Optimal Institutions for Majoritarian Debate

Catherine Hafer (NYU, Politics) and Dimitri Landa (NYU, Politics)

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Abstract

We analyze the informational properties of debates preceding majority voting under a wide class of debate rules in games of persuasion with agents who vary with respect to what arguments they find convincing. We show that there exist institutions that always create incentives for the debaters to conduct a fully revealing debate before voting on the agenda commences. A key condition that guarantees full revelation without delay is that speakers be allowed to restart the debate before every vote on the voting agenda.

1 Introduction

Debates on the choice of a policy or the justifiability of an action are typically envisioned as contests in which each opponent marshals her best, most persuasive arguments in an attempt to sway the audience in favor of her preferred alternative and against the competition. However, with a diverse audience, if arguments are persuasive to some, they are likely to be unpersuasive to others, and the very fact of their potential persuasiveness carries with it the danger that the unpersuaded will turn against the alternative favored by the speaker; after all, they are receiving evidence that the reasons that appear to underlie the appeal of that alternative do not - for them - hold water. This suggests that, contrary to the popular image, making one's best arguments in debate may not always be the best course of action. What should we, then, expect from public debate? Under what institutional and political circumstances will individuals with diverse interests and reasons share their reasons with each other? When will public debate be most informative? The present paper addresses these questions by considering the properties of debates that precede the collective selection of a binding policy choice.

We model persuasion in debate as resulting from the communication of reasons or arguments that resonate (or not) with some relevant subset of the audience. Thus, an anti-abortion debater may argue that the preservation of any form of human life trumps anything else in a conflict of rights. A pro-abortion rights debater may argue that women should have autonomy in dealing with matters concerning their own bodies. Whether one of these arguments resonates for a given listener depends on whether that listener happens to share the speaker's foundational assumptions for it. If she does, then a (coherent) argument will be persuasive, and the speaker may succeed in convincing the listener to see that argument as a reason for her preferred policy choice. If not, then it may become a reason to support a different policy. Interpretively, we can say that for each agent, some of the possible arguments are *active*: she knows whether or not she finds them persuasive, and they determine her perceived optimal choice. Other possible arguments are *latent*: she is initially unsure whether they are persuasive, but finds out when she hears those arguments being made. Persuasion is the effect of hearing such arguments: the "activation" of the corresponding latent reasons, which turns them into active ones.

We show that majoritarian politics creates strong incentives for the participants in debate to make their arguments. In particular, under majority rule with binary sequential agenda, there is always a family of institutions that lead to the making of all potentially consequential informative arguments before the first vote regardless of the debate rule. This family of institutions posits a class of binary sequential agendas and requires that debate be allowed before each vote on the agenda. Under such conditions, all relevant arguments are revealed before the first vote. The possibility of re-starting the debate before subsequent votes is critical to this result even though on the path of play no information is revealed in these later debates. In contrast, when debate is permitted only prior to the first vote on the agenda, no combination of voting agenda and debate rule can guarantee fully revealing debate before the first vote. The relationship between voting agendas, optimal speaking strategies for the debaters and the expected voting outcomes when debates can occur before some but not necessarily all votes on the agenda shows that some of the key intuitions from the complete-information theory of voting do not generalize to the incomplete-information settings with the possibility of debate.

The remainder of the paper is organized as follows. We begin with a somewhat stylized example, which captures some of the key intuition of our results in Section 2. Ins Section 3, we discuss the related literature on deliberation. Section 4, then, introduces the key elements of our model, including the formal definitions of the informational and decision-making environment. In Section 5 we present our results and some illustrative examples. The final section concludes with a brief discussion. The Appendix gathers proofs of the formal results.

1.1 An Informal Example

By way of motivation, consider the following example that abstracts away from a number of substantive and analytical complications, but captures some of the key intuitions behind the main results of the paper. Suppose that a voting body, e.g., U.S. Senate, is considering changing the carried interest tax rate, which is levied on long-term capital gains (current U.S. top rate is 15%). There are two speakers, Libby, who champions left-wing causes and would prefer much higher tax rate and Rich, who is pro-business and prefers the lowest possible rate. These speakers can address the voting body, which must choose by majority rule from the set of three alternative rates: 18%, 25%, and 38%. The sequential binary voting agenda is 18% vs. 38%, and then the winner vs. 25%.

Suppose that members of the voting body do not know their preferences over these alternatives with certainty and would like to learn them by listening to the relevant arguments: e.g., the (un-)desirability of redistribution justified by Rawlsian egalitarianism; (il-)legitimacy of taxation given its effects on individual autonomy; the expectation that government would/would not waste tax revenue; the expectation that raising the tax rate would/would not cause off-shoring; the expectation that raising the tax rate would/would not hurt anti-union private equity (PE) firms, etc. Members' judgments for or against raising the tax rate, then, will turn on whether, on close examination, those arguments resonate with them.[1]

For simplicity, suppose that by this point in the process the only possible new argument is on the effect of raising the tax rate on PE firms; that it is common knowledge what the probability density function over possible preferences is; that *ex ante* 25% is the Condorcet winner; and that a majority currently prefer 38% to 18%.

Does it matter whether Libby and Rich can restart the debate at any stage of agenda (we refer to this institution below as "open-debate voting") or must have it in single round before sequentially voting on the alternatives on the agenda ("single-debate voting")?

Suppose open-debate voting. By assumption, absent any debate, 38% will win in the first vote. Rich knows this, and knows that Libby will make the union argument in the second round, when vote is between 25% and 38%. So, his best move is to make that argument before the first vote, and hope that the voting body turns out to be anti-union. The voters, thus, receive the information before they begin the voting on the agenda.

Suppose now single-debate voting. Libby reasons as follows: absent any argument, 25% is Condorcet winner; I am risk-averse, so I prefer this over a lottery between 18% and 38% (the outcome determined by whether the majority in the room is pro- or anti- union). Rich reasons similarly and they both stay quiet. The voters, thus, must vote on the alternatives on the agenda without the benefit of exposure to this argument.

We will show in a general framework that under open-debate voting, there is always an equilibrium with full information revelation before the first round of votes regardless of the voting agenda, rules of debate, or the distributions of preferences or alternatives; and that this is not the case without open-debate voting.

1.2 Prior Work on Debate and Persuasion

The existing formal literature on deliberation and debate is divided between models of cheap-talk signaling and models of persuasion in which speech can be said to be partially or

[1] An equivalent interpretation may be that the members know their own primitive preferences but their votes are constrained by their expectations of how those arguments may resonate with their constituents who may be the real audience for the debate.

fully verifiable. The latter, which include, e.g., Lipman and Seppi (1995); Lanzi and Mathis (2004); Glazer and Rubinstein (2006); and Patty (2009), are closer to the present model. However, in all those models, the veridicality (truth content) of transmitted messages is the same for all types of receivers - that is, all types of receivers are convinced by the same messages. By contrast, in the model we analyze, the messages have the properties like those in the abortion rights example in the Introduction: their veridicality (truth content) differs across agents, despite those messages being fully verifiable and complete.

The closest models to the one developed in the present paper are Glazer and Rubinstein (2001) and Hafer and Landa (2006 and 2007). The latter analyze related models with deliberation that, like in the present model, satisfies full verifiability and private veridicality. They focus on the welfare properties of the equilibria of a game in which agents choose how to allocate their deliberative resources between receiving (and processing) arguments and making arguments to others and on the aggregate consequences of equilibrium individual choices of deliberative venues, respectively. Glazer and Rubinstein consider the informational properties of debate rules, including the simultaneous speech and one-shot finite sequential speech rules that are covered by our analysis below. Unlike the model in the present paper, their model is one of common veridicality between the speakers and the (single) listener and assumes that speakers are better informed than the listener about what the latter will find persuasive. Although they restrict speakers to making truthful arguments, their setup is closer to the standard cheap-talk model. Austen-Smith (1993) and Krishna and Morgan (2001) analyze the effects of different communication rules within a cheap talk model in which two speakers attempt to influence the actions of a third party. Battaglini (2002) examines a cheap-talk model with multiple senders and a multidimensional state variable.

The analysis of the effects of different voting rules on the informational quality of deliberation has so far focused on cheap-talk signaling in settings that allow for private values. In this literature, Gerardi and Yariv (2006) show that given unrestricted pre-vote communication, all non-unanimity voting rules generate equivalent sets of sequential equilibrium outcomes, but restrictions on communication protocols may lead to different outcomes. Austen-Smith and Feddersen (2006) show that majority rule can do better than unanimity rule in generating informative pre-vote communication and that when unanimity rule generates complete information revelation, the same can be accomplished with other voting rules, including majority rule. Van Wheelden (2008) shows that sequential pre-vote communication reduces incentives for truthful revelation relative to simultaneous messaging. Meirowitz (2006) adopts a mechanism design approach and shows that creation of external incentives can enable the existence of mechanisms generating full information revelation.

2 The Model

2.1 The Informational Framework

We assume that policy choices can be ordered on a left-right ideological spectrum $[0, 1]$ with 0 representing the left and 1 representing the right. Agent i does not know her true ideal policy point $\theta_i \in [0, 1]$ on that spectrum, but, as discussed in the Introduction, there is a relationship between i's ideal point and the arguments that she considers true. In particular, assume that i's true ideal point θ_i probabilistically determines which arguments she will consider to be true. To capture this formally, for each i let $\alpha_i \in \{0, 1\}^n$ indicate which arguments i finds persuasive, where the value of the j^{th} element of that vector, α_i^j, $j = 1, ..., n$ can be interpreted as reflecting whether person i is convinced by a "right-leaning argument" ($\alpha_i^j = 1$) or by "a left-leaning argument" ($\alpha_i^j = 0$) with respect to dimension j, with the prior probability $\Pr(\alpha_i^j = 1) = \theta_i$. Let $p(\theta)$ be the probability density of θ.

