koo and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *The Quarterly Journal of Economics* 102(2):179–221.
- Daley, Brendan and Erik Snowberg. 2011. "Even if it is not bribery: the case for campaign finance reform." *Journal of Law, Economics, and Organization* 27(2):324–349.
- Feld, Lars P and Gebhard Kirchgässner. 2001. "Does Direct Democracy Reduce Public Debt Evidence from Swiss Municipalities." *Public Choice* 109(3-4):347–370.
- Feld, Lars P and John G Matsusaka. 2003. "Budget referendums and government spending: evidence from Swiss cantons." *Journal of Public Economics* 87(12):2703–2724.
- Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50(1/3):5–25.
- Fox, J. and R. Van Weelden. 2012. "Costly transparency." *Journal of Public Economics* 96(1):142–150.
- Fox, Justin. 2007. "Government transparency and policymaking." *Public choice* 131(1-2):23–44.
- Fox, Justin and Matthew C. Stephenson. 2013. "The Welfare Effects of Minority-Protective Judicial Review."
- Gehlbach, Scott. 2006. "Electoral Institutions and the National Provision of Local Public Goods." *Quarterly Journal of Political Science* 2(1):5–25.
- Gerber, E.R. 1996. "Legislative response to the threat of popular initiatives." *American Journal of Political Science* pp. 99–128.
- Gerber, E.R. 1999. *The populist paradox: Interest group influence and the promise of direct legislation*. Princeton Univ Pr.

- Gerber, E.R. and A. Lupia. 1995. "Campaign competition and policy responsiveness in direct legislation elections." *Political Behavior* 17(3):287–306.
- Hatfield, John William and Gerard Padro i Miquel. 2006. "Multitasking, limited liability and political agency."
- Hobolt, S. 2009. "Europe in question: referendums on European integration."
- Holmström, Bengt. 1979. "Moral hazard and observability." *The Bell Journal of Economics* pp. 74–91.
- Holmström, Bengt and Paul Milgrom. 1991. "Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design." *Journal of Law, Economics, and Organization* 7:24.
- Hug, S. 2004. "Occurrence and policy consequences of referendums." *Journal of Theoretical Politics* 16(3):321–356.
- Hug, S. 2009. "Some thoughts about referendums, representative democracy, and separation of powers." *Constitutional Political Economy* 20(3):251–266.
- Hugh-Jones, D. 2010. "Explaining Institutional Change: Why Elected Politicians Implement Direct Democracy."
- Kessler, A.S. 2005. "Representative versus direct democracy: The role of informational asymmetries." *Public Choice* 122(1):9–38.
- Lohmann, Susanne. 1998. "An information rationale for the power of special interests." *American Political Science Review* pp. 809–827.
- Maskin, Eric and Jean Tirole. 2004. "The Politician and the Judge: Accountability in Government." *The American Economic Review* 94(4):1034–1054.

- Matsusaka, J.G. 1992. "Economics of direct legislation." *The Quarterly Journal of Economics* 107(2):541–571.
- Matsusaka, J.G. 1995. "Fiscal effects of the voter initiative: Evidence from the last 30 years." *Journal of Political Economy* pp. 587–623.
- Matsusaka, J.G. 2004. *For the many or the few: The initiative, public policy, and American democracy*. University of Chicago Press.
- Matsusaka, J.G. 2005. "The eclipse of legislatures: Direct democracy in the 21st century." *Public Choice* 124(1):157–177.
- Matsusaka, J.G. 2010. "Popular Control of Public Policy: A Quantitative Approach." *Quarterly Journal of Political Science* 5(2):133–167.
- Matsusaka, J.G. and N.M. McCarty. 2001. "Political resource allocation: Benefits and costs of voter initiatives." *Journal of Law, Economics, and Organization* 17(2):413–448.
- Morelli, Massimo and Richard Van Weelden. 2013. "Re-election Through Division."
- Mueller, Dennis C. 1996. *Constitutional Democracy*. Oxford: Oxford University Press.
- Persson, Torsten, Gérard Roland and Guido Tabellini. 1997. "Separation of Powers and Political Accountability." *Quarterly Journal of Economics* 112(4):1163–1202.
- Persson, Torsten and Guido Tabellini. 1994. "Representative democracy and capital taxation." *Journal of Public Economics* 55(1):53–70.
- Prat, Andrea. 2005. "The Wrong Kind of Transparency." *American Economic Review* 95(3):862–877.
- Prato, C. and B. Strulovici. 2013. "Direct Democracy, Political Delegation, and Responsibility Substitution."

- Redoano, Michela and Kimberly A Scharf. 2004. “The political economy of policy centralization: direct versus representative democracy.” *Journal of Public Economics* 88(3):799–817.
- Romer, Thomas and Howard Rosenthal. 1978. “Political resource allocation, controlled agendas, and the status quo.” *Public choice* 33(4):27–43.
- Romer, Thomas and Howard Rosenthal. 1979. “Bureaucrats versus voters: On the political economy of resource allocation by direct democracy.” *The Quarterly Journal of Economics* 93(4):563–587.
- Seabright, Paul. 1996. “Accountability and decentralisation in government: An incomplete contracts model.” *European economic review* 40(1):61–89.
- Uleri, Pier Vincenzo and Michael Gallagher. 1996. *The referendum experience in Europe*. St. Martin’s Press.

9 Appendix

Let T be the type space of the Incumbent with $T \equiv \{C_{1,\cdot}, C_{-1,\cdot}, N_{1,\cdot}, N_{-1,\cdot}\}$. $C_{1,\cdot}$ denotes the type of the Incumbent which is congruent and observed $\omega_1 = 1$ and so forth. In principle we would also have to distinguish types depending on what value of ω_2 they observed. However, as all the actors in the game observe ω_2 and the game is identical for each value of ω_2 , I suppress this information. Also let P be the set of policies that can be chosen by the Incumbent in the first period for any pair of states of the world (ω_1, ω_2) with $P \equiv \{(p_1 = \omega_1, p_2 = \omega_2), (p_1 = \omega_1, p_2 \neq \omega_2), (p_1 \neq \omega_1, p_2 = \omega_2), (p_1 \neq \omega_1, p_2 \neq \omega_2)\}$. Let p, p' be arbitrary elements in P . Moreover, denote $\eta_{\cdot,\cdot}(p)$ the probability that the non-congruent Incumbent which observes (\cdot, \cdot) plays the policy vector $p \in P$ in the first period.

Proposition A.1. *The following pair of strategies and beliefs constitute the unique perfect Bayesian equilibrium of the baseline model that satisfies criterion D1:*

1. *If $B < 1$ and $q_1 \in [\frac{B}{B+1}, 1]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all ω , while non-congruent Incumbents choose $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ for all ω . The Voter reelects with certainty upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$, or $(p_1 = -1, p_2 = \omega_2)$ and does not reelect otherwise. The Voter's beliefs satisfy $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = 1$ and $\mu(I = c|p_1 \neq \omega_1, p_2 \neq \omega_2) = \mu(I = c|p_1 = 1, p_2 \neq \omega_2) = \mu(I = c|p_1 = -1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_1 = \omega_1, p_2 \neq \omega_2), \mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) \leq \pi$.*

2. *If $B < 1$ and $q_1 \in [0, \frac{B}{B+1}]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all ω ,*

the non-congruent Incumbent who observes $\omega_1 = -1$ chooses $(p_1 = 1, p_2 = \omega_2)$,

the non-congruent Incumbent who observes $\omega_1 = 1$ chooses $(p_1 = -1, p_2 = \omega_2)$ with probability $\frac{1}{\alpha} - 1$ and $(p_1 = -1, p_2 \neq \omega_2)$ with probability $2 - \frac{1}{\alpha}$,

the Voter reelects the Incumbent with certainty upon observing $(p_1 = \omega_1, p_2 = \omega_2)$ or $(p_1 = 1, p_2 = \omega_2)$, with probability $\frac{1}{(1-q_1)(B+1)}$ upon observing $(p_1 = -1, p_2 = \omega_2)$, and does not reelect otherwise.

The Voter's beliefs satisfy $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi$, $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + (1-\alpha)(1-\pi)} = \pi$, $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 \neq \omega_2) = \mu(I = c|p_1 \neq \omega_1 = 1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_1 \neq \omega_1 = -1, p_2 \neq \omega_2), \mu(I = c|p_1 = \omega_1, p_2 \neq \omega_2), \mu(I = c|p_1 = 1, p_2 \neq \omega_2) \leq \pi$.

3. *If $B \geq 1$ and $q_1 \in [\frac{1}{B+1}, 1]$, then congruent and non-congruent Incumbents choose*

$(p_1 = \omega_1, p_2 = \omega_2)$ for all ω . The Voter may reelect with positive probability upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$ or $(p_1 = -1, p_2 = \omega_2)$ and does not reelect otherwise. The Voter's beliefs satisfy $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2)$, $\mu(I = c|\cdot, p_2 \neq \omega_2) \leq \pi$.

4. If $B \geq 1$ and $q_1 \in [0, \frac{1}{B+1}]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all ω ;

the non-congruent Incumbent who observes $\omega_1 = -1$ chooses $(p_1 = 1, p_2 = \omega_2)$;

the non-congruent Incumbent who observes $\omega_1 = 1$ chooses $(p_1 = 1, p_2 = \omega_2)$ with probability $2 - \frac{1}{\alpha}$ and $(p_1 = -1, p_2 = \omega_2)$ with probability $\frac{1}{\alpha} - 1$;

the Voter's reelection strategy is: $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$, $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{(1-q_1)(B+1)} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2)$ with $r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)(B+1)}$, and $r^*(p_1 \neq \omega_1, p_2 = \omega_2) = r^*(\cdot, p_2 \neq \omega_2) = 0$.

