Lecture 4
EVOLUTIONARY GAME THEORY
Toulouse School of Economics

Jörgen Weibull

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1 Stochastic social learning

The evolution of conventions in recurrent play of games, in the style of Nash's mass action interpretation

- Modelling paradigm: individuals most of the time play best replies to "recent history of play"

Young P. (1993): "The evolution of conventions", *Econometrica* 61, 57-84.

Hurkens S. (1995): "Learning by forgetful players", *Games and Economic Behavior* 11, 304-329.

Young (1998): *Individual Strategy and Social Structure*, Princeton University Press, 1998.

1.1 Young's model

- Finite normal-form games

- For each player role i: a population of (arbitrary) finite size N_i

- Recurrent play with (uniform) random matching, each time 1 individual from each player population

- Individuals receive random samples of size k from last m rounds of play

- Markov chain where a state $=$ the m most recent *pure-strategy profiles*

- After each match: add the new action profile, delete the oldest

1. The *unperturbed* process: always play some best reply against your sample of past play

2. The *perturbed* process:

   (a) with probability $1 - \varepsilon$: play a best reply against your sample

   (b) with probability ε: play at random, with positive probability for all your pure strategies

3. The perturbed process is *ergodic* and thus has a unique invariant distribution μ^ε

4. Let $\varepsilon \to 0$. Then $\mu^\varepsilon \to \mu^*$. Pure strategy-profiles used in histories in the support of μ^* are called *stochastically stable*

5. μ^* defines a social *convention*, a statistical description of how the game is usually played

- A finite normal-form game has *property NDBR* (non-degenerate best replies) if, for every player $i \in I$ and pure strategy $h \in S_i$, the set

$$B_{ih} = \{x \in \boxdot(S) : h \in \beta_i(x)\}$$

is either empty or has a non-empty (relative) interior. This is a generic property of finite normal-form games

- For each player role i, let $T_i \subset S_i$ and consider the sub-polyhedron $\boxdot(T) = \times_{i \in N} \Delta(T_i)$: $X = \boxdot(T)$ is *closed under rational behavior (CURB)* [Basu and Weibull, 1991] if

$$\beta[\boxdot(T)] \subset T.$$

Theorem 1.1 (Young (1998)) *Let G be a finite game with the NDBR property. If k/m is small and k large, then the unperturbed process converges with probability one to a minimal CURB set. Moreover, μ^* has support on those minimal CURB sets that have minimal stochastic potential. For generic payoffs this singles out a unique minimal CURB set.*

- The *potential* is a concept defined for so-called perturbed Markov chains, essentially captures both the "size" (and "depth") of "basins of attraction", see

  Freidlin M. and A. Wentzell (1984): *Random Perturbations of Dynamical Systems*, Springer.

Example 1.1 (Coordination game)

	L	R
L	2, 2	0, 0
R	0, 0	1, 1

Two minimal curb sets, $\{L\} \times \{L\}$ *and* $\{R\} \times \{R\}$*. Young's model predicts* (L, L)*. This has a "bigger basin of attraction" than* (R, R)*, and hence* (L, L) *is stochastically stable. The mixed NE is unstable.*

- In any symmetric 2×2-coordination game, (L, L) is said to *risk dominate* (R, R) if (L, L) is the best reply to $x = ((1/2, 1/2), (1/2, 1/2))$ [Harsanyi and Selten,1988]

Example 1.2 (Risk dominance) *Consider the following coordination game, in which (R, R) Pareto dominates (L, L), but (L, L) risk dominates (R, R):*

	L	R
L	2, 2	3, 0
R	0, 3	4, 4

This game has the same best-reply correspondence as the preceding example. Hence, Young's model gives the same prediction: (L, L) as in that game.

Example 1.3 *The game with a unique Nash equilibrium, that, moreover, was strict:*

	L	C	R
T	7, 0	2, 5	0, 7
M	5, 2	3, 3	5, 2
B	0, 7	2, 5	7, 0

The unique minimal curb set is $\{M\} \times \{C\}$ *and hence the unique stochastically stable strategy-profile is* (M, C)*.*

- Application to the Nash demand game: Young (1993), "An evolutionary model of bargaining", *Journal of Economic Theory* 59, 145-168.

- The *Nash demand game* (Nash, 1953): A simultaneous-move two-player game, where each player submits a bid, $x_1, x_2 \in [0, 1]$, with payoffs

$$\pi_i(x) = \begin{cases} x_i & \text{if } x_1 + x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Young (1993): Discretize $[0, 1]$ in order to obtain a finite game: $S_1 = S_2 = \{0, 1/n, 2/n, ..., (n-1)/n, 1\}$. Then let $n \to +\infty$. Remarkable and beautiful result, the generalized Nash Bargaining Solution, where the parties' sample sizes, k_1 and k_2, determine their bargaining power!