Agent i is assumed to make correct inferences about the value of α_i^j upon hearing argument j regardless of whether that argument is right- or left-wing. Thus, in this environment, right- and left-wing arguments are equally informative for any given i.[2] (We comment on

[2]This symmetry may be seen to reflect the conjunction of two assumptions: first, that a right-wing argument can be turned into a left-wing argument by adding "isn't that silly, and therefore, you must support the opposite view" at the end of it (and vice versa), and second, the assumption that there is no bias in the cognitive processing of right- vs. left- wing arguments. Such a bias could be incorporated by assuming that the true policy ideal point is some function of λ_1 and λ_2, where $\Pr(m_i^j = 1|j$ is "right-wing"$) = \lambda_1$, $\Pr(m_i^j = 0|j$ is "right-wing"$) = \lambda_2$, and $\lambda_1 \neq 1 - \lambda_2$ to reflect the fact that, for idiosyncratic

the robustness of our results to the assumption that argument j is more informative for i when i is convinced by it in the Discussion.)

Thus, i can make inferences about her true policy ideal point from arguments that she does not find persuasive (using Bayes' rule), as well as from those that she does; if $\alpha_i^j = 1$ then i's learning dimension j leads to i's greater marginal preference for a right-wing policy, that is it indicates i's posterior belief that θ_i is higher. (Note that, because the value of θ_i is agent-specific and the persuasiveness of each of i's reasons is independently drawn, different members of a society will not necessarily find the same arguments persuasive.)

Although each i is uncertain about θ_i, it is common knowledge that agents are independently drawn from a distribution described by the probability density function $p(\theta)$. Nature further randomly selects and reveals some arguments to i, who may find some of them persuasive and others not. We say that argument j is *active* for i if α_i^j is revealed to i and *latent* if it is not. Let r_i^t be the number of i's active arguments at time t for which $\alpha_i^j = 1$ and l_i^t be the number of active arguments for which $\alpha_i^j = 0$. While the true value of the vector α_i cannot change, i may learn the values of more of its dimensions, and thus r_i^t and/or l_i^t may increase as a result of argumentation.

2.2 The Game Forms

We assume that the population is finite and divided into speakers and listener-voters. Let $\mathcal{R}$ be the set of listeners and $\{0, 1\}$ be the set of speakers. Below, we sometimes refer to speaker 0 as the "left speaker" and to speaker 1 as "the right speaker."

The game form is defined by a combination of a voting rule, a specification of when debate may occur in relation to voting, and a debate protocol. Unless specified otherwise, we consider debate in relation to majoritarian preference aggregation with a finite sequential binary voting agenda and the set of voters $\mathcal{R}$.[3] We consider the set of possible binary voting

reasons not a function of the individual's ideology, i's response to an argument depends not only on its informational content but also on its rhetorical packaging. We do not pursue this possibility in the present paper in order to focus attention on the strategic incentives for speech.

[3]Including speakers in the set of voters complicates the expression of the proofs without altering the substance of the results.

agendas (given a fixed set of alternatives), the possible assignments of debates to votes (i.e. whether or not debate is allowed before voting over the first pair, the second pair, etc.), and a set of possible debate protocols (described in detail below) in which all speakers have an opportunity to speak and may say whatever they wish.

Let $\Pi = [0, 1]$ be the set of alternatives, with finite subset $A \subset \Pi$, $z := |A| - 1$ on the agenda for a collective choice.

The voting agenda is an ordered list of alternatives in A, $a_0, a_1, ..., a_z$ such that the listeners first vote over $\{a_0, a_1\}$, then over $\{w_1, a_2\}$, then over $\{w_2, a_3\}$, and so on, where w_k is the majority rule winner of the k^{th} vote, w_z (the majority choice in the z^{th} vote) is the ultimate collective choice, and $\forall j, k\ a_j \neq a_k$.

Let Ω be a speaker order of length ω such that $\Omega^t \in 2^{\{0,1\}}$ and $\Omega = \{2^{\{0,1\}}\}^\omega$ and such that $\bigcup_{t=1}^{\omega} \Omega^t = \{0, 1\}$. Let K be the set of rules for closing debate and $\sigma \subseteq \{1, ..., z\}$ be a schedule of debates, $\sigma \in 2^{\{1,...,z\}}$. Let T index the vote, with the understanding that debate $T \in \sigma$ is the debate before vote T.

Let $v_i^T \in \{w_{T-1}, a_T\}$ be i's vote at T. Denote with $D_i^{T,t} \subseteq \{1, ..., n\}$ i's speech in time t in debate before vote T; $D^{T,t} = \bigcup_{i \in \Omega^t} D_i^{T,t}$, $D^T = \bigcup_{t=1}^{\infty} D^{T,t}$, $D^\infty = \bigcup_{T=1}^{z} D^T$. (On dimensions $\{1, ..., n\} \backslash D_i^t$, speaker i may be thought to be silent or making content-free speech (e.g., "this is a great country").) Note that if there is no debate before vote T, i.e., $T \notin \sigma$, then $D^T = \varnothing$ by definition.

We assume throughout that all speakers have an opportunity to speak in debate and can send whatever messages they like. After each speech, listeners update their beliefs about their types consistent with Bayes' Rule.

2.3 Preferences and Beliefs

Given agent i's ideal policy θ_i and the collective choice w_z, we assume that i's utility, $\hat{u}_i$, is strictly concave in the distance between the collective choice and θ_i - in particular, $\frac{\partial \hat{u}_i}{\partial(|w_z - \theta_i|)} < 0$ and $\frac{\partial_i^2 \hat{u}}{\partial(|w_z - \theta_i|)^2} < 0$. This guarantees a single-peaked preference profile. Thus, the majority-preferred alternative is determined by the preferences of the listener who is the median voter *given the information available at that point in the game.* Because different voters may respond differently to the same messages, the identity of the median voter may change as new information becomes available.

To keep things simple, we assume that it is common knowledge that $\theta_i \sim U[0,1]$ $\forall i$ and that each speaker $i \in \mathcal{S}$ knows the biases r_j^t, l_j^t $\forall j \in \mathcal{R}$. Assume that $\forall j \in S = \{0,1\}$, it is common knowledge that $\theta_j = j$.[4]

Agent i's complete knowledge of the persuasiveness of arguments does not imply certain knowledge of her preferred policy position θ_i. Accordingly, listener i's beliefs at time t about her own type θ_i can be characterized by the probability density function

$$p(\theta|r_i^t, l_i^t) = \frac{\theta^{r_i^t}(1-\theta)^{l_i^t}}{\int_0^1 \hat{\theta}^{r_i^t}(1-\hat{\theta})^{l_i^t} d\hat{\theta}}, \tag{1}$$

where, as before, r_i^t and l_i^t represent the number of right and left-wing arguments, respectively, known to i at time t. Let $t = 0$ indicate the initial information state. When it is necessary to distinguish between different debates, we use $r_i^{T,t}$ and $l_i^{T,t}$ to represent the numbers of effective right- and left-wing arguments known to i at time t in debate T. Efficient updating upon receiving a message requires agent i to update her beliefs about θ_i using Bayes' Rule. Let $u_i(\pi|r_i^t, l_i^t)$ be i's expected utility from policy π given $p(\theta|r_i^t, l_i^t)$.

3 Analysis

3.1 Optimal Institutions

Our equilibrium concept is Perfect Bayesian Equilibrium in undominated behavioral strategies. In particular, at every point in the game, players' beliefs are derived from their common prior and their active arguments via Bayes Rule; i votes for the alternative $v_i^T \in \{w_{T-1}, a_T\}$

[4] As a matter of interpretation, we suppose that speakers' reasons are active, i.e., that $\mathcal{A}_i = \{1, 2, ..., n\}$ and thus $l_i^0 + m_i^0 = n$ $\forall i \in \mathcal{S}$. The speakers' ability to make potentially persuasive arguments on every issue dimension follows naturally from such a supposition. However, in the technical sense it plays no role in establishing the results that follow and thus is, strictly speaking, unnecessary. Similarly, relaxing the assumption that $i \in \mathcal{S}$ knows θ_j $\forall j \in \mathcal{S}$ has no effect on our substantive results.

at vote T according to strategy that maximizes her expected payoffs given other players' future voting and speaking strategies and expected responses to speech; and i's equilibrium speaking strategy at time t in debate T, $D_i^{T,t*}$, maximizes her expected payoff given other players' speaking strategies $D_j^{T,t*}$, their future voting and speaking strategies and expected responses to speech. The formal definition of the equilibrium conditions is in the Appendix.

Throughout the paper, we restrict our attention to situations in which it is possible for the debate to change the preferences over alternatives (and thus the behavior) of enough voters to produce a policy choice different than the ex ante expected Condorcet winner.[5]

We next introduce a set of definitions to focus our analysis. We say that an argument is *potentially consequential* (at time t in debate j) if there is a positive probability that making it will change the policy outcome relative to the Condorcet winner under the current information state. *Debate j reveals a new argument* if in the course of that debate one of the senders makes an argument that was not made in an earlier debate. *Debate j is fully revealing* if and only if there is no argument that remains to be made that is potentially consequential for the policy outcome. Whether a remaining argument is potentially consequential is not known ex ante but depends on the receivers' responses to the arguments made. The *grand debate is fully revealing* if and only if it reveals all potentially consequential arguments. The *grand debate is fully revealing without delay* if and only if debate 1 is fully revealing. Our key question, then, is whether there exist institutions that generate fully revealing grand debate without delay.

Because we are analyzing institutions in voting games with many voters, it is not surprising that they give rise to multiple equilibria – when voters are indifferent with respect to how to cast their ballots. Consequently, we focus our analysis on the equilibria that generate most revealing grand debate without delay.

We begin with the following theorem, which characterizes what happens when the set of alternatives is binary:

[5]Formally, suppose the median voter prefers y, and let x be the largest alternative less than y and z be the smallest alternative greater than x (so that $x < y < z$). Then this condition amounts to restricting attention to the cases in which either $|\{j \in \mathcal{R} : u_j(z|l_j^0 + |\mathcal{L}_j^0|, m_j^0) \geq u_j(y|l_j^0 + |\mathcal{L}_j^0|, m_j^0)\}| \geq \frac{|\mathcal{R}|}{2}$ or $|\{j \in \mathcal{R} : u_j(x|l_j^0, m_j^0 + |\mathcal{L}_j^0|) \geq u_j(y|l_j^0, m_j^0 + |\mathcal{L}_j^0|)\}| \geq \frac{|\mathcal{R}|}{2}$ (or both).