The Voter's beliefs satisfy $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = \pi$, $\mu(I = c|p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$, $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|\cdot, p_2 \neq \omega_2) \leq \pi$.

The proof proceeds in four steps. In Lemma 1, I show that the pair of strategies and beliefs identified in Proposition A.1 is indeed a perfect Bayesian equilibrium. In Lemma 2, I then show that this equilibrium satisfies a straightforward adaptation of criterion *D1* (Cho and Kreps, 1987). Lemmas 3 and 4 then show that is is the unique equilibrium that satisfies *D1*.

Lemma 1. *The pair of strategies and beliefs identified in proposition A.1 constitute a perfect Bayesian equilibrium.*

Proof. 1. Assume $B < 1$ and $q_1 \geq \frac{B}{B+1}$. Then, the congruent Incumbent always gets re-elected if he chooses $(p_1 = \omega_1, p_2 = \omega_2)$. Hence, $U_C(p_1 = \omega_1, p_2 = \omega_2) = 3 + B$ which is the highest possible payoff that the congruent Incumbent can receive in the game and thus the congruent Incumbent has no incentive to deviate. Moreover, $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) = 2$, while $U_N(p_1 = \omega_1, p_2 \neq \omega_2) = 1 < 2$, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) = 1 + (1 - q_1)(B + 1) \leq 2$ as $q_1 \geq \frac{B}{B+1}$, and $U_N(p_1 = \omega_1, p_2 = \omega_2) = B + 1 < 2$ as $B < 1$. Hence, the non-congruent Incumbent has no incentive to deviate. Moreover, $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = 1 > \pi$, $\mu(I = c|p_1 \neq \omega_1, p_2 \neq \omega_2) = \mu(I = c|p_1 = 1, p_2 \neq \omega_2) = \mu(I = c|p_1 = -1, p_2 \neq \omega_2) = 0$, and out-of-equilibrium beliefs are specified by $\mu(I = c|p_1 = \omega_1, p_2 \neq \omega_2), \mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) \leq \pi$. It follows that the Voter has no incentive to deviate.

2. Assume $B < 1$ and $q_1 < \frac{B}{B+1}$. Then, $C_{1.}$ receives a payoff of $3 + B$ when choosing $(p_1 = \omega_1, p_2 = \omega_2)$ and thus does not have an incentive to deviate. $C_{-1.}$ receives a payoff of $3 + q_1(B + 1)$ when choosing $(p_1 = \omega_1, p_2 = \omega_2)$. Deviating to $(p_1 = 1, p_2 = \omega_2)$ yields $1 + (1 - q_1)(B + 1) < 3$, while deviating to $p_2 \neq \omega_2$ yields at most 1. Hence, $C_{-1.}$ has no incentive to deviate. Moreover, $U_{N_{-1.}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)(B + 1)$, while $U_{N_{-1.}}(p_2 \neq \omega_2) \leq 2 < 1 + (1 - q_1)(B + 1)$ as $q_1 < \frac{B}{B+1}$, and $U_{N_{-1.}}(p_1 = -1, p_2 = \omega_2) = B + 1 < 2$. Hence, $N_{-1.}$ has no incentive to deviate. $N_{1.}$ receives a payoff of 2 from choosing $(p_1 = -1, p_2 = \omega_2)$ as well as from choosing $(p_1 = -1, p_2 \neq \omega_2)$. Deviating to $(p_1 = 1, p_2 = \omega_2)$ yields $B + 1 < 2$, while deviating to $(p_1 = 1, p_2 \neq \omega_2)$ yields $1 < 2$. Hence, $N_{1.}$ has no incentive to deviate. The Voter's beliefs are given by: $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi$ as $\alpha > 1/2$, $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(\frac{1}{\alpha}-1)(1-\pi)} = \pi$ and $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 \neq \omega_2) = \mu(I = c|p_1 \neq \omega_1 = -1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_1 \neq \omega_1 = -1, p_2 \neq \omega_2), \mu(I = c|p_1 = \omega_1, p_2 \neq \omega_2), \mu(I = c|p_1 = 1, p_2 \neq \omega_2) \leq \pi$. It follows that the Voter has no incentive to deviate

from his reelection strategy.

3. Assume $B \geq 1$ and $q_1 \in [\frac{1}{B+1}, 1]$ and that the choice of first-period policy vectors is as specified in proposition 1. Then, $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p) \leq \pi$ for all p . Based on these beliefs the re-election strategy used by the Voter is indeed a best-response. In order for the non-congruent Incumbent not to deviate, the re-election probabilities used by the Voter need to satisfy the following inequalities:

$$\begin{aligned}
U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1, p_2 = \omega_2)(B + 1) \\
&+ (1 - q_1) r^*(p_1 = 1, p_2 = \omega_2)(B + 1) \\
&\geq \begin{cases} 1 + (1 - q_1) r^*(p_1 = -1, p_2 = \omega_2)(B + 1) \\ 2 \end{cases} \\
&\geq \begin{cases} U_{N_1, \cdot}(p_1 = -1, p_2 = \omega_2) \\ U_{N_1, \cdot}(p_1, p_2 \neq \omega_2) \end{cases}
\end{aligned}$$

and

$$\begin{aligned}
U_{N_{-1}, \cdot}(p_1 = -1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)(B + 1) \\
&+ (1 - q_1) r^*(p_1 = -1, p_2 = \omega_2)(B + 1) \\
&\geq \begin{cases} 1 + (1 - q_1) r^*(p_1 = 1, p_2 = \omega_2)(B + 1) \\ 2 \end{cases} \\
&\geq \begin{cases} U_{N_{-1}, \cdot}(p_1 = 1, p_2 = \omega_2) \\ U_{N_{-1}, \cdot}(p_1, p_2 \neq \omega_2). \end{cases}
\end{aligned}$$

Note that there always exists re-election probabilities $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)$, $r^*(p_1 =$

$\omega_1 = -1, p_2 = \omega_2$), $r^*(p_1 = 1, p_2 = \omega_2)$, and $r^*(p_1 = -1, p_2 = \omega_2)$ such that these inequalities are satisfied. Consider for example $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$, and $r^*(p_1 = 1, p_2 = \omega_2) = r^*(p_1 = -1, p_2 = \omega_2)$. It is straightforward to show that for such re-election probabilities the congruent Incumbent has no incentive to deviate either.

4. Assume $B \geq 1$ and $q_1 \in [0, \frac{1}{B+1}]$ and that the choice of first-period policy vectors is as specified in proposition 1. Then, $\mu(I = c|p_1 = \omega_1 = 1, p_2 = \omega_2) = \frac{\pi}{\pi + (2 - \frac{1}{\alpha})(1 - \pi)} > \pi$, $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + [(1 - \alpha) + \alpha(2 - \frac{1}{\alpha})](1 - \pi)} = \pi$, $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1 - \alpha)\pi}{(1 - \alpha)\pi + \alpha(\frac{1}{\alpha} - 1)(1 - \pi)} = \pi$, and $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p) \leq \pi$ for all p . Based on these beliefs the re-election strategy used by the Voter is indeed a best-response.

Hence, $U_{N_{1.}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1(B + 1)$ and $U_{N_{1.}}(p_1 = -1, p_2 = \omega_2) = q_1 + (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1))$. In equilibrium, $N_{1.}$ is mixing between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and hence is indifferent between these two policy vectors. Indifference is satisfied as $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{(1 - q_1)(B + 1)} - \frac{q_1}{1 - q_1} + r^*(p_1 = -1, p_2 = \omega_2)$. As $q_1 < \frac{1}{B + 1}$, this implies $1 \geq r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. Moreover, as $r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1 - q_1)(B + 1)}$, we have $U_{N_{1.}}(p_2 \neq \omega_2) \leq 2 \leq q_1 + (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) = U_{N_{1.}}(p_1 = -1, p_2 = \omega_2)$. Hence, $N_{1.}$ has no incentive to deviate.

To see that $N_{-1.}$ has no incentive to deviate note that, as $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1 - q_1)(B + 1)}$ and as $q_1 < \frac{1}{B + 1}$, we have $U_{N_{-1.}}(p_1 = 1, p_2 = \omega_2) = q_1 + (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) > 2 \geq U_{N_{-1.}}(p_2 \neq \omega_2)$ and $U_{N_{-1.}}(p_1 = 1, p_2 = \omega_2) > (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) + q_1(B + 1) = U_{N_{-1.}}(p_1 = -1, p_2 = \omega_2)$.

Moreover, we have $U_{C_{-1.}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(2 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1))$

$1)) + q_1(B + 3) > 1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) = U_{C_{-1}, \cdot}(p_1 = 1, p_2 = \omega_2)$
 and $U_{C_{1, \cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(2 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) + q_1(B + 3) >$
 $1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) = U_{C_{1, \cdot}}(p_1 = -1, p_2 = \omega_2)$ as $r^*(p_1 = 1, p_2 =$
 $\omega_2) = \frac{1}{(1 - q_1)(B + 1)} - \frac{q_1}{1 - q_1} + r^*(p_1 = -1, p_2 = \omega_2)$. As deviating to $p_2 \neq \omega_2$ yields at most
 a payoff of $1 < 2$, congruent Incumbents have no incentive to deviate.