2 Preference evolution

- "Indirect evolution", initiated by Güth and Yaari (1992): "An evolutionary approach to explain reciprocal behavior in a simple strategic game"

- As if "nature" delegates to individuals to make decisions, but gives them utility functions

- Symmetric two-player games. Random matching of pairs from a large population, treated as a continuum.

- Assume that paired individuals play a Nash equilibrium of the game defined in terms of their utility functions and information

- Utility functions that obtain high average payoffs to their carriers are selected for

Two basic information settings:

1. Preference evolution under complete information ("perfect signals about types")

2. Preference evolution under incomplete information ("no signals")

- Under complete information: one's utility function can serve as a commitment device.
  - Recall initial (verbal) example of Cournot duopoly where managers were given incentive contracts that gave weight to sales, not only profit
  - Utility functions that are not perfectly aligned with payoffs may well be evolutionarily stable (tough bargainers, overconfident competitors etc.)
- Under incomplete information: there no such commitment effect

2.1 Quick glimpses of two models

- Alger, I. and J. Weibull (2010): "Kinship, incentives and evolution," *American Economic Review* 100, 1725-1758.

- Alger, I. and J. Weibull (2012): "Homo moralis - Preference evolution under incomplete information and assortative matching," TSE WP 12-281.

2.1.1 Alger and Weibull (2010): Kinship, incentives and evolution

1. Symmetric two-stage games, stochastic production followed by voluntary *ex-post* transfers

2. Pairs of siblings

3. Biological and/or cultural inheritance from parent generation

4. Type space: family ties defined as degree $\alpha \in (-1, 1)$ of sibling *altruism/spitefulness*:

$$u^{\alpha}(x, y) = \pi(x, y) + \alpha \cdot \pi(y, x)$$

5. Complete information (siblings arguably know each other well...).

6. Each pair plays the unique Nash equilibrium, given their preferences

- Main result: the evolutionarily stable degree of sibling altruism (family ties) depends on the "harshness" of the "production climate": stronger in milder climates (Italy vs. Sweden)

2.1.2 Alger and Weibull (2012): Homo moralis

1. Symmetric two-player games, $\pi : X^2 \rightarrow \mathbb{R}$ continuous, X compact and convex set

2. Type space: *all* continuous functions, $u : X^2 \rightarrow \mathbb{R}$

3. Random matching, but not necessarily uniform. Matching probabilities may depend on types.

4. Incomplete information: paired individuals do not know each other's individual preferences, but behave as if they knew the type distribution in their own matches

Definition 2.1 *In a population state* $s = (\theta, \tau, \varepsilon)$ *with two types* $\theta, \tau \in \Theta$ *in population shares* $1-\varepsilon$ *and* ε*, a strategy pair* $(x^*, y^*) \in X^2$ *is a (***Bayesian) Nash Equilibrium** *if*

$$\begin{cases} x^* \in \arg\max_{x \in X} & \Pr[\theta|\theta, \varepsilon] \cdot u_\theta(x, x^*) + \Pr[\tau|\theta, \varepsilon] \cdot u_\theta(x, y^*) \\ y^* \in \arg\max_{y \in X} & \Pr[\theta|\tau, \varepsilon] \cdot u_\tau(y, x^*) + \Pr[\tau|\tau, \varepsilon] \cdot u_\tau(y, y^*) . \end{cases}$$

Definition 2.2 *A type* $\theta \in \Theta$ *is* **evolutionarily stable against a type** $\tau \in \Theta$ *if there exists an* $\bar{\varepsilon} > 0$ *such that the average payoff to type* θ *is higher than that to type* τ *in all Nash equilibria* (x^*, y^*) *in all population states* $s = (\theta, \tau, \varepsilon)$ *with* $\varepsilon \in (0, \bar{\varepsilon})$.

Main result: under certain regularity conditions, the following preferences emerge as evolutionarily stable:

$$u^{\kappa}(x, y) = (1 - \kappa)\,\pi(x, y) + \kappa \cdot \pi(x, x)$$

for some $\kappa \in [0, 1]$

- Such individuals are torn between two goals:
  - to maximize own payoff
  - to "do the right thing" (cf. Immanuel Kant's categorical imperative)

- We call individuals with such preferences, u^{κ}, *homo moralis*, where $\kappa \in [0, 1]$ is their *degree of morality*

- We prove that $\kappa = \sigma$, the so-called *index of assortativity* of the matching process

  ($\sigma = 0$ under uniform random matching, $\sigma = 1/2$ between siblings if they inherit their preferences from their parents)

THE VERY END

Thanks for your attention, good questions & comments!