Theorem 3.1 *Let $|A| = 2$. Then, for all closing rules $k \in K$, and for all Ω s.t. $\bigcup_{t=1}^{\omega} = \{0,1\}$, equilibrium debate is fully revealing.*

Thus, when there are two alternatives, the debate always produces full revelation.

Suppose next that there are more than two alternatives. Say that a game has *single-debate voting* if it allows debate only before the first vote, $\sigma = \{1\}$. The following example shows that the informational content of debate under the single-debate voting depends on the debate rule.

Example 1. Suppose the set of listener-voters $\mathcal{R} = \{1, 2, 3\}$, $n = 2$ possible arguments, and that for all $i \in \mathcal{R}$, $r = l = 0$ initially - that is, the listeners do not know the persuasiveness of any arguments. Suppose that listeners' preferences are described by quadratic loss functions, so that optimal policy choices in expectation are their expected values of θ, given their information. The set of alternatives is $\{\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}\}$. Let the utilities of the left speaker (0) be $\hat{u}_0(\frac{1}{4}) = 1$, $\hat{u}_0(\frac{1}{2}) = \frac{8}{9}$, $\hat{u}_0(\frac{2}{3}) = \frac{2}{3}$, $\hat{u}_0(\frac{3}{4}) = 0$. The ex-ante Condorcet winner is $\frac{1}{2}$.

1a. Suppose single-debate voting with an exogenous termination rule; first Speaker 0 speaks, then Speaker 1 speaks. 1 prefers making arguments on one dimension to making no arguments, conditional on 0 having chosen not to make arguments. To see this, note that if after speech, a majority has $r = 1$, the outcome is $\frac{2}{3}$, but if a majority has $l = 1$, the outcome is still $\frac{1}{2}$. (In the latter case, their ideal points are $\frac{1}{3}$, but $\frac{1}{3}$ is not an alternative and $\frac{1}{2}$ is closer than $\frac{1}{4}$ to $\frac{1}{3}$.) Speaker 1 also prefers arguing on one dimension to arguing on two - by concavity, given that possible ideal points are $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$. Anticipating that 1 will argue on one dimension if 0 says nothing, 0 prefers saying nothing to making arguments on both dimensions:

$$\frac{1}{2}\hat{u}_0(\frac{1}{2}) + \frac{1}{2}\hat{u}_0(\frac{2}{3}) > \frac{10}{64}\hat{u}_0(\frac{1}{4}) + \frac{44}{64}\hat{u}_0(\frac{1}{2}) + \frac{10}{64}\hat{u}_0(\frac{3}{4}).$$

Further, speaking on one dimension makes 0 worse off because if a majority learns $r = 1$, 1 may prefer to respond by speaking on the other dimension (in which case, 0 is in expectation worse off: $\frac{4}{9}\hat{u}_0(\frac{3}{4}) + \frac{5}{9}\hat{u}_0(\frac{1}{2}) < \hat{u}_0(\frac{2}{3})$), whereas if a majority learn $l = 1$, Speaker 1 says nothing. Thus, in equilibrium, 0 prefers not to make arguments and 1 prefers making arguments on one dimension. Thus, there is partial revelation in equilibrium under this debate rule (regardless of the agenda).

1b. Suppose next an endogenous termination rule, again under single-debate voting. Ex ante, both Speaker 0 and Speaker 1 prefer no revelation to revelation on both dimensions. 0 prefers no revelation to revelation on one dimension. 1 prefers revelation on one dimension to revelation on none, but if 1 speaks on one dimension and a majority of voters learn $l = 1$ from her speech, then, given the updated beliefs of the voters, 0 will prefer to speak on the other dimension in response. If, on the other hand, a majority learn $r = 1$ from Speaker 1's speech, then, given the updated beliefs of the voters, 1 subsequently may prefer to make arguments on the other dimension. Thus, 1 anticipates that revelation on one dimension will lead to revelation on the other dimension at least when the outcome is not in his favor, and possibly even when it is. Thus, 1 prefers making no arguments to making arguments on one dimension, and so no arguments are made in equilibrium under an endogenous termination rule.

More generally, we have the following theorem, which describes the properties of single-debate voting:

Theorem 3.2 *Suppose $|A| > 2$ and single-debate voting ($\sigma = \{1\}$). Then:*

(a) There is no debate rule (Ω, k) such that, in equilibrium, the debate is fully revealing without delay for all A and for all agendas on A.

(b) The extent of revelation in equilibrium depends on the debate rule but is independent of the voting agenda.

Note that the ex ante invariant with respect to the choice of voting agenda comes from the fact that, under single-debate voting, all debate must occur before voting begins. This means that the identity of the Condorcet winner is fixed for the entire sequence of votes, and as a result, the sequence of the votes is irrelevant for the outcome.

Can we systematically obtain full revelation without delay and, hence, implement fully informed voting under a positively responsive voting rule? Our next result provides an affirmative answer for any $|A| > 2$. Say that the institutional environment that schedules a debate before each pairwise vote has *open-debate voting*, $\sigma = \{1, 2, ..., z\}$. We have the following:

Theorem 3.3 *Let $|A| > 2$. Then for any debate rule (Ω, k), any set of alternatives A and any binary sequential agenda on A, there exists an equilibrium in which debate is fully revealing without delay if and only if there is open-debate voting ($\sigma = \{1, 2, ..., z\}$).*

This theorem says that in games with open-debate voting there is full revelation in the first debate - that is, even though those games have open-debate voting, the debates following the first vote on the agenda do not reveal new arguments relevant to the policy choice. In effect, debates before the subsequent votes need never take place, so long as they are permitted - that is, so long as any speaker can insist on having them.

The intuition for the sufficiency part of the theorem is as follows. There is always full revelation in last debate (if not before). In previous debate, either *ex ante* the median strictly prefers that one alternative advance rather than the other, in which case some speaker prefers revelation now to revelation later; or *ex ante* the median voter is indifferent, and can credibly commit, in the absence of new information, to voting in such a way that some speaker prefers revelation now. By induction, this argument can be used to construct an equilibrium with full revelation in the first debate. To see the intuition for necessity, let the last two alternatives on the agenda be far left and far right alternatives, and let the other alternatives be very moderate (close to the median ex ante). Suppose that there is no debate scheduled before the last vote. At the time of the last debate, the three remaining alternatives are the far-left and the far-right alternatives, and a moderate alternative preferred by the median voter ex ante. Thus, in the last debate, both speakers prefer no revelation. Suppose, instead, there is a debate scheduled before the last vote, but there is no debate scheduled before some earlier vote on the binary agenda, so that there is some t s.t. there is no debate between the vote over $\{w_{t-2}, a_{t-1}\}$ and the vote over

$\{w_{t-1}, a_t\}$, where w_{t-1} is the winner of the vote over $\{w_{t-2}, a_{t-1}\}$. If a_t is sufficiently left and a_{t-1} sufficiently right, and $a_0, ..., a_{t-2}$ sufficiently moderate, then there is no revelation until the debate after the vote over $\{w_{t-1}, a_t\}$.

We add two further remarks. First, the agenda-independence of the existence of equilibrium with full revelation without delay under open-debate voting suggests affinities between this theorem and the voting rule-independence results in Gerardi and Yariv (2006). The logic of the those results is, however, fundamentally different. The key observation in Gerardi and Yariv's construction is that voters have no incentive to deviate from a voting profile in which they are not pivotal, and that makes it possible to construct voting profiles for different voting rules that make those voting rules essentially equivalent in the communication stage preceding voting. The argument that establishes Theorem 3.3 turns entirely on the speakers' incentives given the anticipated behavior by the other speaker and strategic voting by the voters who are making weakly undominated choices all the way through the voting agenda.

Second, the possibility of choosing the debate rules to improve delay-free revelation under single-debate voting naturally raises the question of the welfare comparison of equilibria with greatest extent of information-revelation without delay under single- vs. open-debate voting. We address with with the following remark:

Remark 3.1 *Holding fixed agents' preferences and the set of alternatives, the most informative equilibrium grand debate that can be induced by any agenda and debate rule under single-debate voting is always weakly and sometimes strictly less informative than the most informative equilibrium grand debate that can be induced by any agenda and debate rule under open-debate voting.*

To establish this remark, it suffices to show that regardless of the debate rule, for some set of admissible primitives, single-debate voting may not generate a fully revealing debate. This can be done trivially. Suppose $A = \{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}\}$, $r^0 = l^0 = 0$, and $n = 1$. For $\sigma = \{1\}$, both speakers prefer $D = \emptyset$, in which case $w_2 = \frac{1}{2}$ with certainty for all (Ω, k). By Theorem

3.3, for $\sigma = \{1, 2\}$, for any (Ω, k), for any agenda, there is an equilibrium in which debate is fully revealing without delay, yielding $\Pr(w_2 = \frac{1}{3}) = \Pr(w_2 = \frac{2}{3}) = \frac{1}{2}$.

3.2 The world in-between

Single-debate and open-debate voting anchor two ends of the spectrum of debate-scheduling institutions. In the middle are institutions that provide for debate before multiple but not all votes on the agenda. Recall that under single-debate voting, the informativeness of debate depends on the debate rules but not on the voting agenda, and under open-debate voting, full revelation without delay can be implemented regardless of either. For debate-scheduling institutions in-between, we can have both debate rule and voting agenda dependence.

Consider the following example:

Example 2. Let $\mathcal{R} = \{1, 2, 3\}$ with $(r_1^0, l_1^0) = (0, 3)$, $(r_2^0, l_2^0) = (0, 0)$, and $(r_3^0, l_3^0) = (3, 0)$. Suppose that there are 3 possible arguments ($n = 3$). Suppose that $\sigma = \{1, 2\}$, i.e., there are debates before the first two votes but not before the last one, and that the debate rule has speaker 0 speaking first, followed by speaker 1, and the debate closing after that ($\Omega = (\{0\}, \{1\})$, and $k =$ exogenous closing). Let $A = \{\frac{1}{7}, \frac{1}{2}, \frac{4}{7}, \frac{6}{7}\}$ and assume that voters have quadratic preferences.