□

Lemma 2. *The pair of strategies and beliefs identified in proposition A.1 satisfy criterion D1 (Cho and Kreps, 1987).*

Proof. Note that for any policy vector of first period policies $p \in P$, there are two information sets that could be reached: $(p_1, p_2; \cdot, \omega_2)$ and $(p_1, p_2; \omega_1, \omega_2)$. Note that for any retention probability used at any information set there exists a specification of beliefs of the Voter about the types of the Incumbent that makes this retention probability a best-response of the Voter at that information set. Let \mathbf{r} denote the vector of re-election probabilities $(r(p_1, p_2; \cdot, \omega_2), r(p_1, p_2; \omega_1, \omega_2))$ used by the Voter. Now let $D(t, T, p)$ be the set of vectors of probabilities of retention \mathbf{r} that make type t strictly prefer p to his equilibrium strategy policy vector and let $D^0(t, T, p)$ be the set of vectors of retention probabilities that make type t exactly indifferent. Finally, let $D^1(t, T, p) \equiv D(t, T, p) \cup D^0(t, T, p)$. Criterion D1 requires that if for some type t there exists a type t' such that $D^1(t, T, p) \subset D^1(t', T, p)$, then the Voter should not believe that she is facing an Incumbent of type t when she observes p .

Note that in any equilibrium identified in proposition A.1, the equilibrium payoff of any congruent type is strictly greater than the equilibrium payoff of any non-congruent type. Now consider any policy vector $p \neq (p_1 = \omega_1, p_2 = \omega_2)$ and fix a vector of re-election probabilities \mathbf{r} . We have $U_N(p, \mathbf{r}) \geq U_C(p, \mathbf{r})$. It follows that $D^1(C, T, p) \subset D^1(N, T, p)$ at any out-of-equilibrium information set $(p; \omega)$. Therefore, D1 requires that the Voter believes

she is facing a non-congruent type upon observing $(p; \omega)$ and thus should not re-elect at these information sets. Note that so far we only have pinned down out-of-equilibrium beliefs at information sets where the Voter observes both the policy vector of first-period policies and the vector of first-period states of the world. To pin down beliefs at information sets of the kind $(p_1, p_2; \cdot, \omega_2)$, we use the re-election probabilities just specified for corresponding information sets $(p_1, p_2; \omega_1, \omega_2)$. Note that the only information sets to consider are of the kind $(p_1, p_2 \neq \omega_2)$ and that based on the previous step we have $r(p_1 \neq \omega_1, p_2 \neq \omega_2) = 0$. Hence, $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) = 2 + (1 - q_1)r(p_1, p_2 \neq \omega_2)(B + 1)$, while the payoff to a congruent type of choosing the same policy vector $(p_1, p_2 \neq \omega_2)$ is at most $1 + (1 - q_1)r(p_1, p_2 \neq \omega_2)(B + 1)$. Therefore, *D1* requires that the Voter believes she is facing a non-congruent type upon observing $(p_1, p_2 \neq \omega_2)$ at any such out-of-equilibrium information set. \square

Lemma 3. *The pair of strategies and beliefs characterized in proposition A.1 constitute the unique equilibrium in which congruent incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$.*

Proof. 1. Assume $B < 1$ and that congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. As $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) \geq 2 > B + 1 \geq U_N(p_1 = \omega_1, p_2 = \omega_2)$, the non-congruent Incumbent never chooses $(p_1 = \omega_1, p_2 = \omega_2)$ in equilibrium. Moreover, if the non-congruent Incumbent chooses $p_i \neq \omega_i$ ($i \in \{1, 2\}$) with positive probability in equilibrium, the Voter learns that the Incumbent is non-congruent upon observing $p_i \neq \omega_i$ and thus does not reelect whenever ω_i is revealed to her. It follows that there is no equilibrium in which the non-congruent Incumbent chooses $(p_1 = \omega_1, p_2 \neq \omega_2)$ as then $U_N(p_1 = \omega_1, p_2 \neq \omega_2) = 1 < 2 \leq U_N(p_1 \neq \omega_1, p_2 \neq \omega_2)$. Hence, in equilibrium the non-congruent Incumbent chooses between $(p_1 \neq \omega_1, p_2 = \omega_2)$ and $(p_1 \neq \omega_2, p_2 \neq \omega_2)$. Moreover, we have $U_N(p_1 \neq \omega_1, p_2 = \omega_2) \leq 1 + (1 - q_1)(B + 1)$. Hence, if $q_1 > \frac{B}{B+1}$, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) < 2$ and the non-congruent Incumbent chooses $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ in equilibrium.

As $N_{1,\cdot}$ never chooses $(p_1 = 1, p_2 = \omega_2)$ and as $\alpha > 1/2$ we have $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + \eta_{-1,\cdot}(p_1=1, p_2=\omega_2)(1-\alpha)(1-\pi)} > \pi$ for all $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ in any equilibrium. It follows that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)(B + 1)$. Hence, if $q_1 < \frac{B}{B+1}$, we have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > 2$ and $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ in equilibrium.

There is no equilibrium, however, in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with certainty. Assume otherwise, then $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(1-\pi)} < \pi$ and thus $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 < 2$. Similarly, if $q_1 < \frac{B}{B+1}$, there is no equilibrium in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 \neq \omega_2)$ with certainty, as then $\mu(I = c|p_1 = -1, p_2 = \omega_2) = 1$ and thus $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)(B + 1) > 2$. It follows that if $q_1 < \frac{B}{B+1}$ then $N_{1,\cdot}$ mixes between $(p_1 = -1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 \neq \omega_2)$ in any equilibrium, which requires $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) = 2 = U_{N_{1,\cdot}}(p_1 \neq \omega_1, p_2 \neq \omega_2)$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = \frac{1}{(1-q_1)(B+1)} \in (0, 1)$. For the Voter to be willing to re-elect with positive probability, we need $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha\eta_{1,\cdot}(p_1=-1, p_2=\omega_2)(1-\pi)} = \pi$ which implies $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$.

2. Assume $B \geq 1$ and that congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. I first show that there then is no equilibrium in which non-congruent Incumbents choose $(p_1, p_2 \neq \omega_2)$ with positive probability. WLOG assume $N_{1,\cdot}$ plays $(p_1, p_2 \neq \omega_2)$ with positive probability. Then, $\mu(I = c|p_1, p_2 \neq \omega_2) = 0$ and thus $U_{N_{1,\cdot}}(p_1, p_2 \neq \omega_2) \leq 2$. Moreover, $\mu(I = c|p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$ and $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$. Finally, $\mu(I = c|p_1 = 1, p_2 = \omega_2) > \pi$ or $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$. If $\mu(I = c|p_1 = 1, p_2 = \omega_2) > \pi$, then $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B + 1 > 2$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. If $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$, then $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ in equilibrium. Note that $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$ implies $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Thus, if $q_1 \leq \frac{1}{B+1}$, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)(B + 1) \geq B + 1 > 2$ and

$N_{1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$. Moreover, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = B + 1 > 2 \geq U_{N_{-1,\cdot}}(p_1, p_2 \neq \omega_2)$. Hence, $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ implies that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) \geq B + 1$ which implies $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{B}{(1 - q_1)(B + 1)}$. If $q_1 > \frac{1}{B + 1}$, this is impossible, however, as then $\frac{B}{(1 - q_1)(B + 1)} > 1$. It follows that $N_{1,\cdot}$ and $N_{-1,\cdot}$ choose between $(p_1 = \omega_1, p_2 = \omega_2)$ and $(p_1 \neq \omega_1, p_2 = \omega_2)$.

Note moreover, that there is no equilibrium in which $\mu(I = c | p_1 = 1, p_2 = \omega_2) < \pi$. Assume otherwise. Then it must be the case that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ are superior to 0. But then, $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 < 2 = U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$ which contradicts $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$. By a similar argument, there is no equilibrium in which $\mu(I = c | p_1 = -1, p_2 = \omega_2) < \pi$. This implies that in any equilibrium $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$ as otherwise $\mu(I = c | p_1 = -1, p_2 = \omega_2) < \pi$. The fact that in equilibrium $\mu(I = c | p_1 = 1, p_2 = \omega_2)$ and $\mu(I = c | p_1 = -1, p_2 = \omega_2)$ are superior or equal to π implies that $N_{1,\cdot}$ chooses $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically if, and only if $N_{-1,\cdot}$ does so as well.

I now show that if $q_1 > \frac{1}{B + 1}$, then N chooses $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically. To derive a contradiction assume that $N_{1,\cdot}$ is mixing between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$. Then, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1(B + 1) + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) = 1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$. As $q_1 > \frac{1}{B + 1}$ we have $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$. But then $N_{-1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ deterministically, which by the argument made in the previous paragraph implies that $N_{1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically as well. To see this, assume otherwise. Then $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = q_1(B + 1) + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) > 1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$.