Consider the agenda $a_0 = \frac{1}{2}$, $a_1 = \frac{4}{7}$, $a_2 = \frac{1}{7}$, $a_3 = \frac{6}{7}$. Suppose $D^1 = \emptyset$. If $w_1 = \frac{1}{2}$, then $D_0^1 = D_1^2 = \emptyset$ leads to $w_3 = \frac{1}{2}$, which both speakers prefer to the lotteries over $\{\frac{1}{6}, \frac{1}{2}, \frac{1}{7}\}$ that follow from $D^2 \neq \emptyset$. If instead $w_1 = \frac{4}{7}$, then speaker 1 most prefers $D^2 = \emptyset$ but speaker 0 most prefers $|D^2| = 1$. (If $r^2 = 0$, $l^2 = 1$, then the pivotal voter prefers $\frac{4}{7}$, as she would if $r^2 = l^2 = 0$.) However, if D_0^2 s.t. $|D_0^2| = 1$, then regardless of the effects of this information on the pivotal voter's preferences, speaker 1's best response is to reveal further information. (If $r^2 = 0$, $l^2 = 1$ then speaker 1 prefers full revelation and if $r^1 = 1, l^2 = 0$ then speaker 1 prefers to reveal one additional argument in $T = 2$.) Anticipating speaker 1's responses, speaker 0's best response is $D_0^2 = \emptyset$; hence if $w_1 = \frac{4}{7}$, $D_0^2 = D_1^2 = \emptyset$ and $w_3 = w_1 = \frac{4}{7}$. Because the pivotal voter prefers ex ante $w_3 = \frac{1}{2}$ to $w_3 = \frac{4}{7}$, $D^1 = \emptyset$ and so $w_1 = \frac{1}{2}$. Observe that both speakers prefer $D^1 = D^2 = \emptyset$ and $w_3 = \frac{1}{2}$ to the lotteries that correspond to the revelation of 2 or 3 arguments in $T = 1$. It remains to consider $|D^1| = 1$: if $r^1 = 0$, $l^1 = 1$, then $w_1 = \frac{1}{2}$ and speaker 0 prefers additional revelation in $T = 2$. If $r^1 = 1, l^1 = 0$, then $w_1 = \frac{4}{7}$ and speaker 1 prefers additional revelation in $T = 2$. Thus D^1 s.t. $|D^1| = 1$ necessarily leads to more revelation and so ex ante both speakers prefer $D^1 = D^2 = \emptyset$ and $w_3 = \frac{1}{2}$.

Consider now the agenda $a_0 = \frac{1}{7}$, $a_1 = \frac{4}{7}$, $a_2 = \frac{1}{2}$, $a_3 = \frac{6}{7}$. To see that the ex ante equilibrium outcome is different, it is sufficient to show that the first debate is revealing in equilibrium; if information is revealed, then the outcome cannot be a degenerate distribution. Suppose $D^1 = \emptyset$. If $w_1 = \frac{1}{7}$, then $D^1 = D^2 = \emptyset$ leads to $w_3 = \frac{1}{2}$, which both speakers prefer to the lotteries over $\{\frac{1}{7}, \frac{1}{2}, \frac{6}{7}\}$ that correspond to possible $D^2 \neq \emptyset$. If $w_1 = \frac{4}{7}$, and if $D_0^2 = \emptyset$, then speaker 1 prefers D_1^2 s.t. $|D_1^2| = 2$. Anticipating this, speaker 0's best response is $D_0^2 = \{1, 2, 3\}$. Anticipating the speaker's behavior in $T = 2$, voters 2 and 3 prefers $w_1 = \frac{4}{7}$ to $w = \frac{1}{7}$ given $D^1 = \emptyset$. (In the latter case, they obtain $w_3 = \frac{1}{2}$ regardless of α_2; whereas if $w_1 = \frac{4}{7}$, voter 2 learns some part of α_2 and with positive probability prefers either $w_3 = \frac{4}{7}$ or $w_3 = \frac{6}{7}$, making both voters 2 and 3 better off relative to $w_3 = \frac{1}{2}$. Because $D^1 = \emptyset$ leads to $w_1 = \frac{4}{7}$ and $D^2 = \{1, 2, 3\}$, speaker 0 prefers any $D^1 \neq \emptyset$ to $D^1 = \emptyset$. Thus, $D^1 = \emptyset$ cannot occur in equilibrium.[6]

The broader intuition behind this example can be put this way. By putting the relatively extreme alternatives together on the agenda and not permitting debate between the elimination of one and the elimination of the other, the first agenda bundles together both "good" and "bad" consequences of speech for the speakers. In this case, the risk of the "bad" outcome out-weighs the hope of the good one for each speaker, and so neither of them prefers to speak before the "bad" outcome is eliminated from consideration, and because there was no debate between the relevant votes, neither speaker has an opportunity to speak afterward. In the second agenda, one of the relatively extreme alternatives ($\frac{1}{7}$) would be eliminated in the 1st vote unless the first debate were revealing, and that alternative were eliminated, the opposite speaker could only benefit from speaking subsequently. Thus, the

[6] In fact, in equilibrium, under this agenda, debate is fully revelaing without delay and the outcome is $\Pr(w_3 = \frac{1}{7}) = \Pr(w_3 = \frac{6}{7}) = \frac{1}{8}$ and $\Pr(w_3 = \frac{1}{2}) = \Pr(w_3 = \frac{4}{7}) = \frac{3}{8}$.

speaker who prefers $\frac{1}{7}$ faces no effective loss from revealing information in the first debate in an effort to keep $\frac{1}{7}$ in contention.[7]

Example 2 brings out a more general point. To formulate it, we start with the following definition:

Definition 3.1 *The ex ante equilibrium outcome is the probability distribution η over A such that for all $x \in A$, $\eta(x; a, \Omega, k, \sigma)$ is the probability that $w_z = x$, given agenda a, debate rule (Ω, k), and schedule of debates σ.*

The following result follows by the argument above:

Remark 3.2 *Let $\hat{\sigma} \in 2^{\{1,\ldots,z\}} \backslash \{\{1\}, \{1, ..., z\}\}$. Then the ex ante equilibrium outcome is not invariant with respect to voting agenda.*

Aside from underscoring the relevance of debate scheduling for ultimate outcomes, this result has another interesting implication. A key result of the classic complete-information voting theory is that voting outcomes are agenda-invariant if the core of the majority rule is non-empty. In our model, the voters' preferences are single-peaked, yielding a well-defined ex ante majority outcome. The remark above shows that what might be thought of as an incomplete-information analogue of the agenda-invariance result does not hold. The reason is that different agendas give rise to different incentives to reveal information driven by the possibilities of different alternatives being removed from the agenda as a consequence of voting in distinct information states; given that we can associate different lotteries over outcomes with different speaking choices, we can, from the ex ante perspective associate different lotteries over outcomes with different voting agendas.

4 Discussion

This paper analyzes the possibility of full information revelation in debate preceding voting. Our main result shows that there exist institutions that always generate full revelation

[7]We need to add another piece of the argument showing that debate rules matter in the setting of the Example 2.

without delay, regardless of the profiles of prior beliefs on the part of the receivers and the nature of the sets of alternatives.

Our key results suggest that institutional details of the decision-making process, such as the possibility of starting a debate before each vote (that is, single- vs. open-debate voting), the choice of a debate rule, and the voting agenda (the sequence in which the alternatives are voted on) all affect the expected outcome of collective decision-making following debate. However, they do so in relatively subtle ways that point to the importance of considering interactions between them, as we have attempted to do in our analysis. We show that the very possibility of starting a debate before each vote - without that possibility necessarily realized on the path of play - matters for the equilibrium informational properties of debate. Theorems 3.1 and 3.3 suggest that under the open-debate voting, majority rule is highly conducive to eliciting fully informative debates without delay. Under single-debate voting, however, the relevant institutional lever is the debate rules, and their maximal effectiveness cannot be as high as that of open-debate voting.

It is worth noting that our results are robust to relaxing the assumptions on agency maintained above to allow for inefficient updating following an "unconvincing" argument. Following such argument, a Bayesian receiver updates away from the policy position that is supported by that argument or by the possibility that she might be convinced by it. Experimental evidence reported in Dickson, Hafer, and Landa (2008) suggests that subjects may fail to do so in the deliberation games set in a laboratory environment: their prior policy positions are "sticky" or subject to a version of confirmatory bias. In the presence of a single debater in the present environment, such inefficient updating would alter the speaker's strategy because the downside of making an argument is decreasing in the proportion of receivers who update that way. With multiple speakers, however, that effect is neutralized: if a given speaker chooses to make an argument on a given dimension, or is expected to do so, then a speaker with an opposite preference will have a dominant strategy to speak on that dimension, since doing so could only bring on board the inefficient updaters who were unconvinced by the competition.

Finally, while our analysis focused on majoritarian decision-making, it also suggests the value of a more focused comparison across common voting rules, which we have left to future work.

5 Appendix

5.1 Formal Definition of the Equilibrium

The equilibrium requires that at every point in the game, players' beliefs are derived from their common prior and their active arguments via Bayes Rule; i votes for the alternative $v_i^T \in \{w_{T-1}, a_T\}$ at vote T according to strategy

$$v_i^{T*} \in \arg \max_{\{w_{T-1},a_T\}} {}_0^1 p(\hat{\theta}|r_i^{T,\infty}, l_i^{T,\infty})\{\Pr(w_z = v_i^T|\cdot)\hat{u}_i(v_i^T, \hat{\theta}) + \sum\nolimits_{g=T+1}^{z} [\Pr(w_z = a_g|\cdot)\hat{u}_i(a_g, \hat{\theta})]\} d\hat{\theta},$$

where $\Pr(w_z = v_i^T|\cdot)$ and $\Pr(w_z = a_g|\cdot)$ are conditioned on

$$\theta_i = \hat{\theta}, \{(r_j^{T,\infty}, l_j^{T,\infty})\}_{j\in\mathcal{R}}, \bigcup_{j=1}^{T} D^{i,\infty}, \{\{v_j^{k*}(\cdot)\}_{k=T+1}^{z}\}_{j\in\mathcal{R}}, \{\{\{D_h^{j,k*}(\cdot)\}_{k=1}^{\infty}\}_{j=T+1}^{z}\}_{h\in\mathcal{S}}),$$

and $v_j^{k*}(\cdot)$ is conditioned on

$$w_{k-1}, \{a_h\}_{h=k}^{z}, \{(r_h^{k,\infty}, l_h^{k,\infty})\}_{h\in R}, \bigcup_{h=1}^{k} D^{h,\infty}.$$

The equilibrium speaking strategy at time t in debate T is

$$D_i^{T,t*} \in \arg \max_{2^{\{1,\ldots,n\}}} [\Pr(w_z = w_{T-1}|\cdot)u_i(w_{T-1}, \theta_i) +_{h=T}^{z} \Pr(w_z = a_h|\cdot)u_i(a_h, \theta_i)],$$

where $\Pr(w_z = w_{T-1}|\cdot)$ and $\Pr(w_z = a_h|\cdot)$ are conditioned on

$$\{(r_j^{T,t-1}, l_j^{T,t-1})\}_{j\in\mathcal{R}}, \bigcup_{j=1}^{T-1} D^{j,\infty}, \bigcup_{j=1}^{t-1} D^{T,j}, D_i^{T,t}, D_{-i}^{T,t*}, \{D^{T,j*}\}_{j=t+1}^{\infty}, \{D^{j,\infty*}\}_{j=T+1}^{z}, \{\{v_j^{k*}(\cdot)\}_{k=T}^{z}\}_{j\in\mathcal{R}}).$$

5.2 Proofs

Theorem 3.1

Proof. Suppose, without loss of generality, that $a_0 < a_1$. Let δ_1^t be the smallest number of arguments that can change the outcome from a_0 to a_1 given the history of debate and the voters' active arguments at t: δ_1^t is the smallest integer s.t.

$$\begin{aligned}|\{i \quad &\in \quad \mathcal{R} : u_1(a_1|r_i^{t+1} = \min\{n - l_1^t, r_i^t + \delta_1^t\}, l_i^{t+1} = l_i^t) \geq u_i(a_0|r_i^{t+1} = \min\{n - l_1^t, r_i^t + \delta_1^t\}, l_i^{t+1} = l_i^t)\}| \\ &> \quad |\{i \in \mathcal{R} : u_1(a_1|r_i^{t+1} = \min\{n - l_1^t, r_i^t + \delta_1^t\}, l_i^{t+1} = l_i^t) < u_i(a_0|r_i^{t+1} = \min\{n - l_1^t, r_i^t + \delta_1^t\}, l_i^{t+1} = l_i\end{aligned}$$

Likewise, δ_0^t is the smallest integer s.t.

$$\begin{aligned}|\{i \quad &\in \quad \mathcal{R} : u_1(a_0|r_i^{t+1} = r_i^t,\ l_i^{t+1} = \min\{n - r_1^t, l_i^t + \delta_0^t\}) \geq u_i(a_1|r_i^{t+1} = r_i^t,\ l_i^{t+1} = \min\{n - r_1^t, l_i^t + \delta_0^t\})\}| \\ &> \quad |\{i \in \mathcal{R} : u_1(a_0|r_i^{t+1} = r_i^t,\ l_i^{t+1} = \min\{n - r_1^t, l_i^t + \delta_0^t\}) < u_i(a_1|r_i^{t+1} = r_i^t,\ l_i^{t+1} = \min\{n - r_1^t, l_i^t + \delta_0\end{aligned}$$

Note that $\delta_1^t = 0$ iff a_1 is majority preferred to a_0 at beginning of time t, and $\delta_0^t = 0$ iff a_0 is majority preferred to a_1 at beginning of time t.

Case 1: $k =$ endogenous closing rule. Suppose $\delta_0^t = 0$. Then if $n - |\bigcup_{k=1}^{t-1} D^k| \geq \delta_1^t$, speaker 1 prefers any D_1^t s.t. $|D_1^t \setminus \bigcup_{k=1}^{t-1} D^k| \geq \delta_1^t$ to any D_1^t s.t. $|D_1^t \setminus \bigcup_{k=1}^{t-1} D^k| < \delta_1^t$. Suppose $\delta_1^t = 0$. Then if $n - |\bigcup_{k=1}^{t-1} D^k| \geq \delta_q^t$, speaker 0 prefers any D_0^t s.t. $|D_0^t \setminus \bigcup_{k=1}^{t-1} D^k| \geq \delta_0^t$ to any D_0^t s.t. $|D_0^t \setminus \bigcup_{k=1}^{t-1} D^k| < \delta_0^t$. Thus, in equilibrium, $D = \bigcup_{t=1}^{\infty} D^t$ and $\{(r_i^\infty, l_i^\infty)\}_{i \in \mathcal{R}}$ must be s.t. one of the following conditions is true:

(1) $\delta_0^\infty = 0$ and $\delta_1^\infty > n - |D|$;

(2) $\delta_1^\infty = 0$ and $\delta_0^\infty > n - |D|$;

(3) $|D| = n$.

Any one of these three implies full revelation.

Case 2: $k =$ exogenous closing rule. If $\bigcup_{k=1}^{\omega} D^k$ and $\{(r_i^{\omega+1}, l_i^{\omega+1})\}_{i \in \mathcal{R}}$ meet conditions (1), (2), or (3), then full revelation has been achieved. Suppose none of these conditions are met. There are three possibilities:

(a) Suppose $\Omega^\omega = \{0\}$ and let $\omega' < \omega$ be the largest t s.t. $1 \in \Omega^t$. Suppose $\delta_0^{\omega'+1} = 0$. Then $\{D_0^t\}_{t=\omega'+1}^{\omega}$ s.t. $\bigcup_{t=\omega'+1}^{\omega} D_0^t \subseteq \bigcup_{t=1}^{\omega'} D^t$ and $w = a_0$. Suppose instead $\delta_1^{\omega'+1} = 0$. Then $\{D_0^t\}_{t=\omega'+1}^{\omega}$ s.t. $\forall \hat{t} = \omega'+1, ..., \omega$ s.t. $\delta_0^{\hat{t}} = 0$, $D_0^{\hat{t}} \subseteq \bigcup_{t=1}^{\hat{t}-1} D^t$, and $\forall \hat{t} = \omega'+1, ..., \omega$ s.t. $\delta_0^{\hat{t}} \neq 0$, $n - |\bigcup_{t=1}^{\hat{t}-1} D^t| \geq \delta_0^{\hat{t}}$, $D_0^{\hat{t}}$ s.t. $|D_0^{\hat{t}} \backslash \bigcup_{t=1}^{\hat{t}-1} D^t| \geq \delta_0^{\hat{t}}$. Anticipating 0's behavior, 1 prefers $D_1^{\omega'} \supseteq \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega'-1} D^t$ to any $D_1^{\omega'} \not\supseteq \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega'} D^t$, achieving full revelation.

(b) Suppose $\Omega^\omega = \{1\}$. The argument is symmetric to that for $\Omega^\omega = \{0\}$ in (b) above.

(c) Suppose $\Omega^\omega = \{0, 1\}$. Suppose $\delta_0^\omega = 0$. Then 1 prefers $D^{\omega *} = D_0^\omega \cup D_1^\omega$ s.t.

$$\max_{D^\omega \subseteq \{1,...,n\}} \Pr\left(|\sum i \in \mathcal{R} : u_i(a_1|r_i^\omega, l_i^\omega) \geq u_i(a_0|r_i^\omega, l_i^\omega)\}| \geq |\sum i \in \mathcal{R} : u_i(a_0|r_i^\omega, l_i^\omega) \geq u_i(a_1|r_i^\omega, l_i^\omega)\}| \right).$$

If $D^{\omega *} \supseteq \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega-1} D^t$, then $D_1^\omega = D^{\omega *}$ and full revelation is achieved. If $D^{\omega *} \subset \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega-1} D^t$, then 0 prefers some $D_0^\omega \supset D^{\omega *}$ to any D_0^ω s.t. $D^{\omega *}$ is not a proper subset of D_0^ω. Either $D_0^\omega \supseteq \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega-1} D^t$, in which case full revelation is achieved, or else there is some $\hat{D}_1^\omega \supset D_0^\omega$ s.t. 1 prefers $D^\omega = \hat{D}_1^\omega$ to $D^\omega = D_0^\omega$. By induction, $D^\omega \supseteq \{1, ..., n\} \backslash \bigcup_{t=1}^{\omega-1} D^t$.

If $\delta_1^\omega = 0$ instead, then the result follows by symmetry. ■

Theorem 3.2

Proof. Consider $\mathcal{R} = \{1, 2, 3\}$, $n = 2$, $r_i^0 = l_i^0 = 0$ for all $i \in \mathcal{R}$, $A = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$, and $u_0(\frac{1}{4}) = u_1(\frac{3}{4}) = 1$, $u_0(\frac{1}{2}) = u_1(\frac{1}{2}) = \frac{8}{9}$, and $u_0(\frac{3}{4}) = u_1(\frac{1}{4}) = 0$. Note that if $|D^\infty| \leq 1$, $w = \frac{1}{2}$. If $D^\infty = \{1, 2\}$,

$$\Pr(w = \frac{1}{4}|D^\infty = \{1, 2\} = \Pr(w = \frac{3}{4}|D^\infty = \{1, 2\}) = 1 - p,$$

where $p = \Pr(w = \frac{1}{2}|D^\infty = \{1, 2\})$. Thus, for all $i \in S$, $E[u_i(w)|D^\infty \neq \{1, 2\}] = \frac{8}{9}$ and

$$E[u_i(w)|D^\infty = \{1, 2\}] = \frac{8}{9}p + \frac{1}{2}(1 + 0)(1 - p);$$

therefore for all p,

$$E[u_i(w)|D^\infty \neq \{1,2\}] > E[u_i(w)|D^\infty = \{1,2\}]$$

for all $i \in S$. Thus, ex ante, both speakers prefer no revelation or partial revelation to full revelation. This implies that $D_i = \emptyset\ \forall i \in S$ for $\omega = 1$, $\Omega^\omega = \{0,1\}$.