Next I show that if $q_1 < \frac{1}{B+1}$, then $N_{1,\cdot}$ playing $(p_1 = 1, p_2 = \omega_2)$ with positive probability implies that $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically. By arguments made above, it is the case in any equilibrium that $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ with positive probability. This requires that $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)(B+1) + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)(B+1) \geq 1 + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)(B+1) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$. As $q_1 < \frac{1}{B+1}$, $q_1(B+1) < 1$ and thus $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. But then $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)(B+1) > q_1(B+1) + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)(B+1) = U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically. This in turn implies that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) \leq 2 - \frac{1}{\alpha}$ as otherwise $\mu(I = c | p_1 = 1, p_2 = \omega_2) < \pi$. From $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) \leq 2 - \frac{1}{\alpha}$, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$, and $\eta_{1,\cdot}(p_1, p_2 \neq \omega_2) = 0$, we conclude that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 2 - \frac{1}{\alpha}$, and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$ in equilibrium.

□

Lemma 4. *The pair of strategies and beliefs characterized in proposition A.1 constitute the unique equilibrium that satisfies criterion D1.*

Proof. Assume there exists an equilibrium in which $(p_1 = \omega_1, p_2 = \omega_2)$ is not played with positive probability for some state of the world ω . Fix a vector of re-election probabilities \mathbf{r} , then $U_C(p_1 = \omega_1, p_2 = \omega_2, \mathbf{r}) > U_N(p_1 = \omega_1, p_2 = \omega_2, \mathbf{r})$. Moreover, for any policy vector $p \neq (p_1 = \omega_1, p_2 = \omega_2)$ and any vector of equilibrium re-election probabilities \mathbf{r}^* we have $U_C(p, \mathbf{r}^*) \leq U_N(p, \mathbf{r}^*)$. In other words, in any equilibrium in which $(p_1 = \omega_1, p_2 = \omega_2)$ is not played with positive probability the equilibrium payoff of any non-congruent type is at least as high as the equilibrium payoff of any congruent type. It follows that $D^1(N, T, p_1 = \omega_1, p_2 = \omega_2) \subset D(C, T, p_1 = \omega_1, p_2 = \omega_2)$ and the Voter should believe she is facing a

congruent Incumbent upon observing $(p_1 = \omega_1, p_2 = \omega_2)$. Similarly, $D^1(N_{1,\cdot}, T, p_1 = 1, p_2 = \omega_2) \subset D^1(N_{-1,\cdot}, T, p_1 = 1, p_2 = \omega_2) \subset D(C_{1,\cdot}, T, p_1 = 1, p_2 = \omega_2)$ and the Voter should believe she is facing a congruent Incumbent upon observing $(p_1 = 1, p_2 = \omega_2)$. A similar argument holds for $(p_1 = -1, p_2 = \omega_2)$. But then the congruent Incumbent wants to deviate to $(p_1 = \omega_1, p_2 = \omega_2)$ as he then receives a payoff of $3 + B$ which is greater than $2 + B$, the highest payoff the congruent type can get in any equilibrium in which he does not play $(p_1 = \omega_1, p_2 = \omega_2)$. \square

Proposition A.2. *The following pair of strategies and beliefs constitute the equilibrium of the direct democracy model when $q_1 \in [0, 1]$ and $q_2 = 1$.*

1. *If $q_1 \in [\max\{1 - 2B, \frac{1}{B+2}\}, 1]$, then congruent and non-congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all ω , the Voter holds a referendum to set $p_1 = \omega_1$ if $p_1 \neq \omega_1$ is revealed to her, may reelect with positive probability upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$, and $(p_1 = -1, p_2 = \omega_2)$ and does not reelect otherwise. The Voter's beliefs satisfy $\mu(I = c | p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c | p_1 = 1, p_2 = \omega_2) = \mu(I = c | p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(I = c | p_1 \neq \omega_1, p_2 = \omega_2)$, $\mu(I = c | \cdot, p_2 \neq \omega_2) \leq \pi$. If $q_1 \in [1 - 2B, 1 - B]$, the Voter also holds, with probability $R_1 \in [1 - \frac{B}{1-q_1}, \frac{B}{1-q_1}]$, a referendum to set $p_1 = 1$ upon observing $(p_1 = -1, p_2 \neq \omega_2)$ and with probability $R_{-1} \in [1 - \frac{B}{1-q_1}, \frac{B}{1-q_1}]$ a referendum to set $p_1 = -1$ upon observing $(p_1 = 1, p_2 \neq \omega_2)$.*
2. *If $q_1 \in [0, \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$,
The non-congruent Incumbent who observes $\omega_1 = -1$ chooses $(p_1 = 1, p_2 = \omega_2)$;
The non-congruent Incumbent who observes $\omega_1 = 1$ chooses $(p_1 = 1, p_2 = \omega_2)$ with probability $2 - \frac{1}{\alpha}$ and $(p_1 = -1, p_2 = \omega_2)$ with probability $\frac{1}{\alpha} - 1$;
The Voter holds a referendum to set $p_1 = \omega_1$ if $p_1 \neq \omega_1$ is revealed to her and to set*

$p_1 = 1$ upon observing $(p_1 = -1, p_2 \neq \omega_2)$;

The Voter's reelection strategy is: $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$, $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{B+1} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)(B+1)}$ and $r^*(p_1 \neq \omega_1, p_2) = r^*(p_1, p_2 \neq \omega_2) = 0$.

The Voter's beliefs satisfy $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = \pi$, $\mu(I = c|p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$, $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|\cdot, p_2 \neq \omega_2) \leq \pi$.

3. If $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$, there exists an infinity of equilibria. In all these equilibria, congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ and the non-congruent Incumbents play $p_1 = \omega_1$ and $p_2 \neq \omega_2$ with positive probability. For all $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$ there always exists equilibria in which the non-congruent Incumbents choose $p_2 = \omega_2$ with positive probability. If $q_1 \in [\frac{2B+1}{2B+3}, 1 - 2B]$ however, there also exist equilibria in which the non-congruent Incumbents play $p_2 \neq \omega_2$ with certainty.

The Voter holds a referendum to set $p_1 = \omega_1$ when $p_1 \neq \omega_1$. If ω_1 is not revealed, the Voter never holds a referendum when $p_2 = \omega_2$, and holds a referendum with non-degenerate probability whenever $p_2 \neq \omega_2$.

The Voter always reelects with positive probability when $p_2 = \omega_2$ and never reelects when $p_2 \neq \omega_2$.

The proof proceeds in the following steps. First, I show in Lemma 5 that the pair of strategies and beliefs identified in proposition A.2 constitutes a perfect Bayesian equilibrium. In Lemma 6, I then show that this equilibrium survives $D1$. Lemmas 7 to 13 prove that if $q_1 \in [0, \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}]$ this equilibrium is the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. Lemmas 7, 14, 15 then prove the same statement for $q_1 > \max\{1 - 2B, \frac{1}{B+2}\}$. Lemmas 16 and 17 then prove that when $q_1 \in [\frac{2B}{B+1}, 1 - 2B]$,

then in any equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$, the non-congruent Incumbents play $p_1 = \omega_1$ and $p_2 \neq \omega_2$ with positive probability. Finally, Lemma 18 shows that an equilibrium satisfies *D1* if, and only if, congruent Incumbents choose $p_2 = \omega_2$ in equilibrium.

Lemma 5. *The pair of strategies and beliefs identified in proposition A.2 constitutes a perfect Bayesian equilibrium.*

Proof. 1. Assume that $q_1 \in [\max\{1 - 2B, \frac{1}{B+2}\}, 1]$ and that the profile of first-period policy vectors is as specified in proposition 2. Then, $\mu(I = c|p_1 = \omega_1, p_2 = \omega_2) = \mu(I = c|p_1 = 1, p_2 = \omega_2) = \mu(I = c|p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2), \mu(I = c|\cdot, p_2 \neq \omega_2) \leq \pi$. Based on these beliefs the re-election strategy used by the Voter is indeed a best-response. In order for the non-congruent Incumbent not to deviate, the re-election probabilities used by the Voter need to satisfy the following inequalities:

$$\begin{aligned}
U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)(B + 1) \\
&\quad + (1 - q_1) r^*(p_1 = 1, p_2 = \omega_2)(B + 1) \\
&\geq \begin{cases} (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) \\ 2 - q_1 \end{cases} \\
&\geq \begin{cases} U_{N_1, \cdot}(p_1 = -1, p_2 = \omega_2) \\ U_{N_1, \cdot}(p_1, p_2 \neq \omega_2), \end{cases}
\end{aligned}$$

and

$$\begin{aligned}
U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)(B + 1) \\
&\quad + (1 - q_1) r^*(p_1 = -1, p_2 = \omega_2)(B + 1) \\
&\geq \begin{cases} (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) \\ 2 - q_1 \end{cases} \\
&\geq \begin{cases} U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2) \\ U_{N_{-1},\cdot}(p_1, p_2 \neq \omega_2). \end{cases}
\end{aligned}$$

Note that, if $q_1 \geq \max\{1 - B, \frac{1}{B+2}\}$ there always exists re-election probabilities $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)$, $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)$, $r^*(p_1 = 1, p_2 = \omega_2)$, and $r^*(p_1 = -1, p_2 = \omega_2)$ such that these inequalities are satisfied. In any case $r^*(p_1 = \omega_1, p_2 = \omega_2) = r^*(p_1 = 1, p_2 = \omega_2) = r^*(p_1 = -1, p_2 = \omega_2) = 1$ satisfies the conditions. It is straightforward to show that for such re-election probabilities congruent Incumbents have no incentive to deviate either. If $q_1 \in [\max\{1 - 2B, \frac{1}{B+2}\}, 1 - B)$ then $B + 1 < 2 - q_1$, however. If the Voter holds, with probability $R_{-1} \in [1 - \frac{B}{1-q_1}, \frac{B}{1-q_1}]$, a referendum to set $p_1 = -1$ upon observing $(p_1 = 1, p_2 \neq \omega_2)$, then we have $B + 1 \geq 1 + (1 - q_1)R_{-1} = U_{N_{-1},\cdot}(p_1 = 1, p_2 \neq \omega_2)$ and $B + 1 \geq 1 + (1 - q_1)(1 - R_{-1}) = U_{N_{-1},\cdot}(p_1 = 1, p_2 \neq \omega_2)$. Thus, there then always exists re-election probabilities $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)$, $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)$, $r^*(p_1 = 1, p_2 = \omega_2)$, and $r^*(p_1 = -1, p_2 = \omega_2)$ such that non-congruent Incumbents have no incentive to deviate to $(p_1 = 1, p_2 \neq \omega_2)$. A similar remark holds with respect to $(p_1 = -1, p_2 \neq \omega_2)$.