Note next that if $|\bigcup_{k=1}^{t} D^k| = 1$, then 0 prefers $D^\infty = \{1,2\}$ to $D^\infty = \bigcup_{k=1}^{t} D^k$ if $|\{i \in \mathcal{R} : r_i^t = 0,\ l_i^t = 1\}| \geq 2$ and 1 prefers $D^\infty = \{1,2\}$ to $D^\infty = \bigcup_{k=1}^{t} D^k$ if $|\{i \in \mathcal{R} : r_i^t = 1,\ l_i^t = 0\}| \geq 2$. Thus, at time t, if $\bigcup_{k=1}^{t-1} D^k = \emptyset$, and if $\forall i \in S$, $\exists t' > t$ s.t. $i \in \Omega^t$, then each speaker anticipates that D^t s.t. $|D^t| = 1$ will result in $D^\infty = \{1,2\}$. Given that both speakers prefer no revelation to full revelation, $D_i^t = \emptyset\ \forall i \in \Omega^t$ at such t. This implies that with endogenous termination, $D_i^t = \emptyset\ \forall i \in S$, $\forall t$ on the path of play, regardless of Ω.

If, however, $\bigcup_{k=1}^{t-1} D^k = \emptyset$ and $\exists i \in \Omega^t$ s.t. $\forall t' > t\ \Omega^{t'} = \{i\}$ (i.e., $\nexists t' > t$ s.t. $\{1,2\}\backslash\{i\} \subseteq \Omega^{t'}$), then for some such $t < \omega$, $|D_i^t| = 1$. After observing $\{(r_i^t, l_i^t)\}_{i\in\mathcal{R}}$, i chooses $D_i^{t'} \supseteq \{1,2\}\backslash D_i^t$ if a majority of receivers moved closer to him and $D_i^{t'} \subseteq D_i^t$ if they did not, for t' s.t. $t < t' \leq \omega$. Anticipating i's behavior in t and t', $j \in S\backslash\{i\}$ prefers $D_j^{\hat{t}} = \{1,2\}$ at some $\hat{t} < t$ s.t. $j \in \Omega^{\hat{t}}$.

The contrast in equilibrium path of play between this case and the previous one establishes debate-rule dependence. Because under single-debate voting, all debate must occur before voting begins, the identity of the Condorcet winner is fixed for the entire sequence of votes. As a result, the sequence of the votes is irrelevant for the outcome. This establishes part (b) of the theorem.

Returning to part (a), it remains to show that for some other set of alternatives, an exogenous closing rule with Ω s.t. $\Omega^{\omega-1} = \Omega^\omega = \{1\}$ will not generate full revelation on the equilibrium path of play. Consider that debate rule with the set of alternatives $A = \{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}\}$. Suppose the receivers, their information, and the speaker preferences are as in the example above, with $u_1(\frac{1}{3}) = \frac{2}{3}$. Solving by backward induction, it is still true that

$D_1^\omega \supseteq \{1,2\}\backslash \bigcup_{t=1}^{\omega-1} D^t$ if $|\bigcup_{t=1}^{\omega-1} D^t| = 1$ and $|\{i \in \mathcal{R} : r_i^{\omega-1} = 1,\ l_i^{\omega-1} = 0\}| \geq 2$; and $D_1^\omega = \emptyset$ if $\bigcup_{t=1}^{\omega-1} D^t = \emptyset$. It is also true that $D_1^\omega \subseteq \bigcup_{t=1}^{\omega-1} D^t$ if $|\bigcup_{t=1}^{\omega-1} D^t| = 1$ and $|\{i \in \mathcal{R} : r_i^{\omega-1} = 1,$ $l_i^{\omega-1} = 0\}| < 2$. Note that if $|\bigcup_{t=1}^{\omega-1} D^t| = 1$, then for all $i \in \mathcal{R}$ either $(r_i^{\omega-1}, l_i^{\omega-1}) = (1,0)$ or $(r_i^{\omega-1}, l_i^{\omega-1}) = (0,1)$, and suppose that $|\{i \in \mathcal{R} : r_i^{\omega-1} = 1,\ l_i^{\omega-1} = 0\}| = 1$ (the more favorable possibility for speaker 1). If $D_1^\omega \subseteq \bigcup_{t=1}^{\omega-1} D^t$, then $w = \frac{1}{3}$ and $u_1(\frac{1}{3}) = \frac{2}{3}$. If $D_1^\omega \supseteq \{1,2\}\backslash \bigcup_{t=1}^{\omega-1} D^t$, then

$$\Pr\left(w = \frac{1}{4}|\{(r_i^{\omega-1}, l_i^{\omega-1})\}_{i\in\mathcal{R}}, D_1^\omega, \bigcup_{t=1}^{\omega-1} D^t\right) = \left(\frac{2}{3}\right)^2$$

and

$$\Pr\left(w = \frac{1}{2}|\{(r_i^{\omega-1}, l_i^{\omega-1})\}_{i\in\mathcal{R}}, D_1^\omega, \bigcup_{t=1}^{\omega-1} D^t\right) = 1 - \left(\frac{2}{3}\right)^2$$

and $E[u_i(w)|\cdot] = \left(\frac{2}{3}\right)^2(0) + \left(1 - \left(\frac{2}{3}\right)^2\right)\left(\frac{8}{9}\right) = \frac{40}{81} < u_1\left(\frac{1}{3}\right)$.

Now consider $t = \omega - 1$. If $\bigcup_{t=1}^{\omega-2} D^t = \emptyset$, then $D_1^{\omega-1} = D_1^\omega = \emptyset$ yields $w = \frac{1}{2}$ and $u_1(\frac{1}{2}) = \frac{8}{9}$, whereas the strategy $D_1^{\omega-1}$ s.t. $|D_1^{\omega-1}| = 1$, followed by the best response D_1^ω, yields $\frac{1}{2}u_1(\frac{1}{3}) + \frac{1}{2}((\frac{2}{3})^2 u_1(\frac{3}{4}) + (1-(\frac{2}{3})^2)u_i(\frac{1}{2})) = \frac{65}{81} < u_1(\frac{1}{2})$. Thus, if $\bigcup_{t=1}^{\omega-2} D^t = \emptyset$, then $D_1^{\omega-1} = D_1^\omega = \emptyset$. Anticipating 1's best responses in $t = \omega - 1$, ω, and given that

$$u_0(\frac{1}{2}) = \frac{8}{9} > \frac{121}{162} > \frac{1}{2}u_0(\frac{1}{3}) + \frac{1}{2}[(1-(\frac{2}{3})^2)u_0(\frac{1}{2}) + (\frac{2}{3})^2 u_0(\frac{3}{4})] = \frac{1}{2}u_0(\frac{1}{3}) + \frac{20}{81},$$

$D_0^t = \emptyset$ for all t s.t. $0 \in \Omega^t$, given $\bigcup_{k=1}^{t-1} D^t = \emptyset$. Thus, on the path of play, $D^t = \emptyset$ for all $t = 1, ..., \omega$. ■

Theorem 3.3

To prove this theorem, we begin with the following lemma:

Lemma 5.1 *Let $|A| > 2$ and suppose open-debate voting ($\sigma = \{1, 2, ..., z\}$). If there is full revelation in $T+1$, and the majority outcome $w_T \in \{w_{T-1}, a_T\}$ given $D^T \subseteq \bigcup_{k=1}^{T-1} D^k$ is* *certain, then there exists $i \in \{0,1\}$ such that speaker i prefers $D^T \supseteq \{1,...,n\} \backslash \bigcup_{k=1}^{T-1} D^k$ to $D^T \subseteq \bigcup_{k=1}^{T-1} D^k$, i.e., there is always at least one speaker who prefers full revelation to no revelation in T.*

Proof. Suppose that the ex ante median voter under $\bigcup_{k=1}^{T-1} D^{k,\infty}$ is not indifferent between a_j and w_{j-1}. We consider an exhaustive set of cases.

Suppose that the ex ante median voter prefers w_{T-1} to a_T. (The case where she prefers a_T is symmetric.)

1. Suppose a_T such that $\min\{w_{T-1}, a_{T+1}, ..., a_z\} < a_T < \max\{w_{T-1}, a_{T+1}, ..., a_z\}$. Let

$$\begin{aligned} x &= \max\{\pi : \pi \in \{w_{T-1}, a_T, a_{T+1}, ..., a_z\} \text{ and } \pi < a_T\}, \text{ and} \\ y &= \min\{\pi : \pi \in \{w_{T-1}, a_T, a_{T+1}, ..., a_z\} \text{ and } \pi > a_T\}. \end{aligned}$$

Let $\pi_m(A')$ be the full-information median voter's most preferred alternative in $A' \subseteq A$, and define

$$\begin{aligned} p = &\Pr(\pi_m(\{w_{T-1}, a_{T+1}, a_{T+2}, ..., a_z\}) = x | \{(r_i^{T-1}, l_i^{T-1})\}_{i \in \mathcal{R}}) \\ &- \Pr(\pi_m(\{w_{T-1}, a_T, a_{T+1}, ..., a_z\}) = x | \{(r_i^{T-1}, l_i^{T-1})\}_{i \in \mathcal{R}}) \end{aligned}$$

and

$$\begin{aligned} q = &\Pr(\pi_m(\{w_{T-1}, a_{T+1}, a_{T+2}, ..., a_z\}) = y | \{(r_i^{T-1}, l_i^{T-1})\}_{i \in \mathcal{R}}) \\ &- \Pr(\pi_m(\{w_{T-1}, a_T, a_{T+1}, ..., a_z\}) = y | \{(r_i^{T-1}, l_i^{T-1})\}_{i \in \mathcal{R}}). \end{aligned}$$

For speaker $i \in \mathcal{S}$, revelation in T is preferred iff

$$p\hat{u}_i(x) + q\hat{u}_i(y) \leq (p+q)\hat{u}_i(a_j).$$

Dividing through by $(p+q)$, we obtain

$$\frac{p}{p+q}\hat{u}_i(x) + \frac{q}{p+q}\hat{u}_i(y) \leq \hat{u}_i(a_j). \tag{2}$$

We know from the concavity of $\hat{u}_i(\pi)$ that

$$\frac{p}{p+q}\hat{u}_i(x) + \frac{q}{p+q}\hat{u}_i(y) < \hat{u}_i(\frac{p}{p+q}x + \frac{q}{p+q}y). \tag{3}$$

Define b s.t.

$$bx + (1-b)y = a_j. \tag{4}$$

There are three exhaustive possibilities:

(a) $b = \frac{p}{p+q}$. Then

$$\hat{u}_i(\frac{p}{p+q}x + \frac{q}{p+q}y) = \hat{u}_i(a_j)$$

and so from (2), it follows that for all $i \in \mathcal{S}$, i prefers full revelation in j to no revelation in j, given full revelation in $j+1$.