2. Assume that $q_1 \in [0, \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}]$ and that the choice of first-period policy vectors is as specified in proposition 2. Then, $\mu(I = c | p_1 = \omega_1 = 1, p_2 = \omega_2) = \frac{\pi}{\pi + (2 - \frac{1}{\alpha})(1 - \pi)} > \pi$, $\mu(I = c | p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(I = c | p_1 = 1, p_2 = \omega_2) =$

$\frac{\alpha\pi}{\alpha\pi + [(1-\alpha) + \alpha(2 - \frac{1}{\alpha})](1-\pi)} = \pi$, $\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(\frac{1}{\alpha} - 1)(1-\pi)} = \pi$, and $\mu(I = c|p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(I = c|p_2 \neq \omega_2) \leq \pi$. Based on these beliefs the re-election strategy used by the Voter is indeed a best-response.

By lemma 7 below, the Voter never holds a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$. Moreover, we have $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) = \frac{\pi(1-\alpha)}{\pi(1-\alpha) + (\frac{1}{\alpha} - 1)(1-\pi)\alpha} = \pi > 1/2$ and the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$.

Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1(B + 1)$ and $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1))$. In equilibrium, $N_{1,\cdot}$ is mixing between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and hence is indifferent between these two policy vectors. Indifference is satisfied as $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{B+1} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2)$. As $q_1 < \frac{1}{B+2}$, this implies $1 \geq r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. Moreover, as the Voter holds a referendum to set $p_1 = 1$ whenever needed upon observing $p_2 \neq \omega_2$ and as $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)(B+1)}$, we have $U_{N_{1,\cdot}}(p_2 \neq \omega_2) = 1 \leq (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1(B + 1) = U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. Hence, $N_{1,\cdot}$ has no incentive to deviate.

To see that $N_{-1,\cdot}$ has no incentive to deviate, note that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) \geq 2 - q_1 = U_{N_{-1,\cdot}}(p_2 \neq \omega_2)$ as $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)(B+1)}$. Moreover, as $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$ and $q_1 < \frac{1}{B+2}$, we have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) + q_1(B + 1) = U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$.

Moreover, we have $U_{C_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(2 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) + q_1(B + 3) > 1 + q_1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) = U_{C_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$, and $U_{C_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(2 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) + q_1(B + 3) > 1 + q_1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) = U_{C_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. As deviating to

$p_2 \neq \omega_2$ yields at most a payoff of $1 < 2$, congruent Incumbents have no incentive to deviate.

3. A full statement and proof of the equilibria in that range is available upon request. In lemmas 16 and 17 below, I show, however, that there is neither an equilibrium in which non-congruent Incumbents never choose $p_2 \neq \omega_2$ nor an equilibrium in which they never choose $p_1 = \omega_1$ in that range.

□

Lemma 6. *The perfect Bayesian equilibria identified in proposition A.2 survive criterion D1.*

Proof. Note that in any equilibrium identified in proposition A.2, the equilibrium payoff of any congruent type is strictly greater than the equilibrium payoff of any non-congruent type. Now remark that $U_C(p_1 = \omega_1, p_2 \neq \omega_2), U_C(p_1 \neq \omega_1, p_2 \neq \omega_2) \leq 1 + q_1 r(p_1, p_2; \omega_1, \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2; \cdot, \omega_2)(B + 1)$, while $U_N(p_1 = \omega_1, p_2 \neq \omega_2), U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) \geq 1 + q_1 r(p_1, p_2; \omega_1, \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2; \cdot, \omega_2)(B + 1)$. It follows that the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1 = \omega_1, p_2 \neq \omega_2)$ or $(p_1 \neq \omega_1, p_2 \neq \omega_2)$. A similar argument shows that the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1, p_2 \neq \omega_2)$. Next suppose $(p_1 \neq \omega_1, p_2 = \omega_2)$ is out-of-equilibrium. As $U_C(p_1 \neq \omega_1, p_2 = \omega_2) \leq 2 + q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1)$ while $U_C(p_1 = \omega_1, p_2 = \omega_2) = 2 + q_1 r(p_1 = \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1)$, it must be the case that $q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1) \geq q_1 r(p_1 = \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1)$ for C to be willing to deviate to $(p_1 \neq \omega_1, p_2 = \omega_2)$. Moreover, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) \geq q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1)$ while $U_N(p_1 = \omega_1, p_2 = \omega_2) = q_1 r(p_1 = \omega_1, p_2 = \omega_2)(B + 1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B + 1)$. It follows that if $q_1 r(p_1 \neq \omega_1, p_2 =$

$\omega_2)(B+1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B+1) = q_1r(p_1 = \omega_1, p_2 = \omega_2)(B+1) + (1 - q_1)r(p_1, p_2 = \omega_2)(B+1)$ N is willing to deviate to $(p_1 \neq \omega_1, p_2 = \omega_2)$. Hence, if in equilibrium N chooses $(p_1 = \omega_1, p_2 = \omega_2)$, the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1 \neq \omega_1, p_2 = \omega_2)$. \square

Lemmas 7 to 13 prove that if $q_1 \in [0, \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}]$ the pair of strategies and beliefs characterized in proposition A.2 constitute the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. Hence, in the following, I assume throughout that congruent Incumbents play $(p_1 = \omega_1, p_2 = \omega_2)$.

Lemma 7. *If the congruent Incumbent chooses $(p_1 = \omega_1, p_2 = \omega_2)$, then the Voter never holds a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$.*

Proof. $Pr(\omega_1 = 1|p_1 = 1, p_2 = \omega_2) = \frac{(\pi + \eta_{1,\cdot}(p_1=1, p_2=\omega_2)(1-\pi))\alpha}{(\pi + \eta_{1,\cdot}(p_1=1, p_2=\omega_2)(1-\pi))\alpha + \eta_{-1,\cdot}(p_1=1, p_2=\omega_2)(1-\pi)(1-\alpha)}$ which is increasing in $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and decreasing in $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Let $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$ and $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$. Then, $Pr(\omega_1 = 1|p_1 = 1, p_2 = \omega_2) = \frac{\pi\alpha}{\pi\alpha + (1-\pi)(1-\alpha)} > \alpha$ as $\pi > 1/2$. \square

Lemma 8. *There does not exist an equilibrium in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with certainty.*

Proof. Assume otherwise. Then

$$\mu(I = c|p_1 = -1, p_2 = \omega_2) = \frac{(1 - \alpha)\pi}{(1 - \alpha)\pi + (\alpha + (1 - \alpha)\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2))(1 - \pi)} < \pi$$

for all $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2)$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq 1 - q_1 < 1 \leq U_{N_{1,\cdot}}(p_2 \neq \omega_2)$. \square

Lemma 9. *If $q_1 < \frac{2B}{B+1}$, there does not exist an equilibrium in which $N_{1\cdot}$ does not play $(p_1 = 1, p_2 = \omega_2)$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Then, $\mu(I = c|p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$ and $\mu(I = c|p_1 = 1, p_2 = \omega_2) > \pi$ for all $\eta_{-1\cdot}(p_1 = 1, p_2 = \omega_2)$, which implies that $U_{N_{1\cdot}}(p_1 = 1, p_2 = \omega_2) = B + 1$ and $U_{N_{-1\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(B + 2)$. By the previous lemma, there is no equilibrium in which $N_{1\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with certainty. Hence, if $\eta_{1\cdot}(p_1 = 1, p_2 = \omega_2) = 0$ in equilibrium then $N_{1\cdot}$ must be playing $p_2 \neq \omega_2$ with positive probability. $N_{1\cdot}$ is only willing to choose $(p_1, p_2 \neq \omega_2)$ with positive probability if $N_{-1\cdot}$ does so as well, as otherwise the Voter infers that $\omega_1 = 1$ when observing $(p_1, p_2 \neq \omega_2)$ and sets $p_1 = 1$ via referendum if needed. But then, $U_{N_{1\cdot}}(p_1, p_2 \neq \omega_2) = 1 < B + 1 = U_{N_{1\cdot}}(p_1 = 1, p_2 = \omega_2)$. So suppose $N_{1\cdot}$ and $N_{-1\cdot}$ play $(p_1 = 1, p_2 \neq \omega_2)$ with positive probability. For such a behavior to be a best-response, it has to be the case that

$$U_{N_{-1\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq (1 - q_1)(B + 2), \text{ and}$$

$$U_{N_{1\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_{-1} \geq B + 1,$$

as otherwise $N_{-1\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$ and $N_{1\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$ respectively. This implies that $\frac{B}{1 - q_1} \leq R_{-1} \leq \frac{1}{1 - q_1} - B - 1$. But this is impossible as $q_1 < \frac{2B}{B+1}$ implies $\frac{B}{1 - q_1} > \frac{1}{1 - q_1} - B - 1$. A similar argument shows that there is no equilibrium in which $N_{1\cdot}$ and $N_{-1\cdot}$ choose $(p_1 = -1, p_2 \neq \omega_2)$ with positive probability when $q_1 < \frac{2B}{B+1}$. \square

Lemma 10. *If $q_1 < \frac{1}{B+2}$, then there does not exist an equilibrium in which $N_{1\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1\cdot}(p_1 = 1, p_2 = \omega_2) = 1$. Then, $\mu(I = c|p_1 = 1, p_2 = \omega_2) \leq \pi$ and as by lemma 7 the Voter does not

hold a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$, we have $U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1r^*(p_1 = \omega_1, p_2 = \omega_2)(B + 1)$. As $q_1 < \frac{1}{B+2}$, $q_1r^*(p_1 = \omega_1, p_2 = \omega_2)(B + 1) < 1$ and thus $U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2) \geq U_{N_1, \cdot}(p_2 \neq \omega_2) \geq 1$ implies that $r^*(p_1 = 1, p_2 = \omega_2) > 0$ in equilibrium. This requires $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \pi$ and thus $\eta_{-1, \cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Moreover, $\eta_{1, \cdot}(p_1 = 1, p_2 = \omega_2) = 1$ implies $\mu(I = c|p_1 = -1, p_2 = \omega_2) \geq \pi$ and $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) = 1$ which in turn implies that the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. If $\eta_{-1, \cdot}(p_1 = -1, p_2 = \omega_2) < 1$ in equilibrium, then $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$ and $r^*(p_1 = -1, p_2 = \omega_2) = 1$. Hence, $U_{N_1, \cdot}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(B + 2) > B + 1 \geq U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2)$ as $q_1 < \frac{1}{B+2}$. Hence, if $\eta_{1, \cdot}(p_1 = 1, p_2 = \omega_2) = 1$ in equilibrium we have $\eta_{-1, \cdot}(p_1 = -1, p_2 = \omega_2) = 1$ which requires that

$$\begin{aligned} U_{N_{-1}, \cdot}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) \\ &\quad + q_1r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)(B + 1) \\ &\geq (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) \\ &= U_{N_{-1}, \cdot}(p_1 = 1, p_2 = \omega_2) \end{aligned}$$

and

$$\begin{aligned} U_{N_1, \cdot}(p_1 = 1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) \\ &\quad + q_1r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)(B + 1) \\ &\geq (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) \\ &= U_{N_1, \cdot}(p_1 = -1, p_2 = \omega_2) \end{aligned}$$

As $q_1 < \frac{1}{B+2}$ and thus $q_1r^*(p_1 = \omega_1, p_2 = \omega_2)(B + 1) < 1 - q_1$, (1) implies that $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$, whereas (2) implies $r^*(p_1 = -1, p_2 = \omega_2) < r^*(p_1 =$

$1, p_2 = \omega_2$). Contradiction. \square

Lemma 11. *If $q_1 < \frac{1}{B+2}$, then there does not exist an equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1, p_2 \neq \omega_2)$ and does not play $(p_1 = -1, p_2 = \omega_2)$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$, $\eta_{1,\cdot}(p_1, p_2 \neq \omega_2) > 0$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. Case 1: $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Then, $\mu(I = c | p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + \eta_{1,\cdot}(p_1=1, p_2=\omega_2)\alpha(1-\pi)} > \pi$ as $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Thus, $r^*(p_1 = 1, p_2 = \omega_2) = 1$ and $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(B + 2)$. As $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$, we have $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) = 1$ and thus the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Hence, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq B + 1$. As $q_1 < \frac{1}{B+2}$, we then have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ and thus $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. This implies that $\mu(I = c | p_1 = -1, p_2 = \omega_2) = 1$. But then, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(B + 2) > B + 1 \geq U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ and $N_{1,\cdot}$ deviates to $(p_1 = -1, p_2 = \omega_2)$.

Case 2: $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$. Then $\mu(I = c | p_1 = -1, p_2 = \omega_2) > \pi$ as $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(B + 2)$ which, as $q_1 < \frac{1}{B+2}$, is strictly greater than $B + 1 \geq U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. Thus, $N_{1,\cdot}$ deviates to $(p_1 = -1, p_2 = \omega_2)$. \square

Lemma 12. *If $q_1 < \min\{\frac{2B}{B+1}, \frac{1}{B+2}\}$ and $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ in equilibrium, then $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > 0$, yet $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Case 1: $\mu(I = c | p_1 = 1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 0$. Then, $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 - q_1 < 1$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. As $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$, we have $\mu(I = c | p_1 =$

$1, p_2 = \omega_2) > \pi$, contradicting the premise.

Case 2: $\mu(I = c|p_1 = 1, p_2 = \omega_2) = \pi$ which implies that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. As $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$ we have $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 2 - \frac{1}{\alpha}$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$. In turn, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$ implies $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) > \pi$ and thus the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Hence, as $N_{1,\cdot}$ is mixing, we have

$$\begin{aligned} U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1(B + 1) \\ &= (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \end{aligned}$$

As $q_1 < \frac{1}{B+2}$, $q_1(B + 1) < 1 - q_1$ and thus $N_{1,\cdot}$ mixing implies $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. But then,

$$\begin{aligned} U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) &= (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) \\ &> (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) + q_1(B + 1) \\ &= U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) \end{aligned}$$

and thus $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. As $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$, this implies that $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$ which contradicts $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$.

Case 3: $\mu(I = c|p_1 = 1, p_2 = \omega_2) > \pi$. Then, $r^*(p_1 = 1, p_2 = \omega_2) = 1$ and as $q_1 < \frac{1}{B+2}$, $N_{-1,\cdot}$ never plays $(p_1 = -1, p_2 = \omega_2)$. As $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$, this implies that $N_{-1,\cdot}$ plays $p_2 \neq \omega_2$ with positive probability. In equilibrium, this requires $U_{N_{-1,\cdot}}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_1 \geq (1 - q_1)(B + 2) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ or $U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq (1 - q_1)(B + 2) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$, which implies $R_1 \geq B + 2 - \frac{1}{1 - q_1}$ or $R_{-1} \leq \frac{1}{1 - q_1} - B - 1$. In turn, as $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$

with positive probability, it must be the case that

$$B + 1 \geq 1 + (1 - q_1)R_{-1}$$

$$B + 1 \geq 1 + (1 - q_1)(1 - R_1)$$

which implies that $R_{-1} \leq \frac{B}{1-q_1} < 1$ and $R_1 \geq 1 - \frac{B}{1-q_1} > 0$. For the Voter to be willing to adopt such probabilities of holding a referendum, it must be the case that $N_{1,\cdot}$ plays $p_2 \neq \omega_2$ with positive probability. As $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$, this requires $R_{-1} = \frac{B}{1-q_1}$ and $R_1 = 1 - \frac{B}{1-q_1}$. As $q_1 < \frac{2B}{B+1}$, we have $1 - \frac{B}{1-q_1} < B + 2 - \frac{1}{1-q_1}$ and $\frac{B}{1-q_1} > \frac{1}{1-q_1} - B - 1$. But then, $N_{-1,\cdot}$ wants to deviate to $(p_1 = 1, p_2 = \omega_2)$. \square

Lemma 13. *In any equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically, we have $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 2 - \frac{1}{\alpha}$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$.*

Proof. Assume there exists an equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically and assume that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 2 - \frac{1}{\alpha}$. Then, $\mu(I = c | p_1 = 1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 0$. Moreover, $\mu(I = c | p_1 = -1, p_2 = \omega_2) > \pi$ (and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$) and $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) = \frac{\pi(1-\alpha)}{\pi(1-\alpha) + \eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2)(1-\pi)\alpha} > \pi$ as $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$. Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1(B+1) < (1-q_1)(B+2) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ as $q_1 < \frac{1}{B+2}$. But then $N_{1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$.

So assume that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 2 - \frac{1}{\alpha}$. Then, $\mu(I = c | p_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 1$, which implies that $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B + 1$. If $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$, then as $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 2 - \frac{1}{\alpha}$, we have $\eta_{1,\cdot}(p_2 \neq \omega_2) > 0$. As $N_{-1,\cdot}$ never plays $p_2 \neq \omega_2$, the Voter holds a referendum to set $p_1 = 1$ if needed upon observing $p_2 \neq \omega_2$. But then, $U_{N_{1,\cdot}}(p_2 \neq \omega_2) = 1 < B + 1$ which implies that $\eta_{1,\cdot}(p_2 \neq \omega_2) = 0$.

So assume that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > \frac{1}{\alpha} - 1$. Then, $\mu(I = c | p_1 = -1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq 1 - q_1 < B + 1$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. \square

The following two lemmas together with lemma 7 prove that if $q_1 > \max\{1 - 2B, \frac{1}{B+2}\}$ the pair of strategies and beliefs characterized in proposition A.2 constitute the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$.