(b) $b < \frac{p}{p+q}$. Then $x < y$ implies

$$bx + (1-b)y > \frac{p}{p+q}x + \frac{q}{p+q}y.$$

Given that $\frac{\partial \hat{u}_1}{\partial \pi} > 0$,

$$\hat{u}_1(\frac{p}{p+q}x + \frac{q}{p+q}y) < \hat{u}_1(bx + (1-b)y).$$

Combining this inequality with (3) and (4), we have from (3.1) that $i = 1$ prefers full revelation in T to no revelation in T, given full revelation in $T+1$.

(c) $b > \frac{p}{p+q}$. Then by a symmetric argument, $i = 0$ prefers full revelation in T to no revelation in T, given full revelation in $T+1$.

2. Suppose $a_T = \max\{w_{T-1}, a_T, a_{T+1}, ..., a_z\}$. Then $1 \in \mathcal{S}$ must prefer full revelation in T to delay until $(T+1)$.

3. Suppose $a_T = \min\{w_{T-1}, a_T, a_{T+1}, ..., a_z\}$. Then $0 \in \mathcal{S}$ must prefer full revelation in

T to delay until $(T+1)$. ■

We next complete the proof of Theorem 3.3.

Proof. Sufficiency.

From Theorem 3.1, full revelation obtains in debate z. In $(z-1)$, either the median voter given $\{(r_i^{z-2}, l_i^{z-2})\}_{i\in\mathcal{R}}$ has a strict preference over $w_{z-1} = w_{z-2}$, $w_{z-1} = a_{z-1}$, in which case the majority rule outcome after $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^k$ is certain, or she is indifferent. If she is indifferent, then she can credibly commit to voting for $\hat{a} \in \{w_{z-2}, a_{z-1}\}$ if $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^k$, in which case the majority rule outcome after $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^k$ is $w_{z-1} = \hat{a}$. She has a strict preference for making such a commitment, and thereby inducing full revelation in $z-1$, if otherwise full revelation in $z-1$ were not guaranteed. Thus, the conditions of Lemma 5.1 are met, and there is at least one speaker who prefers full revelation in $z-1$ to $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^{k,\infty}$.

Case: endogenous termination with arbitrary Ω. By the above logic, the Lemma applies after any amount of revelation in debate $z-1$. Thus, there is always a speaker who prefers full revelation to the current level of revelation, and given the endogenous termination rule, such a speaker always has the opportunity to make additional arguments. Thus, full revelation is obtained in debate $z-1$. By induction, full revelation is obtained in debate $T=1$.

Case: exogenous termination. From the Lemma, either one speaker prefers full revelation in $z-1$ to $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^{k,\infty}$, or both do. If only one does, then the speakers have opposing preferences over two alternative lotteries: the lottery over the set of alternatives $\{w_{z-1}, a_{z-1}, a_z\}$ that they can obtain by choosing full revelation in $z-1$, and the lottery over $\{w_{z-1}, a_z\}$ that they can obtain by choosing $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^k$, where w_{z-1} is the majority preferred outcome given $\{(r_i^{z-2}, l_i^{z-2})\}_{i\in\mathcal{R}}$. Without loss of generality, let this be w_{z-2}. Given that the difference between these lotteries is solely the assignment of positive weight to $\{w_{z-2}, a_{z-1}, a_z\}\backslash\{w_{z-1}, a_z\}$ at the expense of the other alternatives, the speakers have opposing preferences over the entire set of lotteries corresponding to different $|D^{z-1}\backslash \bigcup_{k=1}^{z-2} D^k|$;

thus Theorem 3.1 implies full revelation in $z-1$.

Suppose instead both speakers prefer full revelation in $z-1$ to $D^{z-1} \subseteq \bigcup_{k=1}^{z-2} D^k$. Then both speakers prefer to reveal arguments until either full revelation is achieved (without changing $w_{z-1} = w_{z-2}$) or they succeed in changing w_{z-1} from w_{z-2} to a_{z-1}. If they succeed in changing w_{z-1} without obtaining full revelation, then they face a different pair of lotteries: the lottery over $\{w_{z-2}, a_{z-1}, a_z\}$ associated with full revelation in $z-1$, and the lottery over $\{a_{z-1}, a_z\}$. Again, from the lemma, at least one speaker i prefers the former. Anticipating this, that speaker insures full revelation in $z-1$ even if he must choose $D_i^{z-1,t} \supseteq \{1,...,n\} \backslash \bigcup_{k=1}^{z-2} D^k$ at $t=1$; i.e., he prefers full revelation not only to no revelation but also to any partial revelation in $z-1$. This, full revelation is obtained in $T = z-1$. By induction, full revelation is obtained in $T=1$.

Necessity.

The result is trivial for $|A| = 3$, as there can be at most two debates. If $\sigma = \{2\}$, then by definition, delay cannot be avoided. If $\sigma = \{1\}$, then Example 1 in the text establishes the result. We therefore consider $|A| = 4$, which fully illustrates the incentives at work, and then generalize. There are two cases to consider.

Case 1: $\sigma = \{1,2\}$. Let $n=2$ and $a_0 = \frac{1}{2}, a_1 = \frac{2}{3}, a_3 = \frac{1}{4}, a_4 = \frac{3}{4}$. Suppose $\forall i \in R$, $u_i(|\theta_i - w_3|) = -(\theta_i - w_3)^2$, $\theta_i \sim U[0,1]$, and $l_i^0 = r_i^0 = 0$. It is clear that uninformed median prefers $w_3 = w_2 = w_1 = \frac{1}{2}$. It is clear also that both speakers prefer $\frac{1}{2}$ to lottery $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$; thus, if $D^1 = \emptyset$, $w_1 = \frac{1}{2}$ and $D^2 = \emptyset$. It remains to show that, anticipating these responses, $D^1 = \emptyset$. Both speakers prefer $D^1 = \emptyset$ to $D^1 = \{1,2\}$. Consider then D^1 s.t. $|D^1| = 1$. With probability $\frac{1}{2}$, $w_1 = \frac{1}{2}$ and with probability $\frac{1}{2}$, $w_1 = \frac{2}{3}$. If $w_1 = \frac{2}{3}$, then 1 reveals other argument in 2. If $w_1 = \frac{1}{2}$, then 0 reveals other argument in 2. Thus, one argument being revealed in 1 means full revelation in 2.

For $i \in S$, ex ante utility from $|D^1| = 1$ and $D^1 \cup D^2 = \{1,2\}$ is

$$\begin{aligned}
& (\frac{1}{2})^3 \left(\frac{1}{2}u_i(\frac{2}{3}) + \frac{1}{2}u_i(\frac{3}{4})\right) + (\frac{1}{2})^3 \left(\frac{1}{2}u_i(\frac{1}{2}) + \frac{1}{2}u_i(\frac{3}{4})\right) \\
& + \frac{3}{8}\left(\frac{4}{9}u_i(\frac{3}{4}) + \frac{5}{9}u_i(\frac{2}{3})\right) + \frac{3}{8}\left(\frac{4}{9}u_i(\frac{1}{4}) + \frac{5}{9}u_i(\frac{1}{2})\right) \\
= \; & \frac{1}{8}\left(\frac{12}{9}u_i(\frac{1}{4}) + \frac{39}{18}u_i(\frac{1}{2}) + \frac{39}{18}u_i(\frac{2}{3}) + \frac{21}{9}u_i(\frac{3}{4})\right). \qquad (5)
\end{aligned}$$

For $i = 0$, this expected utility is $\frac{127}{216}$, which is less than $u_0(\frac{1}{2}) = \frac{8}{9}$. Thus, 0 prefers $D^1 = \emptyset$ to D^1 s.t. $|D^1| = 1$. For $i = 1$, $u_1(\frac{2}{3}) < 1$ and so, evaluating (5), ex ante utility is bounded above by $\frac{347}{432}$, which is less than $u_1(\frac{1}{2}) = \frac{8}{9}$. Thus, 1 also prefers $D^1 = \emptyset$ to D^1 s.t. $|D^1| = 1$.

Case 2: $\sigma = \{1,3\}$. Let $n = 5$; $a_0 = \frac{1}{7}, a_1 = \frac{6}{7}, a_2 = \frac{3}{7}, a_3 = \frac{4}{7}$; $\forall i \in R$, $u_i(|\theta_i - w_3|) = -(\theta_i - w_3)^2$, $\theta_i \sim U[0,1]$. Suppose $l_1^0 = 5$, $r_1^0 = 0$ and $l_3^0 = 0$, $r_3^0 = 5$ and $l_2^0 = r_2^0 = 0$. From Lemma (HERE), full revelation will occur in $T = 3$. Both speakers prefer no revelation in $T = 1$ if $w_2 = a_2$ in the absence of revelation in $T = 1$:

$$\frac{1}{2}u_1(\frac{3}{7}) + \frac{1}{2}u_i(\frac{4}{7}) > pu_1(\frac{1}{7}) + (\frac{1}{2} - p)u_i(\frac{3}{7}) + (\frac{1}{2} - p)u_i(\frac{4}{7}) + pu_i(\frac{6}{7})$$

for all $p > 0$.