Lemma 14. *If $q_1 > \max\{1 - 2B, \frac{1}{B+2}\}$, there does not exist an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Then, $\mu(I = c | p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$.
Case 1: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(I = c | p_1 = 1, p_2 = \omega_2) < \pi$ and $r^*(p_1 = 1, p_2 = \omega_2) = 0$. But then $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 - q_1 < 1 \leq U_{N_{-1,\cdot}}(p_2 \neq \omega_2)$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. But then, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 1$, which is impossible.

Case 2: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(I = c | p_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 1$. Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B + 1 > (1 - q_1)(B + 2) \geq U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ as $q_1 > \frac{1}{B+2}$ and thus $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. In turn, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$ implies that $\eta_{1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ which requires $U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_{-1} \geq B + 1 = U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ or $U_{N_{1,\cdot}}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_1) \geq B + 1 = U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. This, in turn implies that $R_{-1} \geq \frac{B}{1 - q_1} > 0$ or $R_1 \leq 1 - \frac{B}{1 - q_1} < 1$. This however requires $\eta_{-1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{-1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ respectively as otherwise the Voter infers from $(p_1, p_2 \neq \omega_2)$ that $\omega_1 = 1$ and thus sets $R_{-1} = 0$ or $R_1 = 1$. It follows that $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ which implies that

$\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$ and $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Thus, $U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) = B + 1$. Finally, $\eta_{-1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{-1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ requires that $U_{N_{-1},\cdot}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_1 \geq B + 1 = U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2)$ or $U_{N_{-1},\cdot}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq B + 1 = U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2)$ which implies $R_1 \geq \frac{B}{1 - q_1}$ or $R_{-1} \leq 1 - \frac{B}{1 - q_1}$. Hence, in equilibrium, we need $1 - \frac{B}{1 - q_1} \geq \frac{B}{1 - q_1}$ which is impossible as $q_1 > 1 - 2B$.

Case 3: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$ as $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Moreover, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ implies that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$ and thus $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) \geq \pi$. It follows that the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Suppose first that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1 - \eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$, and $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$ and thus $U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) = B + 1 > (1 - q_1)(B + 2) \geq U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2)$ as $q_1 > \frac{1}{B+2}$. But then, $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$, contradiction. So suppose that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 1 - \eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ which implies $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > 0$. It follows that $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ which requires

$$\begin{aligned} U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)(B + 1)) \\ &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)(B + 1) + q_1(B + 1) \\ &= U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2). \end{aligned}$$

As $q_1 > \frac{1}{B+2}$, $q_1(B+1) > 1 - q_1$ and thus we have $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$.

But then,

$$\begin{aligned}
U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)(B + 1) + q_1(B + 1) \\
&> (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)(B + 1)) \\
&= U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2)
\end{aligned}$$

and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Contradiction. \square

Lemma 15. *If $q_1 > \max\{1 - 2B, \frac{1}{B+2}\}$ and $N_{1,\cdot}$ plays $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically, then so does $N_{-1,\cdot}$.*

Proof. Assume otherwise, i.e. assume $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$ and $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$. Then, $\mu(I = c|p_1 = -1, p_2 = \omega_2) > \pi$ and $\mu(I = c|p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$ and $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$. But then, $U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) = B + 1 > (1 - q_1)(B + 2) \geq U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2)$ as $q_1 > \frac{1}{B+2}$. Moreover, as $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$, if $\eta_{-1,\cdot}(p_2 \neq \omega_2) > 0$, the Voter infers that $\omega_1 = -1$ from $p_2 \neq \omega_2$ and sets $p_1 = -1$ via referendum whenever needed. Hence, $U_{N_{-1},\cdot}(p_2 \neq \omega_2) = 1 < B + 1$ and $N_{-1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$. \square

Lemma 16. *If $q_1 \in (\frac{2B}{B+1}, 1 - 2B)$, there does not exist an equilibrium s.t. non-congruent types never choose $p_2 \neq \omega_2$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium s.t. non-congruent types choose $p_2 = \omega_2$ deterministically. Case 1: $q_1 \geq \frac{1}{B+2}$. Then, the highest possible payoff that non-congruent incumbents can achieve when choosing $p_2 = \omega_2$ is $B + 1$. In order for non-congruent types not to want to deviate to $(p_1 = 1, p_2 \neq \omega_2)$ we need $B + 1 \geq 1 + (1 - q_1)R_{-1} = U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$ and $B + 1 \geq 1 + (1 - q_1)(1 - R_{-1}) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$, which implies that $R_{-1} \leq \frac{B}{1 - q_1}$ and $R_{-1} \geq 1 - \frac{B}{1 - q_1}$. But $\frac{B}{1 - q_1} \geq 1 - \frac{B}{1 - q_1}$ implies $q_1 \geq 1 - 2B$.

Case 2: $q_1 \in \left(\frac{2B}{B+1}, \frac{1}{B+2}\right)$. Note first that if $q_1 < \frac{1}{B+2}$, the maximal payoff that $N_{1\cdot}$ can achieve, in an equilibrium where non-congruent incumbents never choose $p_2 \neq \omega_2$, is $B + 1$. To see this, assume there exists an equilibrium such that $U_{N_{1\cdot}}(p_1 = -1, p_2 = \omega_2) > B + 1$. As non-congruent types never choose $p_2 \neq \omega_2$, we have $\eta_{1\cdot}(p_1 = -1, p_2 = \omega_2) = 1$ which implies that $\mu(I = c | p_1 = -1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then, $U_{N_{1\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 - q_1 < 1 \leq U_{N_{1\cdot}}(p_2 \neq \omega_2)$. Contradiction. So assume there exists an equilibrium such that non-congruent Incumbents never choose $p_2 \neq \omega_2$. As the highest payoff that $N_{1\cdot}$ can achieve is $B + 1$ and the highest payoff that $N_{-1\cdot}$ can achieve is $(1 - q_1)(B + 2)$, in equilibrium, it has to be the case that $B + 1 \geq 1 + (1 - q_1)(1 - R_1) = U_{N_{1\cdot}}(p_1 = -1, p_2 \neq \omega_2)$ and $(1 - q_1)(B + 2) \geq 1 + (1 - q_1)R_1 = U_{N_{-1\cdot}}(p_1 = -1, p_2 \neq \omega_2)$. This implies that $R_1 \geq 1 - \frac{B}{1 - q_1}$ and $R_1 \leq B + 2 - \frac{1}{1 - q_1}$ which is impossible as $q_1 > \frac{2B}{B+1}$. \square

Lemma 17. *There does not exist an equilibrium such that $N_{1\cdot}$ (or $N_{-1\cdot}$) chooses $(p_1 = 1, p_2 \neq \omega_2)$ (or $(p_1 = -1, p_2 \neq \omega_2)$) yet $N_{-1\cdot}$ ($N_{1\cdot}$) does not.*

Proof. WLOG, assume there exists an equilibrium such that $N_{1\cdot}$ chooses $(p_1 = 1, p_2 \neq \omega_2)$ with positive probability yet $N_{-1\cdot}$ does not. Then, $Pr(\omega_1 = 1 | p_1 = 1, p_2 \neq \omega_2) = 1$ and the Voter does not hold a referendum upon observing $(p_1 = 1, p_2 \neq \omega_2)$. Hence, $U_{N_{1\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1$. Moreover, as $N_{1\cdot}$ does not play $p_2 = \omega_2$ deterministically, we have $\mu(I = c | p_1 = 1, p_2 = \omega_2) > \pi$ or $\mu(I = c | p_1 = -1, p_2 = \omega_2) > \pi$. In the first case, $U_{N_{1\cdot}}(p_1 = 1, p_2 = \omega_2) = B + 1 > 1$ and $N_{1\cdot}$ wants to deviate from $(p_1 = 1, p_2 \neq \omega_2)$. In the second case, we have $\eta_{-1\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ and thus $\mu(I = c | p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Hence, $U_{N_{-1\cdot}}(p_1 = -1, p_2 = \omega_2) = B + 1$. If $q_1 > \frac{1}{B+2}$, then $B + 1 > (1 - q_1)(B + 2)$ and thus $\eta_{-1\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. But then $\mu(I = c | p_1 = 1, p_2 = \omega_2) > \pi$ and $N_{1\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. Moreover, $\mu(I = c | p_1 = -1, p_2 = \omega_2) > \pi$ implies $\eta_{1\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$ and thus $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) > 1/2$. Hence, if $q_1 \leq \frac{1}{B+2}$, then $U_{N_{1\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(B + 2) \geq B + 1 > 1$. But then, $N_{1\cdot}$ deviates to

$(p_1 = -1, p_2 = \omega_2)$. □

Lemma 18. *The pair of strategies and beliefs characterized in proposition A.2 are the only equilibria to satisfy criterion D1.*

Proof. Assume there exists an equilibrium in which $p_2 = \omega_2$ is not played with positive probability for some $\omega_2 \in \{-1, 1\}$. Fix a vector of re-election probabilities \mathbf{r} and probability R that the Voter holds a referendum to change p_1 when ω_1 is not revealed to her, then $U_C(p_1 = \cdot, p_2 = \omega_2, \mathbf{r}, R) > U_N(p_1 = \cdot, p_2 = \omega_2, \mathbf{r}, R)$. Moreover, for any policy vector p such that $p_2 \neq \omega_2$ and any vector of equilibrium re-election and referendum probabilities (\mathbf{r}^*, R^*) we have $U_C(p, \mathbf{r}^*, R^*) \leq U_N(p, \mathbf{r}^*, R^*)$. In other words, in any equilibrium in which $p_2 = \omega_2$ is not played with positive probability the equilibrium payoff of any non-congruent type is at least as high as the equilibrium payoff of any congruent type. It follows that $D^1(N, T, p_1, p_2 = \omega_2) \subset D(C, T, p_1, p_2 = \omega_2)$ and the Voter should believe she is facing a congruent Incumbent upon observing $p_2 = \omega_2$. But then, if $q_1 > 0$ the congruent Incumbent wants to deviate to $p_2 = \omega_2$ as he then receives a payoff of at least $2 + B + q_1$ which is greater than $2 + B$, the highest payoff the congruent type can get in any equilibrium in which he does not play $p_2 = \omega_2$. Hence, to satisfy D1 it must be the case that C chooses $p_2 = \omega_2$ for all ω_2 in equilibrium. □

9.1 Appendix B

In this section of the appendix, I show that the spillover effects identified, namely that the popular referendum also improves congruence with respect to policy dimensions on which no referendum may be placed, is robust to a specification of the model in which the Voter is

uncertain about which policy is in the public interest on the second policy dimension. Hence, I assume from now on, that the Voter observes the state of the world ω_1 but only observes the state of the world ω_2 with probability $q_2 \in [0, 1]$. Note that in the baseline model policy dimensions p_1 and p_2 are essentially identical up to the probability of feedback. Thus, to see what the equilibrium of the baseline model is in this new setting, all one needs to do is to invert p_1 and p_2 . In the model with direct democracy, the equilibrium depends on the probability of feedback q_2 and the value of holding office B (see Figure 3). That is we have the following result:

Proposition A.3. *The following pair of strategies and beliefs constitute the unique equilibrium of the direct democracy model:*

1. *If $q_2 \in [\frac{1}{B+1}, 1]$, then congruent and non-congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all ω , the Voter holds a referendum if and only if $p_1 \neq \omega_1$ and reelects if and only if $p_1 = \omega_1$. The Voter's beliefs satisfy $\mu(I = c | p_1 = \omega_1) = \pi$, $\mu(I = c | p_1 \neq \omega_1) \leq \pi$.*

2. *If $q_2 \in [0, \frac{1}{B+1}]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$,*

The non-congruent Incumbent who observes $\omega_2 = -1$ chooses $(p_1 = \omega_1, p_2 = 1)$;

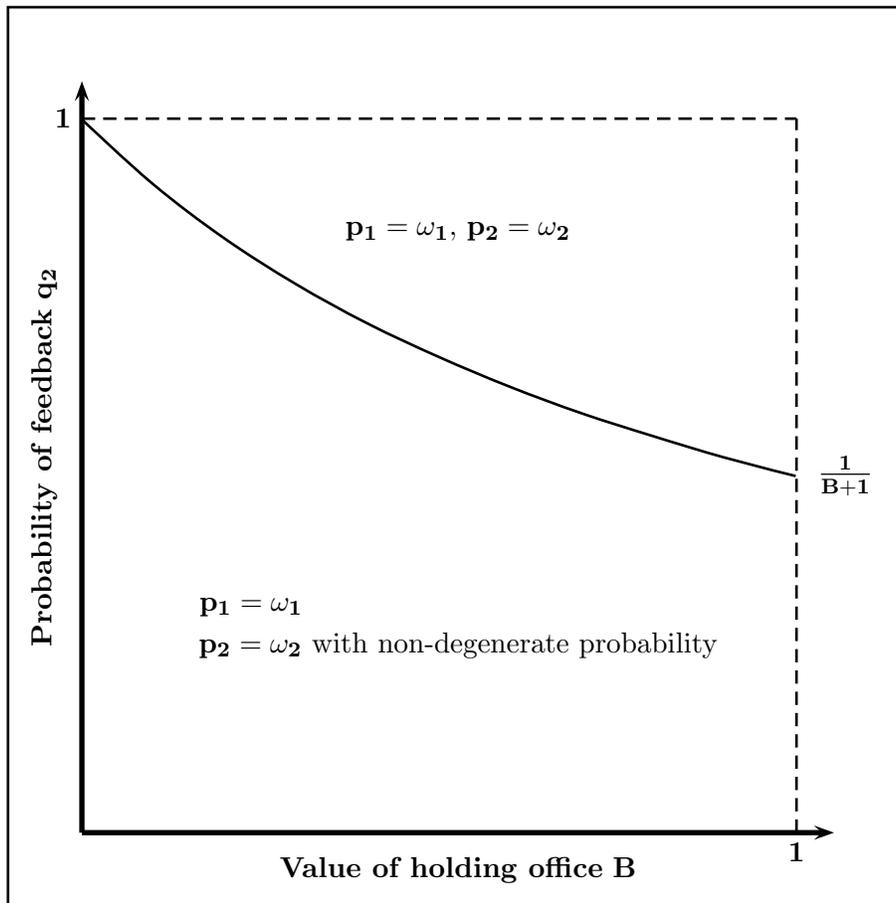
The non-congruent Incumbent who observes $\omega_2 = 1$ chooses $(p_1 = \omega_1, p_2 = -1)$ with probability $\frac{1}{\alpha} - 1$ and $(p_1 = \omega_1, p_2 = 1)$ with probability $2 - \frac{1}{\alpha}$;

The Voter holds a referendum if and only if $p_1 \neq \omega_1$;

The Voter's reelection strategy is: $r^(p_1 = \omega_1, p_2 = \omega_2) = 1$, $r^*(p_1 = \omega_1, p_2 = 1) = \frac{1 - q_2(B+1)}{(1 - q_2)(B+1)} + r^*(p_1 = \omega_1, p_2 = -1)$ and $r^*(p_1 \neq \omega_1) = 0$.*

The Voter's beliefs satisfy $\mu(I = c | (p_1 = \omega_1, p_2 = 1)) = \mu(I = c | (p_1 = \omega_1, p_2 = -1)) = \pi$, $\mu(I = c | (p_1 = \omega_1, p_2 = \omega_2 = 1)) > \pi$, $\mu(I = c | (p_1 = \omega_1, p_2 = \omega_2 = -1)) = 1$, and $\mu(I = c | p_1 \neq \omega_1) \leq \pi$.

Figure 4: Equilibrium Behavior of non-congruent Incumbent in Direct Democracy Model, $q_1 = 1, q_2 \in [0, 1]$



The logic behind this result is as follows. As the Voter knows the state of the world ω_1 , the Voter holds a referendum to set $p_1 = \omega_1$ whenever the Incumbent chooses $p_1 \neq \omega_1$. Hence, whether non-congruent Incumbents choose $p_1 \neq \omega_1$ or $p_1 = \omega_1$, their policy payoff with respect to p_1 is equal to 0 in equilibrium. Choosing $p_1 \neq \omega_1$ then does not yield any policy gains to the non-congruent Incumbent but costs him the reelection, as the Voter then learns his true type. In equilibrium, it is therefore never a best-response for the non-congruent Incumbent to play $p_1 \neq \omega_1$ who thus chooses between $(p_1 = \omega_1, p_2 = \omega_2)$ and $(p_1 = \omega_1, p_2 \neq \omega_2)$.

Choosing $(p_1 = \omega_1, p_2 \neq \omega_2)$ over $(p_1 = \omega_1, p_2 = \omega_2)$ has a gain and a cost: On the one hand the non-congruent Incumbent receives a higher first-period policy payoff, on the other he loses reelection whenever uncertainty about ω_2 is resolved. Note that the magnitude of the cost increases with the probability of feedback q_2 . As the policy gain is equal to 1 and the value of reelection is $B + 1$, the cost of losing the election exceeds the policy gain when the probability of feedback q_2 is high. In such a case, the non-congruent Incumbent is better off mimicking the behavior of congruent Incumbents with respect to all first-period policies. When the probability of feedback q_2 is low, and the non-congruent Incumbent chooses $(p_1 = \omega_1, p_2 \neq \omega_2)$ with sufficiently low probability, the Voter is willing to reelect the Incumbent with positive probability whenever ω_2 is not revealed. In this case, the cost of choosing $(p_1 = \omega_1, p_2 \neq \omega_2)$ over $(p_1 = \omega_1, p_2 = \omega_2)$ is low, and the non-congruent Incumbent implements $(p_1 = \omega_1, p_2 \neq \omega_2)$ with positive probability.

The introduction of the popular referendum also improves congruence in this new setting. In the baseline model, the behavior of non-congruent Incumbents with respect to p_1 depends on the likelihood that the Voter learns the optimality of the policy decision p_2 . If the feedback q_2 is high ($q_2 > \frac{B}{B+1}$), the non-congruent Incumbents never match the policy to the state of the world on dimension 1, i.e. they always choose $p_1 \neq \omega_1$. When feedback is low however ($q_2 \leq \frac{B}{B+1}$), they implement $p_1 = \omega_1$ with non-degenerate probability. Once representative democracy is supplemented by direct democracy however, non-congruent Incumbents always choose $p_1 = \omega_1$. Moreover, as the Voter never holds a referendum in equilibrium, $p_1 = \omega_1$ is now implemented with certainty.

The spillover effects identified also hold. In the baseline model, non-congruent Incumbents never choose $p_2 = \omega_2$, and this independently of the value of q_2 . In the model with direct democracy however, the non-congruent Incumbents always choose $p_2 = \omega_2$ with positive probability and even with certainty, when the probability of feedback q_2 is sufficiently

high ($q_2 \geq \frac{1}{B+1}$). Arguments in sections 5.2 to 5.5 also apply in this case.