It remains to show that ex ante, the median voter strictly prefers the lottery over $\{\frac{3}{7}, \frac{4}{7}\}$ to the lottery over $\{\frac{1}{7}, \frac{4}{7}\}$ and the lottery over $\{\frac{4}{7}, \frac{6}{7}\}$. This establishes that, if $D^1 = \emptyset$, the median voter prefers $w_2 = \frac{3}{7}$ to $w_2 = \frac{1}{7}$ and $w_2 = \frac{6}{7}$. Her expected utility from the lottery over $\{\frac{3}{7}, \frac{4}{7}\}$ is greater than from lottery over $\{\frac{1}{7}, \frac{4}{7}\}$ if

$$\begin{aligned}
\Pr(l_2^2 &= 5)u(\frac{2}{7}) + \Pr(l_2^2 = 4, r_2^2 = 1)u(\frac{1}{7}) \\
+\Pr(l_2^2 &= 3, r_2^2 = 2)u(0) + \Pr(l_2^2 = 2, r_2^2 = 3)u(0) \\
+\Pr(l_2^2 &= 1, r_2^2 = 4)u(\frac{1}{7}) + \Pr(l_2^2 = 0, r_2^2 = 5)u(\frac{2}{7}) \\
&> \Pr(l_2^2 = 5, r_2^2 = 0)u(0) + \Pr(l_2^2 = 4, r_2^2 = 1)u(\frac{1}{7}) \\
+\Pr(l_2^2 &= 3, r_2^2 = 2)u(\frac{1}{7}) + \Pr(l_2^2 = 2, r_2^2 = 3)u(0) \\
+\Pr(l_2^2 &= 1, r_2^2 = 4)u(\frac{1}{7}) + \Pr(l_2^2 = 0, r_2^2 = 5)u(\frac{2}{7}),
\end{aligned}$$

which simplifies to

$$\begin{aligned}
\Pr(l_2^2 &= 5, r_2^2 = 0)u(\frac{2}{7}) + \Pr(l_2^2 = 3, r_2^2 = 2)u(0) \\
&> \Pr(l_2^2 = 5, r_2^2 = 0)u(0) + \Pr(l_2^2 = 3, r_2^2 = 2)u(\frac{1}{7}).
\end{aligned}$$

From an ex ante perspective, $(l_2^2, r_2^2) = (3, 2)$ is 10 times more likely than $(l_2^2, r_2^2) = (5, 0)$; thus, the above inequality is equivalent to

$$10u(\frac{1}{7}) - u(\frac{2}{7}) < 9u(0).$$

Substituting for $u(\cdot)$, we have $-10(\frac{1}{49}) + \frac{4}{49} < 0$. Similarly, her expected utility from the lottery over $\{\frac{4}{7}, \frac{6}{7}\}$ is

$$\begin{aligned}
\Pr(l_2^2 &= 5, r_2^2 = 0)u(\frac{3}{7}) + \Pr(l_2^2 = 4, r_2^2 = 1)u(\frac{2}{7}) \\
+\Pr(l_2^2 &= 3, r_2^2 = 2)u(\frac{1}{7}) + \Pr(l_2^2 = 2, r_2^2 = 3)u(0) \\
+\Pr(l_2^2 &= 1, r_2^2 = 4)u(\frac{1}{7}) + \Pr(l_2^2 = 0, r_2^2 = 5)u(0).
\end{aligned}$$

Clearly, this is less than her expected utility from the lottery over $\{\frac{1}{7}, \frac{4}{7}\}$, which was less than her expected utility from the lottery over $\{\frac{3}{7}, \frac{4}{7}\}$.Thus, if $D^1 = \emptyset$, $w_2 = \frac{3}{7}$.

The construction of these examples can be generalized to agendas of arbitrary lengths and arbitrary number and sequence of omitted debates (relative to open-debate voting). Suppose $\sigma = \{1, ..., z\}\backslash\{y\}$, i.e., there is debate before every vote except the y^{th} vote, between w_{y-1} and a_{y-1}. Let a_{y-1} and a_{y-2} be opposite extremes relative to the median voter, and let alternatives $a_0, ..., a_{y-3}$ be clustered sufficiently closely to the position of the ex ante median, then both speakers prefer silence until debate $T = y + 1$. The same logic applies if multiple non-consecutive debates are omitted from σ. If multiple consecutive debates are omitted, starting with the debate before y, then the agenda that leads to delay can be constructed by letting a_{y-1} and a_{y-2} be opposite extreme alternatives and a_y be sufficiently centrist (close $a_0, ..., a_{y-3}$). ■

6 Appendix B

6.1 Simple Majority vs. Plurality

Although our primary focus is on the properties of debates with binary majority voting, it is instructive to consider briefly whether the properties we have identified distinguish environments with simple majority voting from those with other common voting rules. It is straightforward to see that the logic of all of the results in this paper goes through under super-majoritarian rules (more details to be added).

The following example shows that sequential-agenda majority rule can do strictly better than plurality rule.[8]

Example 3. Suppose the set of listeners/voters $\mathcal{R} = \{1, 2, 3, 4, 5, 6, 7\}$, $n = 5$ possible arguments, and $\hat{u}_i(\pi) = -(\theta_i - \pi)^2$. Let the initial information of the listeners be $(r_1, l_1) = (r_2, l_2) = (0, 4); (r_3, l_3) = (0, 1)$; $(r_4, l_4) = (3, 2)$; $(r_5, l_5) = (1, 0)$; and $(r_6, l_6) = (r_7, l_7) = (4, 0)$. The set of alternatives is $\{\frac{1}{7}, \frac{1}{2}, \frac{6}{7}\}$.

Based on the listeners' initial information, their expected ideal points are, respectively, $\frac{1}{6}; \frac{1}{6}; \frac{1}{3}; \frac{4}{7}; \frac{2}{3}; \frac{5}{6}; \frac{5}{6}$. In the absence of additional information, 1 and 2 prefer $\pi = \frac{1}{7}$; 3, 4, and

[8]Note that although this need not be the case for plurality rule generally, symmetric "strategic voting" in this example is behaviorally equivalent to the "sincere voting" (for one's first best alternative).

5 prefer $\pi = \frac{1}{2}$; and 6 and 7 prefer $\pi = \frac{6}{7}$. Thus $\pi = \frac{1}{2}$ is the ex ante plurality winner. Note that 4 is now fully informed, so her preference is fixed, and so 3 and 5 are the only movable voters. Because their positions are symmetric, the probability distributions over their possible responses to arguments are also symmetric. It follows that the probability that informative speech creates a plurality for $\frac{1}{7}$ is equal to the probability that it creates one for $\frac{6}{7}$. L's expected loss (relative to $\pi = \frac{1}{2}$) from $\pi = \frac{6}{7}$ is greater than her expected gain (relative to $\pi = \frac{1}{2}$) from $\pi = \frac{1}{7}$. Thus, given the symmetry of the distribution of possible outcomes in response to informative speech, Speaker 0 prefers that there be no informative speech, and thus, her weakly dominant strategy is not to speak. Speaker 1's position is symmetric, so no informative speech occurs in equilibrium under plurality rule.[9] This contrasts with the full revelation without delay in equilibrium under open-debate voting.

References

[1] Austen-Smith, David. 1993. "Interested Experts and Policy Advice: Multiple Referrals Under Open Rule." *Games and Economic Behavior* 5, 3-43.

[2] Austen-Smith, David, and Timothy Feddersen. 2006. "Deliberation, Preference Uncertainty, and Voting Rules." *American Political Science Review* 100 (2), 209-217.

[3] Battaglini, Marco. 2002. "Multiple Referrals and Multidimensional Cheap-talk." *Econometrica* 70 (4), 1379-1401.

[4] Crawford, Vincent and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica* 50, 1431-51.

[5] Dekel, Eddie, Barton L. Lipman, and Aldo Rustichini. 2001. "Standard State Space Models Preclude Unawareness." *Econometrica* 66 (1), 159-173.

[9] More generally, given a set of three alternatives $\{A, B, C\}$, let p be the probability that A wins and q be the probability that C wins after debate. There will be no debate under plurality rule if and only if $\forall i \in \mathcal{S}$, $u_i(B) > \frac{p}{p+q}u_i(A) + \frac{q}{p+q}u_i(C)$. It is clear that this condition is not knife-edge.

[6] Dickson, Eric S., Catherine Hafer, and Dimitri Landa. 2008. "Cognition and Strategy: a Deliberation Experiment." *Journal of Politics* 70 (4), 2008.

[7] Gerardi, Dino and Leeat Yariv. 2006. "Deliberative Voting." *Journal of Economic Theory* 134, 317-338.

[8] Glazer, Jacob and Ariel Rubinstein. 2001. "Debates and Decisions: on a Rationale for Argumentation Rules." *Games and Economic Behavior* 36, 158-73.

[9] Glazer, Jacob and Ariel Rubinstein. 2006. "On the Pragmatics of Persuasion: a Game Theoretical Approach." *Theoretical Economics* 1 (4).

[10] Hafer, Catherine and Dimitri Landa. 2007. "Deliberation as Self-Discovery and Institutions for Political Speech." *Journal of Theoretical Politics* 19 (3).

[11] Hafer, Catherine and Dimitri Landa. 2006. "Deliberation and Social Polarization." New York University Mimeo.

[12] Heifetz, Aviad, Martin Meier and Burkhard C. Schipper. 2006. "Interactive Unawareness." *Journal of Economic Theory* 130, 78-94.

[13] Krishna, Vijay and John Morgan. 2001. "A Model of Expertise." *Quarterly Journal of Economics* 116, 747-775.

[14] Lanzi, Thomas and Jerome Mathis. 2004. "Argumentation in Sender-Receiver Games." Mimeographed.

[15] Li, Jing. 2006. "Information Structures with Unawareness." Duke University Mimeo.

[16] Lipman, Barton and Duane J. Seppi. 1995. "Robust Inference in Communication Games with Partial Provability." *Journal of Economic Theory* 66 (2), 370-405.

[17] McKelvey, Richard and Richard Niemi. 1978. *Journal of Economic Theory* 18, 1-22.

[18] Meirowitz, Adam. 2001. "Designing Institutions to Aggregate Preferences and Information." *Quarterly Journal of Political Science* 1(4).

[19] Patty, John. 2009. "Arguments-Based Collective Choice." *Journal of Theoretical Politics.*

[20] Van Wheelden, Richard. 2008. "Deliberation Rules and Voting." *Quarterly Journal of Political Science* 3 (1): 83-88.